FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Thursday 13th April, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

SECTION-A

1. If the system of equations

2x + y – z = 5

2x – 5y + λz = μ

x + 2y – 5z = 7

has infinitely many solutions, then $(\lambda + \mu)^2 + (\lambda - \mu)^2$ is equal to

(1) 916

(2) 912

(3) 920

(4) 904

Official Ans. By NTA (1)

Ans. (1)

Sol.

$$\begin{vmatrix} 2 & 1 & -1 \\ 2 & -5 & \lambda \\ 1 & 2 & -5 \end{vmatrix} = 0$$

2 (25 – 2λ) – (– 10 – λ) – (4 + 5) = 0

50 – 4 λ + 10 + λ – 9 = 0

51 = 3λ ⟹ λ = 17

$$\begin{vmatrix} 2 & 1 & 5 \\ 2 & -5 & \mu \\ 1 & 2 & 7 \end{vmatrix} = 0$$

⟹ 2(– 35 – 2μ) – (14 – μ) + 5 (4 + 5) = 0

– 70 – 4μ – 14 + μ + 45 = 0

– 3μ = 39

– μ = 13

$(\lambda + \mu)^2 + (\lambda - \mu)^2 = 2(\lambda^2 + \mu^2)$

$= 2(17^2 + 13^2) = 916$

2. The coefficient of x^5 in the expansion of $\left(2x^3 - \frac{1}{3x^2}\right)^5$ is

(1) 8

(2) 9

(3) $\frac{80}{9}$

(4) $\frac{26}{3}$

Official Ans. By NTA (3)

Ans. (3)

Sol. $\left(2x^3 - \frac{1}{3x^2}\right)^5$

$$T_{r+1} = {}^5C_r\,(2x^3)^{5-r}\left(\frac{-1}{3x^2}\right)^r = {}^5C_r\,\frac{(2)^{5-r}}{(-3)^r}(x)^{15-5r}$$

∴ 15 – 5r = 5

∴ r = 2

$$T_3 = 10\left(\frac{8}{9}\right)x^5$$

So, coefficient is $\frac{80}{9}$

3. The plane, passing through the points (0, –1, 2) and (–1, 2, 1) and parallel to the line passing through (5, 1, –7) and (1, –1, –1), also passes through the point.

(1) (1, –2, 1)

(2) (0, 5, –2)

(3) (–2, 5, 0)

(4) (2, 0, 1)

Official Ans. By NTA (3)

Ans. (3)

Sol. Points (0, –1, 2) and (–1, 2, 1) parallel to the line of (5, 1, –7) and (1, –1, –1)

$$\text{Normal Vector} = \begin{vmatrix} i & j & k \\ 4 & 2 & -6 \\ -1 & 3 & -1 \end{vmatrix}$$

$\vec{n} = 16\hat{i} + 10\hat{j} + 14\hat{k}$

16x + 10y + 14z = d

Point (0, –1, 2)

0 – 10 + 28 = d ⟹ d = 18

8x + 5y + 7z = 9 is equation of plane.

4. Let α, β be the roots of the equation $x^2-\sqrt{2}x+2=0$. Then $\alpha^{14}+\beta^{14}$ is equal to

(1) $-64\sqrt{2}$

(2) $-128\sqrt{2}$

(3) – 64

(4) – 128

Official Ans. By NTA (4)

Ans. (4)

Sol. $x^2-\sqrt{2}x+2=0$

$$x=\frac{\sqrt{2}\pm\sqrt{2-8}}{2}=\frac{\sqrt{2}\pm\sqrt{6}i}{2}$$

$$\alpha=\frac{\sqrt{2}+\sqrt{6}i}{2}=\sqrt{2}e^{\frac{i\pi}{3}} \,\&\, \beta=\sqrt{2}e^{\frac{-i\pi}{3}}$$

$$\left[e^{\frac{i2\pi}{3}}\right]$$

$$\beta^{14}=128\left[e^{\frac{-i2\pi}{3}}\right]$$

$$\alpha^{14}+\beta^{14}=128(2)\cos\left(\frac{2\pi}{3}\right)=-128$$

5. Let a_1, a_2, a_3, …. be a G.P. of increasing positive numbers. Let the sum of its 6th and 8th terms be 2 and the product of its 3rd and 5th terms be $\frac{1}{9}$. Then 6 (a_2 + a_4) (a_4 + a_6) is equal to

(1) $2\sqrt{2}$

(2) 2

(3) $3\sqrt{3}$

(4) 3

Official Ans. By NTA (4)

Ans. (4)

Sol.

