FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Wednesday 02nd SEPTEMBER, 2020) TIME : 3 PM to 6 PM

MATHEMATICS — TEST PAPER WITH SOLUTION

PARABOLA-XII

1. The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^2 = 8x$, with one of its vertices on the vertex of this parabola, is :

(1) $64\sqrt{3}$ (2) $256\sqrt{3}$

(3) $192\sqrt{3}$ (4) $128\sqrt{3}$

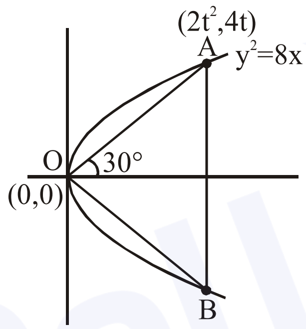
1. एक समबाहु त्रिभुज, जिसका एक शीर्ष, परवलय, $y^2 = 8x$ के शीर्ष पर है, परवलय के अंतर्गत खींचा गया है। तो त्रिभुज का क्षेत्रफल (वर्ग इकाईयों में) है ?

(1) $64\sqrt{3}$ (2) $256\sqrt{3}$

(3) $192\sqrt{3}$ (4) $128\sqrt{3}$

Official Ans. by NTA (3)

Sol.



$\tan 30° = \frac{4t}{2t^2} = \frac{2}{t} \Rightarrow t = 2\sqrt{3}$

$AB = 8t = 16\sqrt{3}$

$\text{Area} = 256.3 \cdot \frac{\sqrt{3}}{4} = 192\sqrt{3}$

P & C-XI

2. Let $n > 2$ be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red number of blue lines, then the value of n is :-

(1) 199 (2) 101

(3) 201 (4) 200

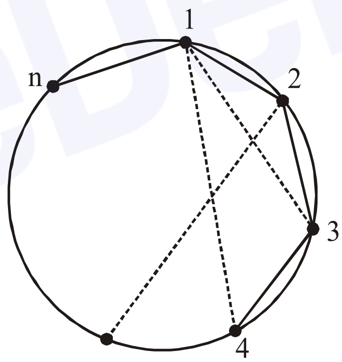
2. माना $n > 2$ एक पूर्णांक है तथा एक शहर में n मेट्रो स्टेशन है, जो एक वृत्ताकार पथ पर स्थित है। प्रत्येक दो स्टेशन एक सीधे ट्रैक (Track) से जोड़े गए है। इसके अतिरिक्त, प्रत्येक दो निकटतम स्टेशन ब्लू लाईन (Blue line) से तथा अन्य सभी दो स्टेशन रेड लाईन (Red line) से जोड़े गए है। यदि रेड लाईन्स की संख्या ब्लू लाईन्स की संख्या का 99 गुना है, तो n का मान है -

(1) 199 (2) 101

(3) 201 (4) 200

Official Ans. by NTA (3)

Sol.



Number of blue lines = Number of sides = n

Number of red lines = number of diagonals

$= {}^nC_2 - n$

${}^nC_2 - n = 99n \Rightarrow \frac{n(n-1)}{2} - n = 99n$

$\frac{n-1}{2} - 1 = 99 \Rightarrow n = 201$

T.E.-XI

3. If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$ has real

(1) $\left[-\frac{3}{2},-\frac{5}{4}\right]$ (2) $\left(-\frac{1}{2},-\frac{1}{4}\right]$

(3) $\left(-\frac{5}{4},-1\right)$ (4) $\left[-1,-\frac{1}{2}\right]$

3. यदि समीकरण $\cos^4\theta+\sin^4\theta + \lambda = 0$ के θ में वास्तविक हल है, तो λ निम्न में से किस अन्तराल में स्थित है?

(1) $\left[-\frac{3}{2},-\frac{5}{4}\right]$ (2) $\left(-\frac{1}{2},-\frac{1}{4}\right]$

(3) $\left(-\frac{5}{4},-1\right)$ (4) $\left[-1,-\frac{1}{2}\right]$

Official Ans. by NTA (4)

Sol. $\lambda = -(\sin^4\theta + \cos^4\theta)$

$\lambda = -[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta]$

$$\lambda = \frac{\sin^2 2\theta}{2} - 1$$

$$\frac{\sin^2 2\theta}{2} \in \left[0, \frac{1}{2}\right]$$

$$\lambda \in \left[-1, -\frac{1}{2}\right]$$

Q.E.-XI

4. Let f(x) be a quadratic polynomial such that f(–1) + f(2) = 0. If one of the roots of f(x) = 0 is 3, then its other root lies in :

(1) (–3, –1) (2) (1, 3)

(3) (–1, 0) (4) (0, 1)

4. माना f(x) एक द्विघात बहुपद है जिसके लिए f(–1) + f(2) = 0 है। यदि f(x) = 0 का एक मूल 3 है, तो दूसरा मूल निम्न में से किस अन्तराल में स्थित है?

