FINAL JEE–MAIN EXAMINATION – JANUARY, 2023

(Held On Tuesday 24th January, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

SECTION-A

1. Two long straight wires P and Q carrying equal current 10A each were kept parallel to each other at 5 cm distance. Magnitude of magnetic force experienced by 10 cm length of wire P is F_1. If distance between wires is halved and currents on them are doubled, force F_2 on 10 cm length of wire P will be :

(1) $8\ F_1$ (2) $10\ F_1$

(3) $F_1/8$ (4) $F_1/10$

Official Ans. by NTA (1)

Ans. (1)

Sol. Force per unit length between two parallel straight wires $= \frac{\mu_0 i_1 i_2}{2\pi d}$

$$\frac{F_1}{F_2} = \frac{\frac{\mu_0 (10)^2}{2\pi(5\text{cm})}}{\frac{\mu_0 (20)^2}{2\pi\left(\frac{5\text{cm}}{2}\right)}} = \frac{1}{8}$$

$\Rightarrow F_2 = 8F_1$

2. Given below are two statements :

Statement-I : An elevator can go up or down with uniform speed when its weight is balanced with the tension of its cable.

Statement-II : Force exerted by the floor of an elevator on the foot of a person standing on it is more than his/her weight when the elevator goes down with increasing speed.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both statement I and statement II are false

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

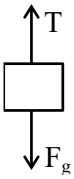
(4) Statement I is false but Statement II is true

Official Ans. by NTA (2)

Ans. (2)

Sol. **Statement-1**

When elevator is moving with uniform speed $T = F_g$



Statement-2

When elevator is going down with increasing speed, its acceleration is downward.

Hence

$$W - N = \frac{W}{g} \times a$$

$$N = W\left(1 - \frac{a}{g}\right)$$ i.e. less than weight.

3. From the photoelectric effect experiment, following observations are made. Identify which of these are correct

A. The stopping potential depends only on the work function of the metal.

B. The saturation current increases as the intensity of incident light increases.

C. The maximum kinetic energy of a photo electron depends on the intensity of the incident light.

D. Photoelectric effect can be explained using wave theory of light.

Choose the correct answer from the options given below:

(1) B, C only (2) A, C, D only

(3) B only (4) A, B, D only

Official Ans. by NTA (3)

Ans. (3)

Sol. (A) Stopping potential depends on both frequency of light and work function.

(B) Saturation current $\propto$ intensity of light

(C) Maximum KE depends on frequency

(D) Photoelectric effect is explained using particle theory

4. The weight of a body at the surface of earth is 18 N. The weight of the body at an altitude of 3200 km above the earth's surface is (given, radius of earth $R_e = 6400$ km)

(1) 9.8 N (2) 4.9 N

(3) 19.6 N (4) 8 N

Official Ans. by NTA (4)

Ans. (4)

Sol. Acceleration due to gravity at height h

$$g' = \frac{g}{\left[1 + \frac{h}{R}\right]^2}$$

So weight at given height

$$mg' = \frac{mg}{\left[1 + \frac{h}{R}\right]^2} = \frac{18}{\left[1 + \frac{1}{2}\right]^2} = 8\text{N}$$

5. A 100 m long wire having cross-sectional area 6.25×10^{-4} m^2 and Young's modulus is 10^{10} Nm^{-2} is subjected to a load of 250 N, then the elongation in the wire will be :

(1) 6.25×10^{-3} m (2) 4×10^{-4} m

(3) 6.25×10^{-6} m (4) 4×10^{-3} m

Official Ans. by NTA (4)

Ans. (4)

Sol. Elongation in wire $\delta = \dfrac{F\ell}{AY}$

$$\delta = \frac{250 \times 100}{6.25 \times 10^{-4} \times 10^{10}}$$

$$\delta = 4 \times 10^{-3}\,m$$

6. 1g of a liquid is converted to vapour at 3×10^5 Pa pressure. If 10% of the heat supplied is used for increasing the volume by 1600 cm^3 during this phase change, then the increase in internal energy in the process will be :

(1) 4320 J (2) 432000 J

(3) 4800 J (4) 4.32×10^8 J

Official Ans. by NTA (1)

Ans. (1)

Sol. Work done = $P\Delta V$

$= 3 \times 10^5 \times 1600 \times 10^{-6}$

= 480 J

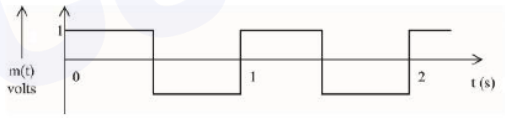
Only 10% of heat is used in work done.