$ar^5 + ar^7 = 2$

$(ar^2)(ar^4)=\frac{1}{9}$

$a^2r^6=\frac{1}{9}$

Now, r > 0

$ar^5(1+r^2)=2$

Now, $ar^3=\frac{1}{3}$ or $-\frac{1}{3}$ (rejected)

$r^2 = 2$

$r=\sqrt{2}$

$a=\frac{1}{6\sqrt{2}}$

Now, 6 (a_2 + a_4) (a_4 + a_6)

$6(ar+ar^3)(ar^3+ar^5)$

$6a^2r^4(1+r^2)$

$6\left(\frac{1}{36.2}\right)(4)(9)=3$

6. Let (α, β) be the centroid of the triangle formed by the lines 15x – y = 82, 6x – 5y = – 4 and 9x + 4y = 17. Then α + 2β and 2α – β are the roots

of the equation

(1) $x^2 - 7x + 12 = 0$

(2) $x^2 - 13x + 42 = 0$

(3) $x^2 - 14x + 48 = 0$

(4) $x^2 - 10x + 25 = 0$

Official Ans. By NTA (2)

Ans. (2)

Sol. upon solving we get coordinates as (6, 8), (1, 2) and (5, –7)

So centroid : (α, β) is

$$\alpha=\frac{6+1+5}{3}=4$$

$$\beta=\frac{8+}{3}\quad 1$$

α + 2β = 6

2α – β = 7

Ans. $x^2 - 13x + 42 = 0$

7. **Let** $\left|\vec{a}\right|=2,\ \left|\vec{b}\right|=3$ e angle between the vectors $\vec{a}$ and $\vec{b}$ be $\frac{\pi}{4}$. Then $\left|\left(\vec{a}+2\vec{b}\right)\times\left(2\vec{a}-3\vec{b}\right)\right|^2$ is equal to

(1) 482

(2) 441

(3) 841

(4) 882

Official Ans. By NTA (4)

Ans. (4)

Sol. $|\vec{a}|=2, |\vec{b}|=3$

$$\left|\left(\vec{a}+2\vec{b}\right)\times(2\vec{a}-3\vec{b})\right|^2$$

$$\left|-3\vec{a}\times\vec{b}+4\vec{b}\times\vec{a}\right|^2$$

$$\left|-3\vec{a}\times\vec{b}-4\vec{a}\times\vec{b}\right|^2$$

$$\left|-7\vec{a}\times\vec{b}\right|^2$$

$$\left(-7|\vec{a}|\times|\vec{b}|\sin\left(\frac{\pi}{4}\right)\right)^2$$

$$49\times4\times9\times\frac{1}{2}=882$$

8. Let N be the foot of perpendicular from the point P (1, –2, 3) on the line passing through the points (4, 5, 8) and (1, –7, 5). Then the distance of N from the plane $2x - 2y + z + 5 = 0$ is

(1) 6

(2) 9

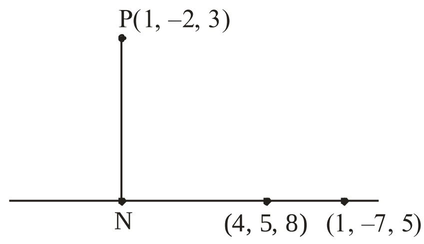
(3) 7

(4) 8

Official Ans. By NTA (3)

Ans. (3)

Sol.