(1) (–3, –1) (2) (1, 3)

(3) (–1, 0) (4) (0, 1)

Official Ans. by NTA (3)

$f(2) = a(\alpha - 2)$

$f(-1) = 4a(1 + \alpha)$

$f(-1) + f(2) = 0 \Rightarrow a(\alpha - 2 + 4 + 4\alpha) = 0$

$a \neq 0 \Rightarrow 5\alpha = -2$

$$\alpha = -\frac{2}{5} = -0.4$$

$\alpha \in (-1, 0)$

FUNCTION-XI

5. Let f : R → R be a function which satisfies $f(x + y) = f(x) + f(y) \forall x,y \in R$. If f(1) = 2 and $g(n) = \sum_{k=1}^{(n-1)} f(k), n \in N$ then the value of n, for which g(n) = 20, is :

(1) 5 (2) 9

(3) 20 (4) 4

5. माना एक फलन f : R → R प्रत्येक x, y ∈ R के लिए f(x + y) = f(x) + f(y) को संतुष्ट करता है। यदि f(1) = 2 तथा $g(n) = \sum_{k=1}^{(n-1)} f(k), n \in N$ है, तो n का वह मान जिसके लिए g(n) = 20 हैं, है –

(1) 5 (2) 9 (3) 20 (4) 4

Official Ans. by NTA (1)

Sol. $f(x + y) = f(x) + f(y)$

$\Rightarrow f(n) = nf(1)$

$f(n) = 2n$

$$g(n) = \sum_{k=1}^{n-1} 2n = 2\left(\frac{(n-1)n}{2}\right) = n(n-1)$$

$g(n) = 20 \Rightarrow n(n - 1) = 20$

$n = 5$

MATRIX-XII

6. Let a, b, c∈R be all non-zero and satisfy

3 3 3

$$A=\begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$$

satisfies $A^TA = I$, then a value of abc can be :

(1) $\frac{2}{3}$ (2) $-\frac{1}{3}$

(3) 3 (4) $\frac{1}{3}$

6. माना a, b, $c \in R$ तथा सभी अशून्य है और $a^3 + b^3 + c^3 = 2$ को संतुष्ट करते है। यदि आव्यूह

$A=\begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$ के लिए $A^TA = I$ है, तो abc का एक मान हो सकता है?

(1) $\frac{2}{3}$ (2) $-\frac{1}{3}$

(3) 3 (4) $\frac{1}{3}$

Official Ans. by NTA (4)

Sol. $A^TA = I$

$\Rightarrow a^2 + b^2 + c^2 = 1$

and $ab + bc + ca = 0$

Now, $(a + b + c)^2 = 1$

$\Rightarrow a + b + c = \pm 1$

So, $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= \pm 1 (1 - 0) = \pm 1$

$\Rightarrow 3abc = 2 \pm 1 = 3, 1$

$\Rightarrow abc = 1, \frac{1}{3}$

MONOTONICITY (A.O.D.)-XII

7. Let $f : (-1, \infty) \to R$ be defined by $f(0) = 1$ and $f(x) = \frac{1}{x}\log_e(1+x), x \neq 0$. Then the function f:

(1) decreases in $(-1, \infty)$

(2) decreases in $(-1, 0)$ and increases in $(0, \infty)$

(3) increases in $(-1, \infty)$

(4) increases in $(-1, 0)$ and decreases in $(0, \infty)$

7. यदि $f : (-1, \infty) \to R$, $f(0) = 1$ तथा $f(x) = \frac{1}{x}\log_e(1+x), x \neq 0$ द्वारा परिभाषित है, तो फलन f :

(1) $(-1, \infty)$ में ह्रासमान है।

(2) $(-1, 0)$ में ह्रासमान है तथा $(0, \infty)$ में वर्धमान है।

(3) $(-1, \infty)$ में वर्धमान है।

(4) $(-1, 0)$ में वर्धमान है तथा $(0, \infty)$ में ह्रासमान है।

Official Ans. by NTA (1)

Sol. $f'(x) = \dfrac{\frac{x}{1+x} - \ell n(1+x)}{x^2}$

$= \dfrac{x-(1+x)\ell n(1+x)}{x^2(1+x)}$

Suppose $h(x) = x - (1 + x)\ \ell n(1+x)$

$\Rightarrow h'(x) = 1 - \ell n(1+x) - 1 = -\ell n(1+x)$

$h'(x) > 0,\ \forall\ x \in (-1, 0)$

$h'(x) < 0,\ \forall\ x \in (0, \infty)$

$h(0) = 0 \Rightarrow h'(x) < 0\ \forall\ x \in (-1, \infty)$

$\Rightarrow f'(x) < 0\ \forall\ x \in (-1, \infty)$

$\Rightarrow$ f(x) is a decreasing function for all $x \in (-1, \infty)$

A.P.-XII

8. If the sum of first 11 terms of an A.P., a_1 a_2, a_3,... is 0 ($a_1 \neq 0$), then the sum of the A.P., a_1, a_3, a_5,...,a_{23} is ka_1, where k is equal to :

(1) $\frac{121}{10}$ (2) $-\frac{72}{5}$

(3) $\frac{72}{5}$ (4) $-\frac{121}{10}$

8. यदि A.P. a_1 a_2, a_3,... के प्रथम 11 पदों का योगफल 0 ($a_1 \neq 0$) है और A.P., a_1, a_3, a_5,...,a_{23} का योगफल ka_1 है, तो k बराबर है –