Hence ΔQ = 4800 J

The rest goes in internal energy, which is 90% of heat.

Change in internal energy = 0.9 × 4800 = 4320 J

7. A modulating signal is a square wave, as shown in the figure.



If the carrier wave is given as c(t) = 2 sin (8πt) volts, the modulation index is :

(1) 1/4 (2) 1

(3) 1/3 (4) 1/2

Official Ans. by NTA (4)

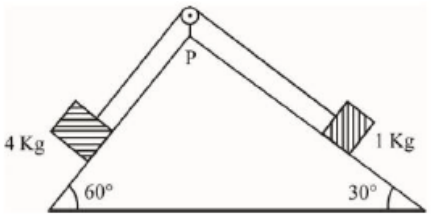
Ans. (4)

Sol. Modulation index

$$= \frac{\text{Amplitude of modulating signal}}{\text{Amplitude of carrier wave}}$$

$$\mu = \frac{1}{2}$$

8. As per given figure, a weightless pulley P is attached on a double inclined frictionless surface. The tension in the string (massless) will be (if g = 10 m/s^2)



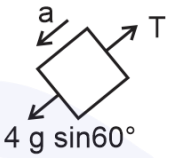
(1) $(4\sqrt{3}+1)$N (2) $4(\sqrt{3}+1)$N

(3) $4(\sqrt{3}-1)$N (4) $(4\sqrt{3}-1)$N

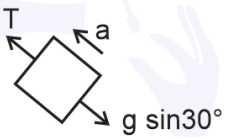
Official Ans. by NTA (2)

Ans. (2)

Sol.



$$4g\sin 60° - T = 4a \quad \ldots(1)$$



$$T - g\sin 30° = a \quad \ldots(2)$$

Solving (1) and (2) we get.

$$20\sqrt{3} - T = 4T - 20$$

$$T = 4(\sqrt{3}+1)N$$

9. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : Photodiodes are preferably operated in reverse bias condition for light intensity measurement.

Reason R : The current in the forward bias is more than the current in the reverse bias for a p – n junction diode.

In the light of the above statement, choose the correct answer from the options given below :

(1) **A** is false but **R** is true

(2) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

(3) **A** is true but **R** is false

(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

Official Ans. by NTA (2)

Ans. (2)

Sol. Photodiodes are operated in reverse bias as fractional change in current due to light is more easy to detect in reverse bias.

10. If $\vec{E}$ and $\vec{K}$ nt electric field and propagation vectors of the EM waves in vacuum, then magnetic field vector is given by : (ω - angular frequency) :

(1) $\frac{1}{\omega}(\vec{K}\times\vec{E})$ (2) $\omega(\vec{E}\times\vec{K})$

(3) $\omega(\vec{K}\times\vec{E})$ (4) $\vec{K}\times\vec{E}$

Official Ans. by NTA (1)

Ans. (1)

Sol. Magnetic field vector will be in the direction of $\hat{K}\times\hat{E}$

magnitude of $B=\frac{E}{C}=\frac{K}{\omega}E$

Or $\vec{B}=\frac{1}{\omega}(\vec{K}\times\vec{E})$

11. A circular loop of radius r is carrying current I A. The ratio of magnetic field at the centre of circular loop and at a distance r from the center of the loop on its axis is :

(1) $1:3\sqrt{2}$ (2) $3\sqrt{2}:2$

(3) $2\sqrt{2}:1$ (4) $1:\sqrt{2}$

Official Ans. by NTA (3)

Ans. (3)

Sol. Magnetic field due to current carrying circular loop on its axis is given as

$$\frac{\mu_0 i r^2}{2(r^2+x^2)^{3/2}}$$

At centre, x = 0, $B_1=\frac{\mu_0 i}{2r}$

At x = r, $B_2=\frac{\mu_0 i}{2\times 2\sqrt{2}r}$

$$\frac{B_1}{B_2}=2\sqrt{2}$$

12. A travelling wave is described by the equation
$y(x, t) = [0.05 \sin (8x - 4t] m$
The velocity of the wave is : [all the quantities are in SI unit]

