CollegeDekho

FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Wednesday 02nd SEPTEMBER, 2020) TIME : 9 AM to 12 PM

MATHEMATICS — TEST PAPER WITH SOLUTION

1. If $|x| < 1, |y| < 1$ and $x \neq y$, then the sum to infinity of the following series

$(x+y) + (x^2+xy+y^2) + (x^3+x^2y + xy^2+y^3)+.....$

(1) $\frac{x+y-xy}{(1-x)(1-y)}$ (2) $\frac{x+y-xy}{(1+x)(1+y)}$

(3) $\frac{x+y+xy}{(1+x)(1+y)}$ (4) $\frac{x+y+xy}{(1-x)(1-y)}$

Official Ans. by NTA (1)

Sol. $|x| < 1, |y| < 1, x \neq y$

$(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + ...$

By multiplying and dividing $x - y$:

$$\frac{(x^2-y^2)+(x^3-y^3)+(x^4-y^4)+......}{x-y}$$

$$=\frac{(x^2+x^3+x^4+......)-(y^2+y^3+y^4+......)}{x-y}$$

$$=\frac{\frac{x^2}{1-x}-\frac{y^2}{1-y}}{x-y}$$

$$=\frac{(x^2-y^2)-xy(x-y)}{(1-x)(1-y)(x-y)}$$

$$=\boxed{\frac{x+y-xy}{(1-x)(1-y)}}$$

2. Let $\alpha > 0, \beta > 0$ be such that $\alpha^3 + \beta^2 = 4$. If the maximum value of the term independent of x in the binomial expansion of $\left(\alpha x^{\frac{1}{9}}+\beta x^{-\frac{1}{6}}\right)^{10}$ is 10k, then k is equal to :

(1) 176 (2) 336

(3) 352 (4) 84

Official Ans. by NTA (2)

Sol. Let t_{r+1} denotes

th $\left(\alpha x^{\frac{1}{9}}+\beta x^{-\frac{1}{6}}\right)^{10}$

$$t_{r+1} = {}^{10}C_r \alpha^{10-r}(x)^{\frac{10-r}{9}}.\beta^r x^{-\frac{r}{6}}$$

$$= {}^{10}C_r\, \alpha^{10-r}\, \beta^r\, (x)^{\frac{10-r}{9}-\frac{r}{6}}$$

If t_{r+1} is independent of x

$\frac{10-r}{9}-\frac{r}{6}=0 \Rightarrow r = 4$

maximum value of t_5 is 10 K (given)

$\Rightarrow {}^{10}C_4\, \alpha^6\, \beta^4$ is maximum

By AM $\geq$ GM (for positive numbers)

$$\frac{\frac{\alpha^3}{2}+\frac{\alpha^3}{2}+\frac{\beta^2}{2}+\frac{\beta^2}{2}}{4} \geq \left(\frac{\alpha^6\beta^4}{16}\right)^{\frac{1}{4}}$$

$\Rightarrow \boxed{\alpha^6\beta^4 \leq 16}$

So, $10\text{ K} = {}^{10}C_4 16$

$\Rightarrow$ K = 336

3. If a function f(x) defined by

$$f(x)=\begin{cases} ae^x+be^{-x}, & -1\leq x<1 \\ cx^2 , & 1\leq x\leq 3 \\ ax^2+2cx , & 3<x\leq 4 \end{cases}$$

be continuous for some a, b, c $\in$ R and $f'(0) + f'(2) = e$, then the value of of a is :

(1) $\frac{e}{e^2-3e-13}$ (2) $\frac{e}{e^2+3e+13}$

(3) $\frac{1}{e^2-3e+13}$ (4) $\frac{e}{e^2-3e+13}$

Official Ans. by NTA (4)

Sol. $$f(x)=\begin{cases} ae^x+be^{-x}, & -1\leq x<1 \\ cx^2, & 1\leq x\leq 3 \end{cases}$$

For continuity at x = 1

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

$\Rightarrow$ $\boxed{ae + be^{-1} = c}$ $\Rightarrow$ $\boxed{b = ce - ae^2}$ ...(1)