Equation of line

$$\frac{x-4}{4-1}=\frac{y-5}{5-(-7)}=\frac{z-8}{8-5}$$

$$\frac{x-4}{3}=\frac{y-5}{12}=\frac{z-8}{3}$$

Let point N(3λ + 4, 12λ + 5, 3λ + 8)

$$\overrightarrow{PN}=(3\lambda+4-1)\hat{i}+(12\lambda+5-(-2))\hat{j}+(3\lambda+8-3)\hat{k}$$

$$\overrightarrow{PN}=(3\lambda+3)\hat{i}+(12\lambda+7)\hat{j}+(3\lambda+5)\hat{k}$$

And parallel vector to line (say $\vec{a}=3\hat{i}+12\hat{j}+3\hat{k}$)

Now, $\overrightarrow{PN}\cdot\vec{a}=0$

$(3\lambda + 3)3 + (12\lambda + 7)12 + (3\lambda + 5)3 = 0$

$$162\lambda + 108 = 0 \Rightarrow \lambda = \frac{-108}{162}=\frac{-2}{3}$$

So point N is (2, –3, 6)

$$\text{Now distance is} = \left|\frac{2(2)-2(-3)+6+5}{\sqrt{4+4+1}}\right| = 7$$

9. If $\lim_{x\to0}\frac{e^{ax}-\cos(bx)-\frac{cxe^{-cx}}{2}}{1-\cos(2x)}=17$, then $5a^2 + b^2$ is equal to

(1) 72

(2) 76

(3) 68

(4) 64

Official Ans. By NTA (3)

Ans. (3)

Sol. $\lim_{x\to 0}\dfrac{\dfrac{e^{ax}-\cos(bx)-\dfrac{cxe^{-cx}}{2}}{\dfrac{(1-\cos 2x)}{4x^2}\times 4x^2}}{}=17$

On expansion,

$$\lim_{x\to 0}\frac{\left(1+ax+\frac{a^2x^2}{2}\right)-\left(1-\frac{b^2x^2}{2}\right)-\frac{cx}{2}(1-cx)}{2x^2}=17$$

$$\lim_{x\to}\frac{\left(a-\frac{c}{2}\right)x+x^2\left(\frac{a^2}{2}+\frac{b^2}{2}+\frac{c^2}{2}\right)}{}=17$$

For limit to be exist $a-\dfrac{c}{2}=0$

$$a=\frac{c}{2}$$

and $\dfrac{a^2+b^2+c^2}{4}=17$

$a^2+b^2+4a^2=17\times 4$

$5a^2+b^2=68$

10. Let the centre of a circle C be (α, β) and its radius r < 8. Let 3x + 4y = 24 and 3x – 4y = 32 be two tangents and 4x + 3y =1 be a normal to C. Then (α – β + r) is equal to

(1) 7

(2) 9

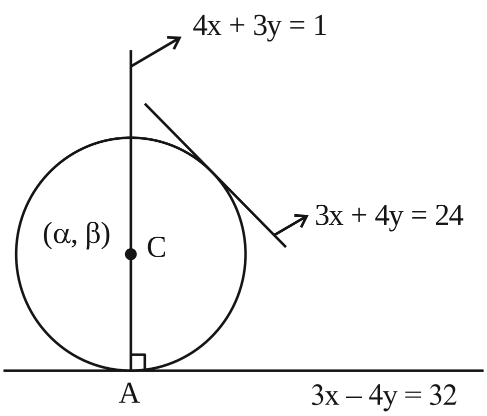
(3) 5

(4) 6

Official Ans. By NTA (1)

Ans. (1)

Sol.



First find point A by solving 4x + 3y = 1 and 3x –4y = 32

After solving, point A is (4, –5)

centre (α, β) lie on 4x + 3y = 1

$$4\alpha+3\beta=1\Rightarrow\beta=\frac{1-4\alpha}{3}$$

Now distance from centre to line 3x – 4y – 32 = 0 and 3x + 4y – 24 = 0 are equal.

$$\left|\frac{3\alpha-4\left(\frac{1-4\alpha}{3}\right)-32}{5}\right|=\left|\frac{3\alpha+4\left(\frac{1-4\alpha}{3}\right)-24}{5}\right|$$

after solving α = 1 and $\alpha=\dfrac{28}{3}$

For α = 1, centre (1, –1) ⇒ radius = 5

For $\alpha=\dfrac{28}{3}$, centre $\left(\dfrac{}{3},\dfrac{}{2}\right)$

⇒ radius ≈ 49.78 (rejected)

Hence, α = 1, β = –1, r = 5

α – β + r = 7

11. All words, with or without meaning, are made using all the letters of the word MONDAY. These words are written as in a dictionary with serial numbers. The serial number of the word MONDAY is

(1) 327

(2) 326

(3) 328

(4) 324

Official Ans. By NTA (1)

Ans. (1)

Sol. First arrange in alphabetical order

i.e. ADMNOY

A _ _ _ _ _ = 5!