(1) $\frac{121}{10}$ (2) $-\frac{72}{5}$

(3) $\frac{72}{5}$ (4) $-\frac{121}{10}$

Official Ans. by NTA (2)

Sol. $a_1 + a_2 + a_3 + \ldots + a_{11} = 0$

$\Rightarrow (a_1 + a_{11}) \times \frac{11}{2} = 0$

$\Rightarrow a_1 + a_{11} = 0$

$\Rightarrow a_1 + a_1 + 10d = 0$

where d is common difference

$\Rightarrow \boxed{a_1 = -5d}$

$a_1 + a_3 + a_5 + \ldots + a_{23}$

$= (a_1 + a_{23}) \times \frac{12}{2} = (a_1 + a_1 + 22d) \times 6$

$= \left(2a_1 + 22\left(\frac{-a_1}{5}\right)\right) \times 6$

$= -\frac{72}{5}a_1 \Rightarrow K = \frac{-72}{5}$

COMPLEX NUMBER-XII

9. The imaginary part of

$\left(3+2\sqrt{-54}\right)^{1/2} - \left(3-2\sqrt{-54}\right)^{1/2}$ can be :

(1) $-2\sqrt{6}$ (2) 6

9. $\left(3+2\sqrt{-54}\right)^{1/2} - \left(3-2\sqrt{-54}\right)^{1/2}$ का काल्पनिक भाग हो सकता है –

(1) $-2\sqrt{6}$ (2) 6

(3) $\sqrt{6}$ (4) $-\sqrt{6}$

Official Ans. by NTA (1)

Sol. $\left(3+2\sqrt{-54}\right) = 3 + 2 \times 3 \times \sqrt{6}\, i$

$= \left(3+\sqrt{6}\, i\right)^2$

$\left(3-2\sqrt{54}\right) = \left(3-\sqrt{6}\, i\right)^2$

$\left(3+2\sqrt{-54}\right)^{1/2} + \left(3-2\sqrt{-54}\right)^{1/2}$

$= \pm\left(3+\sqrt{6}\, i\right) \pm \left(3-\sqrt{6}\, i\right)$

$= 6, -6, 2\sqrt{6}i, -2\sqrt{6}i,$ (four possible answer)

LIMIT-XII

10. $\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}$ is equal to :

(1) 2 (2) e

(3) 1 (4) e^2

10. $\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}$ बराबर है -

(1) 2 (2) e

(3) 1 (4) e^2

Official Ans. by NTA (4)

Sol. $\lim_{x\to 0}\left\{\tan\left(\frac{\pi}{4}+x\right)\right\}^{1/x}$

$= \lim_{x\to 0}\frac{1}{x}\left\{\tan\left(\frac{\pi}{4}+x\right)-1\right\}$

$$= e^{\lim_{x\to 0}\left(\frac{1+\tan x-1+\tan x}{x(1-\tan x)}\right)}$$

$$= e^{\lim_{x\to 0}\frac{2\tan x}{x(1-\tan x)}}$$

$= e^2$

TANGENT & NORMAL (A.O.D.)-XII

11. The equation of the normal to the curve $y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$ at $x = 0$ is :

(1) $y = 4x + 2$ (2) $x + 4y = 8$

(3) $y + 4x = 2$ (4) $2y + x = 4$

11. $x = 0$ पर, वक्र $y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$ पर खींचे गये अभिलम्ब का समीकरण है –

(1) $y = 4x + 2$ (2) $x + 4y = 8$

(3) $y + 4x = 2$ (4) $2y + x = 4$

Official Ans. by NTA (2)

Sol. Given equation of curve

$y = (1 + x)^{2y} + \cos^2(\sin^{-1}x)$

at $\boxed{x=0}$

$y = (1 + 0)^{2y} + \cos^2(\sin^{-1}0)$

$y = 1 + 1$

$\boxed{y=2}$

So we have to find the normal at (0, 2)

Now $y = e^{2y\ln(1+x)} + \cos^2\left(\cos^{-1}\sqrt{1-x^2}\right)$

$$y = e^{2y\ln(1+x)} + \left(\sqrt{1-x^2}\right)^2$$

$y = e^{2y\ln(1+x)} + (1-x^2)$ ...(1)

Now differentiate w.r.t. x

$$y' = e^{2y\ln(1+x)}\left[2y\cdot\left(\frac{1}{1+x}\right) + \ln(1+x).2y'\right] - 2x$$

Put $x = 0$ & $y = 2$

$$y' = e^{2\times 2\ln 1}\left[2\times 2\left(\frac{1}{1+0}\right) + \ln(1+0).2y'\right] - 2\times 0$$

so slope of normal to the curve $= -\frac{1}{4}$ $\{m_1m_2 = -1\}$

Hence equation of normal at (0, 2) is

$$y - 2 = -\frac{1}{4}(x-0)$$

$\Rightarrow 4y - 8 = -x$

$\Rightarrow \boxed{x+4y=8}$

HYPERBOLA-XII

12. For some $\theta \in \left(0, \frac{\pi}{2}\right)$, if the eccentricity of the hyperbola, $x^2 - y^2\sec^2\theta = 10$ is $\sqrt{5}$ times the eccentricity of the ellipse, $x^2\sec^2\theta + y^2 = 5$, then the length of the latus rectum of the ellipse, is:

(1) $\sqrt{30}$ (2) $\frac{4\sqrt{5}}{3}$

(3) $2\sqrt{6}$ (4) $\frac{2\sqrt{5}}{3}$

12. किसी $\theta \in \left(0, \frac{\pi}{2}\right)$ के लिए, यदि अतिपरवलय $x^2 - y^2\sec^2\theta = 10$ को उत्केन्द्रता, दीर्घवृत्त, $x^2\sec^2\theta + y^2 = 5$ की उत्केन्द्रता का $\sqrt{5}$ गुणा है, तो दीर्घवृत्त की नाभिलम्ब जीवा की लम्बाई बराबर है –

(1) $\sqrt{30}$ (2) $\frac{4\sqrt{5}}{3}$

(3) $2\sqrt{6}$ (4) $\frac{2\sqrt{5}}{3}$

Official Ans. by NTA (2)

Sol. Given $\theta \in \left(0, \frac{\pi}{2}\right)$

equation of hyperbola $\Rightarrow x^2 - y^2\sec^2\theta = 10$

$$\Rightarrow \frac{x^2}{10} - \frac{y^2}{10\cos^2\theta} = 1$$

Hence eccentricity of hyperbola

$$(e_H) = \sqrt{1 + \frac{10\cos^2\theta}{10}} \quad \text{...(1)}$$

Now equation of ellipse $\Rightarrow x^2\sec^2\theta + y^2 = 5$

$$\Rightarrow \frac{x^2}{5\cos^2\theta} + \frac{y^2}{5} = 1 \qquad \left\{e = \sqrt{1 - \frac{a^2}{b^2}}\right\}$$

Hence eccenticity of ellipse

$$(e_E) = \sqrt{1 - \frac{5\cos^2\theta}{5}}$$

$$(e_E) = \sqrt{1 - \cos^2\theta} = |\sin\theta| = \sin\theta \qquad \ldots(2)$$

$$\left\{\because \theta \in \left(0, \frac{\pi}{2}\right)\right\}$$

given $\Rightarrow e_H = \sqrt{5}\, e_e$

Hence $1 + \cos^2\theta = 5\sin^2\theta$

$1 + \cos^2\theta = 5(1 - \cos^2\theta)$

$1 + \cos^2\theta = 5 - 5\cos^2\theta$

$6\cos^2\theta = 4$

$\cos^2\theta = \frac{2}{3}$ $\qquad \ldots(3)$

Now length of latus rectum of ellipse

$$= \frac{2a^2}{b} = \frac{10\cos^2\theta}{\sqrt{5}} = \frac{20}{3\sqrt{5}} = \frac{4\sqrt{5}}{3}$$

EXTRA MATHEMATICAL REASONING -XII

13. Which of the following is a tautology ?

(1) $(\sim p) \wedge (p \vee q) \rightarrow q$ (2) $(q \rightarrow p) \vee \sim(p \rightarrow q)$

(3) $(p \rightarrow q) \wedge (q \rightarrow p)$ (4) $(\sim q) \vee (p \wedge q) \rightarrow q$

13. निम्न में से कौनसा कथन पुनरूक्ति है?

(1) $(\sim p) \wedge (p \vee q) \rightarrow q$ (2) $(q \rightarrow p) \vee \sim(p \rightarrow q)$

(3) $(p \rightarrow q) \wedge (q \rightarrow p)$ (4) $(\sim q) \vee (p \wedge q) \rightarrow q$

Official Ans. by NTA (1)

Sol. Option (1) is

$\sim p \wedge (p \vee q) \rightarrow q$

$\equiv (\sim p \wedge p) \vee (\sim p \wedge q) \rightarrow q$

$\equiv C \vee (\sim p \wedge q) \rightarrow q$

$\equiv (\sim p \wedge q) \rightarrow q$

$\equiv \sim(\sim p \wedge q) \vee q$

$\equiv (p \vee \sim q) \vee q$

$\equiv (p \vee q) \vee (\sim q \vee q)$

$\equiv (p \vee q) \vee t$

so $\sim p \wedge (p \vee q) \rightarrow q$ is a tautology

3D-XII

14. A plane passing through the point (3, 1,1) contains two lines whose direction ratios are 1, –2, 2 and 2, 3, –1 respectively. If this plane also passes through the point $(\alpha, -3, 5)$, then α is equal to:

(1) –10 (2) 5

(3) 10 (4) –5

14. बिन्दु (3, 1,1) से होकर जाने वाले समतल में दो सरल रेखाएं स्थित है, जिनके दिक् अनुपात (direction ratios) क्रमश: 1, –2, 2 तथा 2, 3, –1 है। यदि यह समतल बिन्दु $(\alpha, -3, 5)$ से भी होकर जाता हे, तो α बराबर है –