(1) 4 ms^{-1} (2) 2 ms^{-1}

(3) 0.5 ms^{-1} (4) 8 ms^{-1}

Official Ans. by NTA (3)

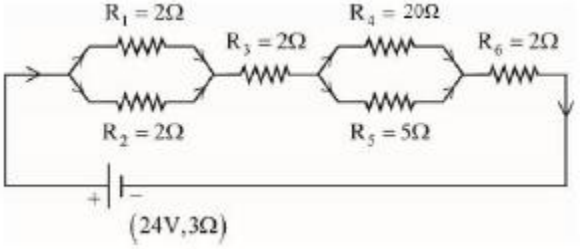
Ans. (3)

Sol. From the given equation k = 8 m^{-1} and ω = 4 rad/s

Velocity of wave = $\frac{\omega}{k}$

$$=\frac{4}{}=0.5 m/s$$

13. As shown in the figure, a network of resistors is connected to a battery of 24 V with an internal resistance of 3Ω. The currents through the resistors R_4 and R_5 are I_4 and I_5 respectively. The values of I_4 and I_5 are :



(1) $I_4=\frac{8}{5}A$ and $I_5=\frac{2}{5}A$

(2) $I_4=\frac{24}{5}A$ and $I_5=\frac{6}{5}A$

(3) $I_4=\frac{6}{5}A$ and $I_5=\frac{24}{5}A$

(4) $I_4=\frac{2}{5}A$ and $I_5=\frac{8}{5}A$

Official Ans. by NTA (4)

Ans. (4)

Sol. Equivalent resistance of circuit
$R_{eq} = 3 + 1 + 2 + 4 + 2$
$= 12\Omega$

Current through battery $i=\frac{24}{12}=2A$

$$\frac{R_5}{R_4+R_5}\times 2=\frac{5}{20+5}\times 2=\frac{2}{5}A$$

$$I_5=2-\frac{2}{5}=\frac{8}{5}A$$

14. Given below are two statements :

Statement I : If the Brewster's angle for the light propagating from air to glass is θ_B, then Brewster's angle for the light propagating from glass to air is $\frac{\pi}{2}-\theta_B$.

Statement II : The Brewster's angle for the light propagating from glass to air is $\tan^{-1}(\mu_g)$ where μ_g is the refractive index of glass.

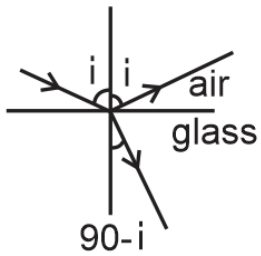
In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both Statements I and Statement II are true.
(2) Statement I is true but Statement II is false.
(3) Both Statement I and Statement II are false.
(4) Statement I is false but Statement II is true.

Official Ans. by NTA (2)

Ans. (2)

Sol.



$$\mu_a \sin i_1 = \mu_g \sin(90 - i_1)$$

$$\tan i_1 = \frac{\mu_g}{\mu_a}$$

When going from glass to air

$$\tan i_2 = \frac{\mu_a}{\mu_g} = \cot i_1$$

Hence

$$i_2 = \frac{\pi}{2} - i_1$$

15. If two charges q_1 and q_2 are separated with distance 'd' and placed in a medium of dielectric constant K. What will be the equivalent distance between charges in air for the same electrostatic force ?

(1) $d\sqrt{k}$ (2) $k\sqrt{d}$

(3) $1.5d\sqrt{k}$ (4) $2d\sqrt{k}$

Official Ans. by NTA (1)

Ans. (1)

Sol. $F = \frac{1}{(4\pi\varepsilon_0)} \frac{q_1 q_2}{kd^2}$ (in medium)

$$F_{Air} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{d'^2}$$

$F = F_{Air}$

$$\frac{q_1 q_2}{4\pi\varepsilon_0 k d^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 d'^2}$$

$$d' = d\sqrt{k}$$

16. Consider the following radioactive decay process

$$^{218}_{84}A \xrightarrow{\alpha} A_1 \xrightarrow{\beta^-} A_2 \xrightarrow{\gamma} A_3 \xrightarrow{\alpha} A_4 \xrightarrow{\beta^+} A_5 \xrightarrow{\gamma} A_6$$

The mass number and the atomic number A_6 are given by :