D _ _ _ _ _ = 5!

M A _ _ _ _ = 4!

M D _ _ _ _ = 4!

M N _ _ _ _ = 4!

M O A _ _ _ = 3!

M O D _ _ _ = 3!

M O N A _ _ = 2!

M O N D A Y = 1

= 327

12. The range of $f(x)=4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$ is

(1) $[0, \pi]$

(2) $[0, 2\pi)$

(3) $[0, \pi)$

(4) $[0, 2\pi]$

Official Ans. By NTA (2)

Ans. (2)

Sol. $f(x) = 4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$

$$\frac{x^2}{x}=1-\frac{1}{x^2+1}\Rightarrow[0,1)$$

Range of $f(x) = [0, 2\pi)$

13. The statement $(p\wedge(\sim q)\vee((\sim p)\wedge q)\vee((\sim p)\wedge(\sim q))$ is equivalent to

(1) $(\sim p)\vee(\sim q)$

(2) $p\vee(\sim q)$

(3) $(\sim p)\vee q$

(4) $p\vee q$

Official Ans. By NTA (1)

Ans. (1)

Sol. $(p\wedge(\sim q)\vee((\sim p)\wedge q)\vee((\sim p)\wedge(\sim q))$

$(p \wedge (\sim q)) \vee ((\sim p) \wedge (q \vee (\sim q)))$

$(p \wedge (\sim q)) \vee ((\sim p) \wedge t)$

$(p \wedge (\sim q)) \vee (\sim p)$

$(\sim p) \vee (p \wedge \sim q)$

$(\sim p \vee p) \wedge (\sim p \vee \sim q)$

$t \wedge (\sim p \vee \sim q)$

$= \sim p \vee \sim q$

14. The random valuable X follows binomial distribution B (n, p) for which the difference of the mean and the variance is 1. If $2P(X = 2) = 3P(X = 1)$, then $n^2 P(X > 1)$ is equal to

(1) 12

(2) 15

(3) 11

(4) 16

Official Ans. By NTA (3)

Ans. (3)

Sol. $np - npq = 1$

$\Rightarrow np^2 = 1$

$2\,{}^nC_2p^2q^{n-2} = 3\,{}^nC_1pq^{n-1}$

$\Rightarrow np - p = 3q \qquad (\therefore q = 1 - p)$

$\Rightarrow p = \frac{1}{2}$

Hence n = 4

$P(x > 1) = 1 - (p(x = 0) + p(x = 1))$

$$= 1-\left({}^4C_0\left(\frac{1}{2}\right)^4+{}^4C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3\right)=\frac{11}{16}$$

15. Let for $A=\begin{bmatrix}1 & 2 & 3\\ a & 3 & 1\\ 1 & 1 & 2\end{bmatrix}$, |A| = 2. If |2adj (2adj (2A))| $= 32^n$, then 3n + α is equal to

(1) 10

(2) 9

(3) 12

(4) 11

Official Ans. By NTA (4)

Ans. (4)

Sol. $A=\begin{bmatrix}1 & 2 & 3\\ a & 3 & 1\\ 1 & 1 & 2\end{bmatrix}$ $|A| = 2$,

$1(6-1) - 2(2\alpha - 1) + 3(\alpha - 3) = 2$

$5 - 4\alpha + 2 + 3\alpha - 9 = 2$

$-\alpha - 4 = 0$

$\alpha = -4$

$8|\text{Adj}(2\text{Adj}(2A))|$

$8|\text{Adj}(2\times 2^2 \text{ Adj }(A))|$

$8|\text{Adj}(2^3\text{Adj}A)|$

$8|2^6\text{Adj}(\text{Adj}A)|$

$2^3(2^6)^3|\text{Adj}(\text{Adj})|$

$2^3 \cdot 2^{18} |A|^4$

$2^{21} \cdot 2^4 = 2^{25} = (2^5)^5 = (32)^5$

$n = 5$

$\alpha = -4$

16. Let $S=\{Z \in C : \bar{z} = i(z^2 + \text{Re}(\bar{z}))\}$. Then $\sum_{z\in S} |z|^2$ is equal to

(1) $\frac{7}{2}$

(2) 4

(3) $\frac{5}{2}$

(4) 3

Official Ans. By NTA (2)