For continuity at x = 3

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$$

$\Rightarrow$ 9c = 9a + 6c

$\Rightarrow$ c = 3a ...(2)

$f'(0) + f'(2) = e$

$(ae^x - be^x)_{x=0} + (2cx)_{x=2} = e$

$\Rightarrow$ $\boxed{a - b + 4c = e}$ ...(3)

From (1), (2) & (3)

$a - 3ae + ae^2 + 12a = e$

$\Rightarrow a(e^2 + 13 - 3e) = e$

$$\Rightarrow a = \frac{e}{e^2 - 3e + 13}$$

4. Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is :

(1) $\frac{8}{17}$ (2) $\frac{2}{3}$

(3) $\frac{4}{17}$ (4) $\frac{2}{5}$

Official Ans. by NTA (1)

Sol. Let B_1 be the event where Box–I is selected.

& $B_2 \rightarrow$ where box-II selected

$$P(B_1) = P(B_2) = \frac{1}{2}$$

Let E be the event where selected card is non prime.

For B_1 : Prime numbers :

{2, 3, 5, 7, 11, 13, 17, 19, 23, 29}

For B_2 : Prime numbers :

$$P(E) = P(B_1) \times P\left(\frac{E}{B_1}\right) + P(B_2)P\left(\frac{E}{B_2}\right)$$

$$= \frac{1}{2} \times \frac{20}{30} + \frac{1}{2} \times \frac{15}{20}$$

Required probability :

$$P\left(\frac{B_1}{E}\right) = \frac{\frac{1}{2} \times \frac{20}{30}}{\frac{1}{2} \times \frac{20}{30} + \frac{1}{2} \times \frac{15}{20}} = \frac{\frac{2}{3}}{\frac{2}{3} + \frac{3}{4}} = \frac{8}{17}$$

5. Area (in sq. units) of the region outside $\frac{|x|}{2} + \frac{|y|}{3} = 1$ and inside the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is :

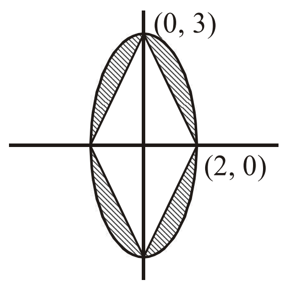
(1) $3(4 - \pi)$ (2) $6(\pi - 2)$

(3) $3(\pi - 2)$ (4) $6(4 - \pi)$

Official Ans. by NTA (2)

Sol. $\frac{|x|}{2} + \frac{|y|}{3} = 1$

$\frac{x^2}{4} + \frac{y^2}{9} = 1$



Area of Ellipse = $\pi ab = 6\pi$

Required area,

$= \pi \times 2 \times 3$ – (Area of quadrilateral)

$$= 6\pi - \frac{1}{2} 6 \times 4$$

$= 6\pi - 12$

6. Let S be the set of all $\lambda \in R$ for which the system of linear equations

$2x - y + 2z = 2$

$x - 2y + \lambda z = -4$

$x + \lambda y + z = 4$

has no solution. Then the set S

(1) contains more than two elements.

(2) is a singleton.

(3) contains exactly two elements.

(4) is an empty set.

Official Ans. by NTA (3)

Sol. $2x - y + 2z = 2$

$x - 2y + \lambda z = -4$

$x + \lambda y + z = 4$

For no solution :

$$D = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & \lambda \\ 1 & \lambda & 1 \end{vmatrix} = 0$$

$\Rightarrow 2(-2 - \lambda^2) + 1(1 - \lambda) + 2(\lambda + 2) = 0$

$\Rightarrow -2\lambda^2 + \lambda + 1 = 0$

$\Rightarrow \lambda = 1, -\frac{1}{2}$

$$D_x = \begin{vmatrix} 2 & -1 & 2 \\ -4 & 2 & \lambda \\ 4 & \lambda & 1 \end{vmatrix} = 2\begin{vmatrix} 1 & -1 & 2 \\ -2 & -2 & \lambda \\ \lambda & \lambda & 1 \end{vmatrix}$$