(1) 210 and 82 (2) 210 and 84

(3) 210 and 80 (4) 211 and 80

Official Ans. by NTA (3)

Ans. (3)

Sol. $^{218}_{84}A \xrightarrow{\alpha} {}^{214}_{82}A_1 \xrightarrow{\beta^-} {}^{214}_{83}A_2 \xrightarrow{\gamma} {}^{214}_{83}A_3$

$$^{214}_{83}A_3 \xrightarrow{\alpha} {}^{210}_{81}A_4 \xrightarrow{\beta^+} {}^{210}_{80}A_5 \xrightarrow{\gamma} {}^{210}_{80}A_6$$

17. Given below are two statements :

Statements I : The temperature of a gas is – 73°C. When the gas is heated to 527°C, the root mean square speed of the molecules is doubled.

Statement II : The product of pressure and volume of an ideal gas will be equal to translational kinetic energy of the molecules.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are false

(4) Statement I is false but Statement II is true

Official Ans. by NTA (2)

Ans. (2)

Sol. **Statement-I**

$T_1 = -73°C = 200\ K$

$T_2 = 527°C = 800\ K$

$$\frac{V_1}{V_2} = \frac{\sqrt{\frac{3RT_1}{M}}}{\sqrt{\frac{3RT_2}{M}}} \quad \sqrt{\frac{}{T_2}}$$

$$= \sqrt{\frac{200}{800}} = \frac{1}{2}$$

$V_2 = 2V_1$ (True)

Statement-II

$PV = nRT$

Translational $KE = \frac{3}{2}nRT$ (False)

18. The maximum vertical height to which a man can throw a ball is 136 m. The maximum horizontal distance upto which he can throw the same ball is

(1) 192 m (2) 136 m

(3) 272 m (4) 68 m

Official Ans. by NTA (3)

Ans. (3)

Sol. $H_{max} = \frac{v^2}{2g} = 136m$

$$R_{max} = \frac{v^2}{g} = 2H_{max}$$

$= 2(136)$

$= 272$ m

19. A conducting loop of radius $\frac{10}{\sqrt{\pi}}$ cm is placed perpendicular to a uniform magnetic field of 0.5T. The magnetic field is decreased to zero in 0.5 s at a steady rate. The induced emf in the circular loop at 0.25 s is:

(1) emf = 1 mV (2) emf = 10 mV

(3) emf = 100 mV (4) emf = 5 mV

Official Ans. by NTA (2)

Ans. (2)

Sol. $\text{EMF} = \frac{d\phi}{dt} = \frac{BA - 0}{t}$

$A = \pi r^2 = \pi\left(\frac{0.1^2}{\pi}\right) = 0.01$

$B = 0.5$

$\text{EMF} = \frac{(0.5)(0.01)}{0.5} = 0.01\text{V} = 10\text{ mV}$

20. Match List I with List II

LIST I		LIST II	
A.	Planck's constant (h)	I.	$[M^1 L^2 T^{-2}]$
B.	Stopping potential (Vs)	II.	$[M^1 L^1 T^{-1}]$
C.	Work function (Ø)	III.	$[M^1 L^2 T^{-1}]$
D.	Momentum (p)	IV.	$[M^1 L^2 T^{-3} A^{-1}]$

(1) A-III, B-I, C-II, D-IV

(2) A-III, B-IV, C-I, D-II

(3) A-II, B-IV, C-III, D-I

(4) A-I, B-III, C-IV, D-II

Official Ans. by NTA (2)

Ans. (2)

Sol. (A) Planck's constant

$h\nu = E$

$h = \frac{E}{\nu} = \frac{M^1L^2T^{-2}}{T^{-1}} = M^1L^2T^{-1}$ (III)

(B) $E = qV$

$V = \frac{E}{q} = \frac{M^1L^2T^{-2}}{A^1T^1} = M^1L^2T^{-3}A^{-1}$ (IV)

(C) ϕ (work function) = energy

$= M^1L^2T^{-2}$ (I)

(D) Momentum (p) = F.t

$= M^1L^1T^{-2}T^1$

$= M^1L^1T^{-1}$ (II)

SECTION-B

21. A spherical body of mass 2 kg starting from rest acquires a kinetic energy of 10000 J at the end of 5th second. The force acted on the body is ____ N.