Ans. (2)

Sol. Let Z = x + iy, $x \in R$, $y \in R$

$x - iy = i(x^2 - y^2 + (2xy)i + x)$

$\mathbf{x = -2xy}$ **.....(1)**

$\mathbf{-y = -y^2 + x^2 + x}$ **.....(2)**

$\Rightarrow x = 0,\ y = -\frac{1}{2}$ (from (1))

If $x \neq 0$, then y = 0, 1

If $y = -\frac{1}{2}$, then $x = \frac{1}{2}, -\frac{3}{2}$

$Z = 0 + i0,\ 0 + i,\ \frac{1}{2} - \frac{i}{2},\ -\frac{3}{2} - \frac{i}{2}$

17. The area of the region

$\{(x, y) : x^2 \leq \quad | \quad ^2 \quad |$ is

(1) $\frac{3}{4}\left(4\sqrt{2}-1\right)$

(2) $\frac{4}{3}\left(4\sqrt{2}-1\right)$

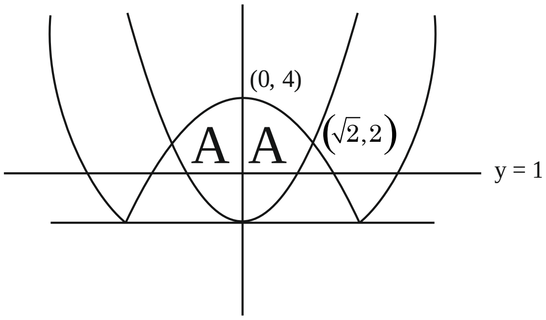
(3) $\frac{4}{3}\left(4\sqrt{2}+1\right)$

(4) $\frac{3}{4}\left(4\sqrt{2}+1\right)$

Official Ans. By NTA (2)

Ans. (2)

Sol.



Required area = $2\left[\int_1^2 \sqrt{y}\,dy + \int_2^4 \sqrt{4-y}\,dy\right] = \frac{4}{3}\left[4\sqrt{2}-1\right]$

18. Let for a triangle ABC,

$\overrightarrow{AB} = -2\hat{i} + \hat{j} + 3\hat{k}$

$\overrightarrow{CB} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

$\overrightarrow{CA} = 4\hat{i} + 3\hat{j} + \delta\hat{k}$

If $\delta > 0$ and the area of the triangle ABC is $5\sqrt{6}$, then $\overrightarrow{CB}.\overrightarrow{CA}$ is equal to

(1) 60
(2) 120
(3) 108
(4) 54

Official Ans. By NTA (1)

Ans. (1)

Sol. $\overline{AB} + \overline{BC} + \overline{CA} = \vec{0}$

$\alpha = 2, \beta = 4, \gamma - \delta = 3$

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$(\delta - 9)^2 + (2\delta + 12)^2 + 100 = 600$

$\Rightarrow \delta = 5, \gamma = 8$

Hence $\overline{CB} \cdot \overline{CA} = 60$

19. The line, that is coplanar to the line $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$, is

(1) $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z-5}{5}$

(2) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$

(3) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$

(4) $\frac{x-1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$

Official Ans. By NTA (2)

Ans. (2)

Sol. Condition of co-planarity

$$\begin{vmatrix} x_2 - x_1 & a_1 & a_2 \\ y_2 - y_1 & b_1 & b_2 \\ z_2 - z_1 & c_1 & c_2 \end{vmatrix} = 0$$

Where a_1, b_1, c_1 are direction cosine of 1st line and a_2, b_2, c_2 are direction cosine of 2nd line.