(1) –10 (2) 5

(3) 10 (4) –5

Official Ans. by NTA (2)

Sol. Hence normal is $\perp^r$ to both the lines so normal vector to the plane is

$$\vec{n} = (\hat{i} - 2\hat{j} + 2\hat{k}) \times (2\hat{i} + 3\hat{j} - \hat{k})$$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix} = \hat{i}(2-6) - \hat{j}(-1-4) + \hat{k}(3+4)$$

$$\vec{n} = -4\hat{i} + 5\hat{j} + 7\hat{k}$$

Now equation of plane passing through (3,1,1) is

$\Rightarrow -4(x-3) + 5(y-1) + 7(z-1) = 0$

$\Rightarrow -4x + 5y + 7z = 0$ ...(1)

Plane is also passing through (α, –3, 5) so this point satisfies the equation of plane so put in equation (1)

$-4\alpha + 5 \times (-3) + 7 \times (5) = 0$

$\Rightarrow -4\alpha - 15 + 35 = 0$

$\Rightarrow \boxed{\alpha = 5}$

PROBABILITY-XII

15. Let E^C denote the complement of an event E. Let E_1, E_2 and E_3 be any pairwise independent events with $P(E_1) > 0$ and $P(E_1 \cap E_2 \cap E_3) = 0$.

Then $P(E_2^C \cap E_3^C / E_1)$ is equal to :

(1) $P(E_3^C) - P(E_2)$ (2) $P(E_2^C) + P(E_3)$

(3) $P(E_3^C) - P(E_2^C)$ (4) $P(E_3) - P(E_2^C)$

15. माना E^C घटना E का पूरक है। यदि कोई तीन घटनाएं E_1, E_2 तथा E_3 युग्मों में स्वतंत्र है, तथा $P(E_1) > 0$ तथा $P(E_1 \cap E_2 \cap E_3) = 0$ तो $P(E_2^C \cap E_3^C / E_1)$ बराबर है –

(1) $P(E_3^C) - P(E_2)$ (2) $P(E_2^C) + P(E_3)$

(3) $P(E_3^C) - P(E_2^C)$ (4) $P(E_3) - P(E_2^C)$

Official Ans. by NTA (1)

Sol. Given E_1, E_2, E_3 are pairwise indepedent events so $P(E_1 \cap E_2) = P(E_1).P(E_2)$

and $P(E_2 \cap E_3) = P(E_2).P(E_3)$

and $P(E_3 \cap E_1) = P(E_3).P(E_1)$

& $P(E_1 \cap E_2 \cap E_3) = 0$

Now $P\left(\frac{\bar{E}_2 \cap \bar{E}_3}{E_1}\right) = \frac{P\left[E_1 \cap (\bar{E}_2 \cap \bar{E}_3)\right]}{P(E_1)}$

$= \frac{P(E_1) - [P(E_1 \cap E_2) + P(E_1 \cap E_3) - P(E_1 \cap E_2 \cap E_3)]}{P(E\ )}$

$= \frac{P(E_1) - P(E_1).P(E_2) - P(E_1)P(E_3) - 0}{P(E_1)}$

$= 1 - P(E_2) - P(E_3)$

$= [1 - P(E_3)] - P(E_2)$

$= P(E_3^C) - P(E_2)$

MATRIX-XII

16. Let $A = \{X = (x, y, z)^T : PX = 0$ and $x^2 + y^2 + z^2 = 1\}$ where $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$,

then the set A :

(1) is a singleton

(2) contains exactly two elements

(3) contains more than two elements

(4) is an empty set

16. यदि $A = \{X = (x, y, z)^T : PX = 0$ तथा $x^2 + y^2 + z^2 = 1\}$ जबकि $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$

है, तो A :

(1) एक ही अवयव वाला समुच्चय है।

(2) में मात्र दो अवयव है।

(3) में दो से अधिक अवयव है।

(4) एक रिक्त समुच्चय है।

Official Ans. by NTA (2)

Sol. Given $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & 1 \end{bmatrix}$, Here |P| = 0 & also

given PX = 0

$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$$\left.\begin{aligned} x+2y+z&=0\\ \Rightarrow -2x+3y-4z&=0\\ x+9y-z&=0 \end{aligned}\right\} \because\ D=0,\text{ so system have}$$

infinite many solutions,

By solving these equation

we get $x=\frac{-11\lambda}{2}; y=\lambda; z=\frac{7\lambda}{2}$

Also given, $x^2 + y^2 + z^2 = 1$

$$\Rightarrow \left(\frac{-11\lambda}{2}\right)^2+(\lambda)^2+\left(\frac{7\lambda}{2}\right)^2=1$$

$$\Rightarrow \lambda=\pm\frac{1}{\sqrt{\frac{121}{4}+1+\frac{49}{4}}}$$

so, there are 2 values of λ.

$\therefore$ so, there are 2 solution set of (x,y,z).

AUC-XII

17. Consider a region $R = \{(x, y) \in R^2 : x^2 \le y \le 2x\}$. If a line $y = \alpha$ divides the area of region R into two equal parts, then which of the following is true?