Official Ans. by NTA (40)

Ans. (40)

Sol. $\frac{1}{2} \times 2 \times v^2 = 10000$

$\Rightarrow v^2 = 10000$

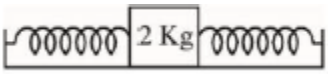
$\Rightarrow v = 100$ m/s

$\Rightarrow v = at = a \times 5 = 100$

$\Rightarrow a = 20\text{ m/s}^2$

$F = ma = 2 \times 20 = 40$ N

22. A block of mass 2 kg is attached with two identical springs of spring constant 20 N/m each. The block is placed on a frictionless surface and the ends of the springs are attached to rigid supports (see figure). When the mass is displaced from its equilibrium position, it executes a simple harmonic motion. The time period of oscillation is $\frac{\pi}{\sqrt{x}}$ in SI unit. The value of x is _____ .



Official Ans. by NTA (5)

Ans. (5)

Sol. $F = -2kx,\ a = -\frac{2kx}{m}, \omega = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2\times 20}{2}}$

$= \sqrt{20}\text{ rad/s}$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{20}} = \frac{\pi}{\sqrt{5}}$

$x = 5$

23. A hole is drilled in a metal sheet. At 27°C, the diameter of hole is 5 cm. When the sheet is heated to 177°C, the change in the diameter of hole is $d \times 10^{-3}$ cm. The value of d will be _____ if coefficient of linear expansion of the metal is 1.6×10^{-5}/°C.

Official Ans. by NTA (12)

Ans. (12)

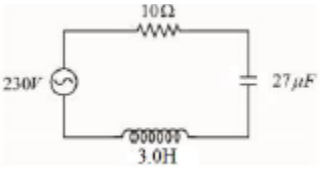
Sol. d_0 at 27°C & d_1 at 177°C

$d_1 = d_0(1 + \alpha\Delta T)$

$d_1 - d_0 = 5 \times 1.6 \times 10^{-5} \times 150$ cm

$= 12 \times 10^{-3}$ cm

24. In the circuit shown in the figure, the ratio of the quality factor and the band width is _____ s.



Official Ans. by NTA (10)

Ans. (10)

Sol. $\Delta\omega = \frac{R}{L}$

$Q = \frac{\omega_0}{\Delta\omega} = \omega_0 \frac{L}{R}$

$\omega_0 = \frac{1}{\sqrt{3\times 27\times 10^{-6}}} = \frac{1}{9\times 10^{-3}}$

$\frac{Q}{\Delta\omega} = \frac{\omega_0 \frac{L}{R}}{\frac{R}{L}} = \omega_0 \frac{L^2}{R^2} = \sqrt{\frac{1}{LC}} \frac{L^2}{R^2}$

$= \frac{1}{9\times 10^{-3}} \times \frac{9}{100} = 10\text{s}$

25. A hollow cylindrical conductor has length of 3.14 m, while its inner and outer diameters are 4 mm and 8 mm respectively. The resistance of the conductor is $n \times 10^{-3}\ \Omega$.
If the resistivity of the material is $2.4 \times 10^{-8}\ \Omega m$. The value of n is _____ .

Official Ans. by NTA (2)

Ans. (2)

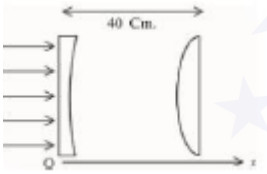
Sol. $R = \rho\frac{\ell}{A}$, the cross-sectional area is $\pi(b^2 - a^2)$

$$R = \rho\frac{\ell}{\pi(b^2 - a^2)} = \frac{2.4\times10^{-8}\times3.14}{3.14\times(4^2-2^2)\times10^{-6}}$$

$$= 2\times10^{-3}\Omega$$

$\rightarrow n = 2$

26. As shown in the figure, a combination of a thin plano concave lens and a thin plano convex lens is used to image an object placed at infinity. The radius of curvature of both the lenses is 30 cm and refraction index of the material for both the lenses is 1.75. Both the lenses are placed at distance of 40 cm from each other. Due to the combination , the image of the object is formed at distance x = _____ cm, from concave lens.