Now, solving options

Point (–3, 1, 5) & point (–1, 2, 5)

(1) $\begin{vmatrix} -3 & 1 & 5 \\ 1 & 2 & 5 \\ -2 & -1 & 0 \end{vmatrix}$

$= -3(5) - (10) + 5(-1 + 4)$

$= -15 - 10 + 15 = -10$

(2) Point (–1, 2, 5)

$$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 5 \\ -2 & -1 & 0 \end{vmatrix}$$

$= 3(5) - (10) + 5(1 + 4)$

$-25 + 25 = 0$

(3) Point (–1, 2, 5)

$$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 4 \\ -2 & -1 & 0 \end{vmatrix}$$

$-3(4) - (8) + 5(1 + 4)$

$-12 - 8 + 25 = 5$

(4) Point (–1, 2, 5)

$$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 5 \\ 4 & 1 & 0 \end{vmatrix}$$

$-3(-5) - (-20) + 5(-1 - 8)$

$15 + 20 - 45 = -10$

20. The value of $\dfrac{e^{-\frac{\pi}{4}}+\int_0^{\frac{\pi}{4}} e^{-x}\tan^{50}x\,dx}{\int_0^{\frac{\pi}{4}} e^{-x}\left(\tan^{49}x+\tan^{51}x\right)dx}$ is

(1) 50

(2) 49

(3) 51

(4) 25

Official Ans. By NTA (1)

Ans. (1)

Sol. $\int_0^{\pi/4} e^{-x}\tan^{50}x\,dx$

$$\left[-e^{-y}(\tan x)^{50}\right]^{\pi/4}+\int_0^{\pi/4} e^{-x}\,(50)\,(\tan x)^{49}\sec^2 x$$

$$=-e^{-\pi/4}+0+50\int_0^{\pi/4} e^{-x}(\tan x)^{49}(\tan^2 x+1)$$

$$=-e^{-\pi/4}+50\left(\int_0^{\pi/4} e^{-x}(\tan x)^{51}+(\tan x)^{49}\right)dx$$

Now, $\dfrac{-e^{-\pi/4}+\int_0^{\pi/4} e^{-x}(\tan x)^{50}dx}{\int_0^{\pi/4} e^{-x}(\tan^{49}x+\tan^{51}x)dx}$

$$\frac{50\int_0^{\pi/4} e^{-x}\left((\tan x)^{51}+(\tan x)^{49}\right)dx}{\int_0^{\pi/4} e^{-x}(\tan^{49}x+\tan^{51}x)dx}=50$$

SECTION-B

21. The mean and standard deviation of the marks of 10 students were found to be 50 and 12 respectively. Later, it was observed that two marks 20 and 25 were wrongly read as 45 and 50 respectively. Then the correct variance is _______.

Official Ans. by NTA (269)

Ans. (269)

Sol. $\bar{x}$ = 50

$\sum x_i$ = 500

$\sum x_{i\ correct}$ = 500+ 20 + 25 – 45 – 50 = 450

σ^2 = 144

$\dfrac{\sum x_i^2}{10}-(50)^2=144$

$\sum x_{i correct}^2=\left(144+(50)^2\right)\times 10-(45)^2-(50)^2+(20)^2+(25)^2$

= 22940

Correct variance = $\dfrac{\sum(x_{i correct})^2}{10}-\left(\dfrac{\sum x_{i correct}}{10}\right)^2$

= 2294 – $(45)^2$

= 2294 – 2025 = 269

22. Let A= {–4, –3, –2, 0, 1, 3, 4} and R = {(a, b) ∈ A × A : b = |a| or b^2 = a + 1} be a relation on A. Then the minimum number of elements, that must be added to the relation R so that it becomes reflexive and symmetric, is_____.

Official Ans. by NTA (7)

Ans. (7)

Sol. R = [(–4, 4), (–3, 3), (3, –2), (0, 1), (0, 0), (1, 1), (4, 4), (3, 3)}

For reflexive, add ⇒ (–2, –2), (–4, –4), (–3, –3)

For symmetric, add ⇒ (4, –4), (3, –3), (–2, 3), (1, 0)

23. Let $f(x) = \sum_{k=1}^{10} kx^k$, $x \in R$. If $2f(2) + f'(2) = 119(2)^n$ + 1 then n is equal to ______.

Official Ans. by NTA (10)

Ans. (10)

Sol. $f(x) = \sum_{k=1}^{10} k\, x^k$

$f(x) = x + 2x^2 + \ldots\ldots\ldots + 10\, x^{10}$

$f(x).\ x = x^2 + 2x^3 + \ldots\ldots\ldots + 9\, x^{10} + 10x^{11}$

$f(x)\,(1 - x) = x + x^2 + x^3 + \ldots\ldots\ldots + x^{10} - 10\, x^{11}$

$$f(x) = \frac{x(1-x^{10})}{(1-x)^2} - \frac{10x^{11}}{(1-x)}$$

$$f(x) = \frac{x - x^{11} - 10x^{11} + 10x^{12}}{(1-x)^2} \Rightarrow \frac{10x^{12} - 11x^{11} + x}{(1-x)^2}$$

Hence $2f(2) + f'(2) = 119.2^{10} + 1$

$\Rightarrow$ So, n = 10

24. Total numbers of 3-digit numbers that are divisible by 6 and can be formed by using the digits 1, 2, 3, 4, 5 with repetition, is____.