(1) $\alpha^3-6\alpha^2 + 16 = 0$ (2) $3\alpha^2 - 8\alpha + 8 = 0$

(3) $\alpha^3 - 6\alpha^{3/2}-16 = 0$ (4) $3\alpha^2 -8\alpha^{3/2}+8 = 0$

17. एक क्षेत्र $R = \{(x, y) \in R^2 : x^2 \le y \le 2x\}$ पर विचार कीजिए। यदि एक सरल रेखा $y = \alpha$, क्षेत्र R के क्षेत्रफल को दोबराबर भागों में बांटती है, तो निम्न में से कौनसा सत्य है?

(1) $\alpha^3-6\alpha^2 + 16 = 0$

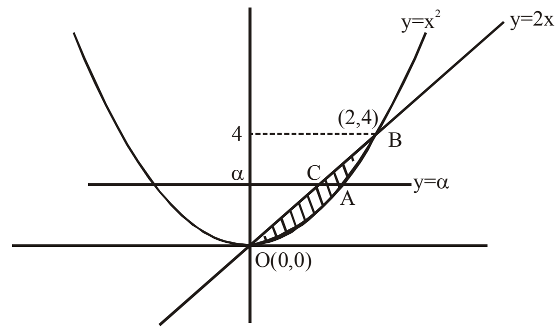
(2) $3\alpha^2 - 8\alpha + 8 = 0$

(3) $\alpha^3 - 6\alpha^{3/2} - 16 = 0$

(4) $3\alpha^2 -8\alpha^{3/2} + 8 = 0$

Official Ans. by NTA (4)

Sol.



* $y \ge x^2 \Rightarrow$ upper region of $y = x^2$

$y \le 2x \Rightarrow$ lower region of $y = 2x$

According to ques, area of OABC = 2 area of OAC

$$\Rightarrow \int_0^4\left(\sqrt{y}-\frac{y}{2}\right)dy=2\int_0^{\alpha}\left(\sqrt{y}-\frac{y}{2}\right).dy$$

$$\Rightarrow \frac{4}{3}=2\left[\frac{2}{3}\alpha^{3/2}-\frac{1}{4}.\alpha^2\right]$$

$$\Rightarrow \boxed{3\alpha^2-8\alpha^{3/2}+8=0}$$

DE-XII

18. If a curve $y = f(x)$, passing through the point (1,2), is the solution of the differential equation, $2x^2dy= (2xy + y^2)dx$, then $f\left(\frac{1}{2}\right)$ is equal to :

(1) $\frac{1}{1-\log_e 2}$ (2) $\frac{1}{1+\log_e 2}$

(3) $\frac{-1}{1+\log_e 2}$ (4) $1+ \log_e 2$

18. यदि एक वक्र, $y = f(x)$ बिन्दु (1,2) से होकर जाता है तथा अवकल समीकरण $2x^2dy= (2xy + y^2)dx$ का हल है, तो $f\left(\frac{1}{2}\right)$ बराबर है –

(1) $\frac{1}{1-\log_e 2}$ (2) $\frac{1}{1+\log_e 2}$

−1

Official Ans. by NTA (2)

Sol. $2x^2dy = (2xy + y^2)\,dx$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy + y^2}{2x^2} \quad \{\text{Homogeneous D.E.}\}$$

$$\left\{\begin{array}{l}\text{let } y = xt \\ \Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}\end{array}\right\}$$

$$\Rightarrow t + x\frac{dt}{dx} = \frac{2x^2t + x^2t^2}{2x^2}$$

$$\Rightarrow t + x\frac{dt}{dx} = t + \frac{t^2}{2}$$

$$\Rightarrow x\frac{dt}{dx} = \frac{t^2}{2}$$

$$\Rightarrow 2\int \frac{dt}{t^2} = \int \frac{dx}{x}$$

$$\Rightarrow 2\left(-\frac{1}{t}\right) = \ell n(x) + C \quad \left\{\text{Put } t = \frac{y}{x}\right\}$$

$$\Rightarrow -\frac{2x}{y} = \ell n x + C \quad \left\{\begin{array}{l}\text{Put } x = 1 \,\&\, y = 2 \\ \text{then we get } C = -1\end{array}\right\}$$

$$\Rightarrow \frac{-2x}{y} = \ell n(x) - 1$$

$$\Rightarrow y = \frac{2x}{1 - \ell n x}$$

$$\Rightarrow f(x) = \frac{2x}{1 - \log_e x}$$

so, $\boxed{f\left(\frac{1}{2}\right) = \frac{1}{1 + \log_e 2}}$

S & S-XII

19. Let S be the sum of the first 9 terms of the series: $\{x + ka\} + \{x^2 + (k + 2)a\} + \{x^3+(k+4)a\}+\{x^4+(k + 6)a\}+.....$ where $a \neq 0$ and $x \neq 1$. If $S = \frac{x^{10} - x + 45a(x-1)}{x-1}$, then k is equal to :