Official Ans. by NTA (120)

Ans. (120)

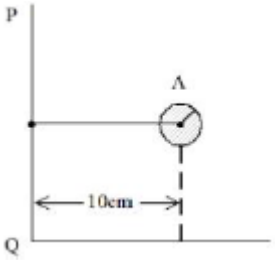
Sol. $$\frac{1}{f_1} = (1.75-1)\left(-\frac{1}{30}\right)$$

$\Rightarrow f_1 = -40$ cm

$$\frac{1}{f_2} = (1.75-1)\left(\frac{1}{30}\right) \Rightarrow f_2 = 40\text{cm}$$

Image from L_1 will be virtual and on the left of L_1 at focal length 40 cm. So the object for L_2 will be 80 cm from L_2 which is 2f. Final image is formed at 80 cm from L_2 on the right.
So x = 120

27. Solid sphere A is rotating about an axis PQ. If the radius of the sphere is 5 cm then its radius of gyration about PQ will be $\sqrt{x}$ cm. The value of x is _____ .



Official Ans. by NTA (110)

Ans. (110)

Sol. $I_{cm} = \frac{2}{5}MR^2$

$I_{PQ} = I_{cm} + md^2$

$I_{PQ} = \frac{2}{5}mR^2 + m(10\text{cm})^2$

For radius of gyration
$I_{PQ} = mk^2$

$k^2 = \frac{2}{5}R^2 + (10\text{cm})^2$

$= \frac{2}{5}(5)^2 + 100$

= 10 + 100 = 110

$k = \sqrt{110}\text{cm}$

x = 110

28. Vectors $a\hat{i} + b\hat{j} + \hat{k}$ and $2\hat{i} - 3\hat{j} + 4\hat{k}$ are perpendicular to each other when 3a + 2b = 7, the ratio of a to b is $\frac{x}{2}$. The value of x is _____ .

Official Ans. by NTA (1)

Ans. (1)

Sol. For two perpendicular vectors

$$(a\hat{i} + b\hat{j} + \hat{k})\cdot(2\hat{i} - 3\hat{j} + 4\hat{k}) = 0$$

2a – 3b + 4 = 0

On solving, 2a – 3b = –4

Also given

3a + 2b = 7

We get a = 1, b = 2

$$\frac{a}{b} = \frac{x}{2} \Rightarrow x = \frac{2a}{b} = \frac{2\times1}{2}$$

$\Rightarrow$ x = 1

29. Assume that protons and neutrons have equal masses. Mass of a nucleon is 1.6×10^{-27} kg and radius of nucleus is $1.5 \times 10^{-15}\ A^{1/3}$ m. The approximate ratio of the nuclear density and water density is $n \times 10^{13}$. The value of n is _____ .

Official Ans. by NTA (11)

Ans. (11)

Sol. density of nuclei = $\frac{\text{mass of nuclei}}{\text{volume of nuclei}}$

$$\rho = \frac{1.6\times10^{-27}A}{\frac{4}{3}\pi(1.5\times10^{-15\ 3}}$$

$$= \frac{1.6\times10^{-27}}{14.14\times10^{-45}} = 0.113\times10^{18}$$

$\rho_w = 10^3$

$$\frac{\rho}{\rho_w} = 11.31\times10^{13}$$

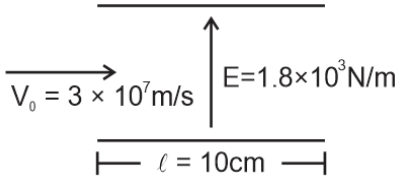
Hence

30. A stream of a positively charged particles having $\frac{q}{m} = 2\times10^{11}\,\frac{C}{kg}$ and velocity $\vec{v}_0 = 3\times10^{7}\,\hat{i}\ m/s$ is deflected by an electric field $1.8\hat{j}kV/m$. The electric field exists in a region of 10 cm along x direction. Due to the electric field, the deflection of the charge particles in the y direction is _____ mm.

Official Ans. by NTA (2)

Ans. (2)

Sol.



$$a = \frac{F}{m} = \frac{qE}{m} = (2\times10^{11})(1.8\times10^{3})$$

$= 3.6 \times 10^{14}\ m/s^2$

Time to cross plates $= \frac{d}{v}$

$$t = \frac{0.10}{3\times10^{7}}$$

$$y = \frac{1}{2}at^2 = \frac{1}{2}(3.6\times10^{14})\left(\frac{0.01}{9\times10^{14}}\right)$$

$= 0.2 \times 0.01$

$= 0.002m$

$= 2\ mm$