Official Ans. by NTA (16)

Ans. (16)

Sol. For number to be divisible by '6' unit digit should be even and sum of digit is divisible by 3.

(2, 1, 3), (2, 3, 4), (2, 5, 5), (2, 2, 5), (2, 2, 2), (4, 1, 1), (4, 4, 1), (4, 4, 4), (4, 3, 5)

2, 1, 3 $\Rightarrow$ 312, 132

2, 3, 4 $\Rightarrow$ 342, 432, 234, 324

2, 5, 5 $\Rightarrow$ 552

2, 2, 5 $\Rightarrow$ 252, 522

2, 2, 2 $\Rightarrow$ 222

4, 1, 1 $\Rightarrow$ 114

4, 4, 1 $\Rightarrow$ 414, 144

4, 4, 4 $\Rightarrow$ 444

4, 3, 5 $\Rightarrow$ 354, 534

Total 16 numbers.

25. Let $[\alpha]$ denote the greatest integer $\le \alpha$. Then $[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \ldots.. + [\sqrt{120}]$ is equal to.

Official Ans. by NTA (825)

Ans. (825)

Sol. $[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \ldots\ldots. [\sqrt{120}]$

$\Rightarrow$ 1 + 1 + 1 + 2 + 2 + 2 + 2 + 2 + 3 + 3 + ... + 3 = 7 times

+ 4 + 4 + ... + 4 = 9 times + ... 10 + 10 + ... + 10 = 21 times

$$\Rightarrow \sum_{r=1}^{10} (2r+1)\,.r$$

$$\Rightarrow 2\sum_{r=1}^{10} r^2 + \sum_{r=1}^{10} r$$

$$\Rightarrow 2 \times \frac{10 \times 11 \times 21}{6} + \frac{10 \times 11}{2}$$

$\Rightarrow$ 770 + 55

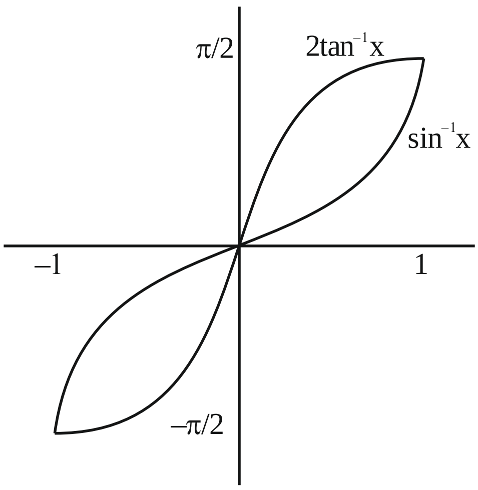
$\Rightarrow$ 825

26. For $x \in (-1, 1]$, the number of solutions of the equation $\sin^{-1}x = 2\tan^{-1}x$ is equal to

Official Ans. by NTA (2)

Ans. (2)

Sol.



27. If y = y (x) is the solution of the differential equation $\frac{dy}{dx}+\frac{4x}{(x^2-1)}y=\frac{x+2}{(x^2-1)^{\frac{5}{2}}}$, x > 1 such that $y(2)=\frac{2}{9}\log_e(2+\sqrt{3})$ and $y(\sqrt{2})$ = $\alpha\log_e(\sqrt{\alpha}+\beta)+\beta-\sqrt{\gamma},\alpha,\beta,\gamma\in N$, then αβγ is equal to ____.