(1) –5 (2) 1

(3) –3 (4) 3

19. माना श्रेणी $\{x + ka\} + \{x^2 + (k + 2)a\} + \{x^3+(k+4)a\}+\{x^4+(k + 6)a\}+.....$ के प्रथम 9 पदों का योगफल S के बराबर है, जबकि $a \neq 0$ तथा $x \neq 1$ है। यदि $S = \frac{x^{10} - x + 45a(x-1)}{x-1}$ है, तो k बराबर है -

(1) –5 (2) 1

(3) –3 (4) 3

Official Ans. by NTA (3)

Sol. $S = [x + ka + 0] + [x^2 + ka + 2a] + [x^3 + ka + 4a] + [x^4 + ka + 6a] + ...9$ terms

$\Rightarrow S = (x + x^2 + x^3 + x^4+.....9 \text{ terms}) + (ka + ka + ka + ka +.......9 \text{ terms}) + (0 + 2a + 4a + 6a + ...9 \text{ terms})$

$$\Rightarrow S = x\left[\frac{x^9 - 1}{x - 1}\right] + 9ka + 72a$$

$$\Rightarrow S = \frac{(x^{10} - x) + (9k + 72)a(x-1)}{(x-1)}$$

Compare with given sum, then we get, $(9k + 72) = 45$

$\Rightarrow \boxed{k = -3}$

STRAIGHT LINE-XII

20. The set of all possible values of θ in the interval $(0, \pi)$ for which the points $(1, 2)$ and $(\sin\theta, \cos\theta)$ lie on the same side of the line $x + y = 1$ is :

(1) $\left(0, \frac{\pi}{4}\right)$ (2) $\left(0, \frac{3\pi}{4}\right)$

(3) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (4) $\left(0, \frac{\pi}{2}\right)$

20. (0, π) θ

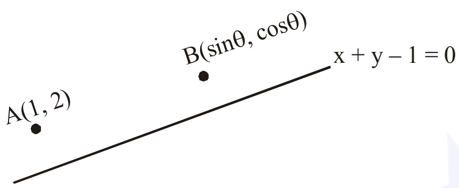
जिसके लिए दोनों बिन्दु (1, 2) तथा (sin θ, cosθ) सरल रेखा x + y = 1 के एक ही तरफ स्थित है, है –

(1) $\left(0, \frac{\pi}{4}\right)$ (2) $\left(0, \frac{3\pi}{4}\right)$

(3) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (4) $\left(0, \frac{\pi}{2}\right)$

Official Ans. by NTA (4)

Sol. Given that both points (1, 2) & (sinθ, cosθ) lie on same side of the line x + y – 1 = 0



So, $\begin{pmatrix}\text{Put (1,2) in} \\ \text{given line}\end{pmatrix}\begin{pmatrix}\text{Put } (\sin\theta, \cos\theta \text{ in} \\ \text{given line}\end{pmatrix} > 0$

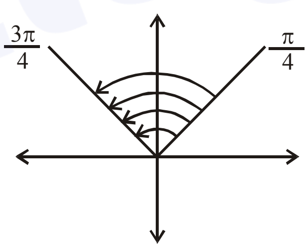
$\Rightarrow (1 + 2 - 1)(\sin\theta + \cos\theta - 1) > 0$

$\Rightarrow \sin\theta + \cos\theta > 1 \;\{\div \text{by}\sqrt{2}\}$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta > \frac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{\pi}{4} < \theta + \frac{\pi}{4} < \frac{3\pi}{4}$



$\Rightarrow \boxed{0 < \theta < \frac{\pi}{2}}$

STATISTICS-XII

21. If the variance of the terms in an increasing A.P., $b_1, b_2, b_3, \ldots b_{11}$ is 90, then the common difference of this A.P. is_______.

21. $b_1, b_2, b_3, \ldots b_{11}$ A.P.

Official Ans. by NTA (3.00)

Sol. Let a be the first term and d be the common difference of the given A.P. Where d > 0

$$\bar{X} = a + \frac{0 + d + 2d + \ldots + 10d}{11}$$

$$= a + 5d$$

$$\Rightarrow \text{varience} = \frac{\Sigma(\bar{X} - x_i)^2}{11}$$

$\Rightarrow 90 \times 11 = (25d^2 + 16d^2 + 9d^2 + 4d^2) \times 2$

$\Rightarrow d = \pm 3 \Rightarrow d = 3$

MOD-XII

22. If $y = \sum_{k=1}^{6} k\cos^{-1}\left\{\frac{3}{5}\cos kx - \frac{4}{5}\sin kx\right\}$,

then $\frac{dy}{dx}$ at x = 0 is________.

22. यदि $y = \sum_{k=1}^{6} k\cos^{-1}\left\{\frac{3}{5}\cos kx - \frac{4}{5}\sin kx\right\}$,

तो x = 0 पर $\frac{dy}{dx}$ का मान है –

Official Ans. by NTA (91)

Sol. Put $\cos\alpha = \frac{3}{5}, \sin\alpha = \frac{4}{5} \quad 0 < \alpha < \frac{\pi}{2}$

Now $\frac{3}{5}\cos kx - \frac{4}{5}\sin kx$

= cos α . cos kx – sin α . sin kx

= cos(α + kx)

As we have to find derivate at x = 0

We have $\cos^{-1}(\cos(\alpha + kx))$

= (α + kx)

$$\Rightarrow y = \sum_{k=1}^{6}(\alpha + kx)$$

$$\left.\frac{dy}{dx}\right| = \sum^{6} k = \frac{6 \times 7 \times 13}{} = 91$$

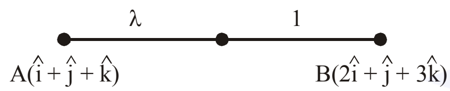
VECTOR-XII

23. Let the position vectors of points 'A' and 'B' be $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+3\hat{k}$, respectively. A point 'P' divides the line segment AB internally in the ratio $\lambda : 1$ ($\lambda > 0$). If O is the origin and $\overrightarrow{OB}\cdot\overrightarrow{OP}-3|\overrightarrow{OA}\times\overrightarrow{OP}|^2=6$, then λ is equal to______.

23. माना बिन्दुओं 'A' तथा 'B' के स्थिति सदिश क्रमशः $\hat{i}+\hat{j}+\hat{k}$ तथा $2\hat{i}+\hat{j}+3\hat{k}$ है। एक बिन्दु P, रेखाखण्ड AB को अन्तः अनुपात $\lambda : 1$ ($\lambda > 0$) में विभाजित करता है। यदि O मूल बिन्दु है तथा $\overrightarrow{OB}\cdot\overrightarrow{OP}-3|\overrightarrow{OA}\times\overrightarrow{OP}|^2=6$ है, तो λ बराबर है –

Official Ans. by NTA (0.8)

Sol.



Using section formula we get

$$\overline{OP}=\frac{2\lambda+1}{\lambda+1}\hat{i}+\frac{\lambda+1}{\lambda+1}\hat{j}+\frac{3\lambda+1}{\lambda+1}\hat{k}$$

Now $\overline{OB}\cdot\overline{OP}=\dfrac{4\lambda+2+\lambda+1+9\lambda+3}{\lambda+1}$

$$=\frac{14\lambda+6}{\lambda+1}$$

$$\overline{OA}\times\overline{OP}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ \dfrac{2\lambda+1}{\lambda+1} & 1 & \dfrac{3\lambda+1}{\lambda+1}\end{vmatrix}$$

$$=\frac{2\lambda+1}{\lambda+1}\hat{i}+\frac{-\lambda}{\lambda+1}\hat{j}+\frac{-\lambda}{\lambda+1}\hat{k}$$

$$|\overline{OA}\times\overline{OP}|^2=\frac{(2\lambda+1)^2+\lambda^2+\lambda^2}{(\lambda+1)^2}$$

$$=\frac{6\lambda^2+1}{(\lambda+1)^2}$$

$$\Rightarrow \frac{14\lambda+6}{\lambda+1}-3\times\frac{(6\lambda^2+1)}{(\lambda+1)^2}=6$$

$\Rightarrow 10\lambda^2 - 8\lambda = 0$

$\Rightarrow \lambda = 0,\ \dfrac{8}{10}=0.8$

$\Rightarrow \lambda = 0.8$

BINOMIAL THEOREM-XI

24. For a positive integer n, $\left(1+\frac{1}{x}\right)^n$ is expanded in increasing powers of x. If three consecutive coefficients in this expansion are in the ratio, 2 : 5 : 12, then n is equal to________.

24. एक घन पूर्णांक n के लिए, $\left(1+\frac{1}{x}\right)^n$ को x की बढ़ती घातों में प्रसारित किया गया है। यदि इस प्रसार में तीन क्रमागत गुणांकों का अनुपात, 2 : 5 : 12 है, तो n बराबर है –

Official Ans. by NTA (118)

Sol. ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 2:5:12$

Now $\dfrac{{}^nC_{r-1}}{{}^nC_r}=\dfrac{2}{5}$

$\Rightarrow 7r = 2n + 2$ …(1)

$\dfrac{{}^nC_r}{{}^nC_{r+1}}=\dfrac{5}{12}$

$\Rightarrow 17r = 5n - 12$ …(2)

On solving (1) & (2)

$\Rightarrow n = 118$

D.I.-XII

25. Let [t] denote the greatest integer less than or equal to t. Then the value of $\int_1^2 |2x-[3x]|\,dx$ is_______.

25. यदि [t] महत्तम पूर्णांक $\le t$ है, तो $\int_1^2 |2x-[3x]|\,dx$ का मान बराबर है –

Official Ans. by NTA (1.0)

Sol. $3 < 3x < 6$

Take cases when $3 < 3x < 4$, $4 < 3x < 5$, $5 < 3x < 6$;

Now $\displaystyle\int_1^2 |2x-[3x]|\,dx$

$$=\int_1^{4/3}(3-2x)dx+\int_{4/3}^{5/3}(4-2x)\,dx+\int_{5/3}^{2}(5-2x)dx$$

$$=\frac{2}{9}+\frac{3}{9}+\frac{4}{9}=1$$