Official Ans. by NTA (6)

Ans. (6)

Sol. $\frac{dy}{dx}+\frac{4x}{(x^2-1)}y=\frac{x+2}{(x^2-1)^{\frac{5}{2}}}$, $x>1$

I.F. = $e^{\int\frac{4x}{x^2-1}dx}$

I.F. = $(x^2-1)^2$

$\Rightarrow d\left(y.(x^2-1)^2\right)=\frac{x+2}{(x^2-1)^{\frac{5}{2}}}.(x^2-1)^2$

$\Rightarrow \int d\ y.(x^2-1)^2=\int\frac{x+2}{(x^2-1)^{\frac{1}{2}}}dx$ ______ (1)

$y(x^2-1)^2=\sqrt{x^2-1}+2\ \ln\left(x+\sqrt{x^2-1}\right)+C$

$\Rightarrow C=-\sqrt{3}$

So $(x^2-1)^2=\sqrt{x^2-1}+2\ln\left(x+\sqrt{x^2-1}\right)-\sqrt{3}$

$\Rightarrow\alpha\beta\gamma=6$

28. The foci of a hyperbola are (±2,0) and its eccentricity is $\frac{3}{2}$. A tangent, perpendicular to the line 2x + 3y = 6, is drawn at a point in the first quadrant on the hyperbola. If the intercepts made by the tangent on the x- and y-axes are a and b respectively, then |6a| + |5b| is equal to_____.

Official Ans. by NTA (12)

Ans. (12)

Sol. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

ae = 2 & $e=\frac{3}{2}\Rightarrow a=\frac{4}{3}$

also $b^2=a^2e^2-a^2\Rightarrow 4-\frac{16}{9}$

$\Rightarrow b^2=\frac{20}{9}$

Slope of tangent = $\frac{3}{2}$

So tangent equation will be

$y=mx\pm\sqrt{a^2m^2-b^2}$

$\Rightarrow y=\frac{3x}{2}\pm\sqrt{\frac{16}{9}.\frac{9}{4}-\frac{20}{9}}$

$\Rightarrow y=\frac{3x}{2}\pm\frac{4}{3}\Rightarrow |x_{intercept}|=\frac{8}{9}$

$|y_{intercept}|=\frac{4}{3}$

$\Rightarrow |6a|+|5b|=\frac{48}{9}+\frac{60}{9}=\frac{109}{9}=12$

29. Let $f_n=\int_0^{\frac{\pi}{2}}\left(\sum_{k=1}^{n}\sin^{k-1}x\right)\left(\sum_{k=1}^{n}(2k-1)\sin^{k-1}x\right)\cos x\ dx$, n ∈ N. Then $f_{21}-f_{20}$ is equal to____.

Official Ans. by NTA (41)

Ans. (41)

Sol.

$f_n(x)=\int_0^{\frac{\pi}{2}}(1+\sin x+\sin^2x+\sin^3x+...+\sin^{n-1}(x))$

$(1+3\sin x+5\sin^2x+...+(2n-1))\sin^{n-1}x.\cos x dx$

Multiply & divide by $\sqrt{\sin x}$

$\int_0^{\frac{\pi}{2}}\left((\sin x)^{\frac{1}{2}}+(\sin x)^{\frac{3}{2}}+(\sin x)^{\frac{5}{2}}+(\sin x)^{\frac{7}{2}}+...(\sin x)^{\frac{2n-1}{2}}\right)$

$(1+3\sin x+5\sin^2x+...+(2n-1)\sin^{n-1}(x))\frac{\cos x}{\sqrt{\sin x}}dx$

Put $(\sin x)^{1/2}+(\sin x)^{3/2}+(\sin x)^{5/2}+...+(\sin x)^{n-1/2}=t$

$\frac{1}{2}\frac{(1+3\sin x+5\sin^2x+...(2n-1)\sin^{n-1}x)}{\sqrt{\sin x}}\cos x dx=dt$

$f_n=2\int_0^n t dt$

$f_n=n^2$

$f_{21}-f_{20}=(21)^2-(20)^2$

$=441-400$

$=41$

30. The remainder, when 7^{103} is divided by 17 is ___.

Official Ans. by NTA (12)

Ans. (12)

Sol. $7^{103} = 7 \times 7^{102}$

$= 7 \times (49)^{51}$

$= 7 \times (51 - 2)^{51}$

Remainder :- $7 \times (-2)^{51}$

$\Rightarrow -7 (2^3 . (16)^{12})$

$\Rightarrow -56 (17 - 1)^{12}$

Remainder $= -56 \times (-1)^{12} = -56 + 68 = 12$