$= 2(1 + \lambda)$

whichis not equal to zero for

$\lambda = 1, -\frac{1}{2}$

7. Let A be a 2×2 real matrix with entries from $\{0, 1\}$ and $|A| \neq 0$. Consider the following two statements :

(P) If $A \neq I_2$, then $|A| = -1$

(Q) If $|A| = 1$, then $tr(A) = 2$,

where I_2 denotes 2×2 identity matrix and tr(A) denotes the sum of the diagonal entries of A. Then:

(1) (P) is true and (Q) is false

(2) Both (P) and (Q) are false

(3) Both (P) and (Q) are true

(4) (P) is false and (Q) is true

Official Ans. by NTA (4)

Sol. $|A| \neq 0$

For (P) : $A \neq I_2$

$$\text{So, } A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \text{ or } \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\text{or } \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$|A|$ can be -1 or 1

So (P) is false.

For (Q); $|A| = 1$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$\Rightarrow tr(A) = 2$

$\Rightarrow$ Q is true

8. The contrapositive of the statement "If I reach the station in time, then I will catch the train" is :

(1) If I will catch the train, then I reach the station in time.

(2) If I do not reach the station in time, then I will not catch the train.

(3) If I will not catch the train, then I do not reach the station in time.

(4) If I do not reach the station in time, then I will catch the train.

Official Ans. by NTA (3)

Sol. Let p denotes statement

q : I will catch the train.

Contrapositive of $p \rightarrow q$

is $\sim q \rightarrow \sim p$

$\sim q \rightarrow \sim p$: I will not catch the train, then I do not reach the station in time.

9. Let y = y(x) be the solution of the differential equation,

$\frac{2+\sin x}{y+1} \cdot \frac{dy}{dx} = -\cos x, y > 0, y(0) = 1$. If $y(\pi) = a$ and $\frac{dy}{dx}$ at $x = \pi$ is b, then the ordered pair (a, b) is equal to :

(1) (2, 1) (2) $\left(2, \frac{3}{2}\right)$

(3) (1, –1) (4) (1, 1)

Official Ans. by NTA (4)

Sol. $\frac{2+\sin x}{y+1}\frac{dy}{dx} = -\cos x,\ y > 0$

$$\Rightarrow \frac{dy}{y+1} = \frac{-\cos x}{2+\sin x}dx$$

By integrating both sides :

$\ell n|y+1| = -\ell n|2+\sin x| + \ell nK$

$$\Rightarrow y + 1 = \frac{K}{2+\sin x} \quad (y + 1 > 0)$$

$$\Rightarrow y(x) = \frac{K}{2+\sin x} - 1$$

Given y(0) = 1 $\Rightarrow$ K = 4

$$\text{So, } y(x) = \frac{4}{2+\sin x} - 1$$

$a = y(\pi) = 1$

$$b = \left.\frac{dy}{dx}\right]_{x=\pi} = \left.\frac{-\cos x}{2+\sin x}(y(x)+1)\right]_{x=\pi} = 1$$

So, (a, b) = (1, 1)

10. Let $X = \{x \in N : 1 \le x \le 17\}$ and $Y = \{ax + b: x \in X \text{ and } a, b \in R, a > 0\}$. If mean and variance of elements of Y are 17 and 216 respectively then a + b is equal to :

(1) –7 (2) 7

(3) 9 (4) –27

Official Ans. by NTA (1)

Sol. σ^2 = variance

μ = mean

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}$$

$\mu = 17$

$$\Rightarrow \frac{\sum_{x=1}^{17}(ax+b)}{17} = 17$$

$\Rightarrow$ 9a + b = 17 ....(1)

$\sigma^2 = 216$

$$\Rightarrow \frac{\sum_{x=1}^{17}(ax+b-17)^2}{17} = 216$$

$$\Rightarrow \frac{\sum_{x=1}^{17}a^2(x-9)^2}{17} = 216$$

$\Rightarrow a^2 81 - 18 \times 9a^2 + a^2\, 3 \times (35) = 216$

$\Rightarrow a^2 = \frac{216}{24} = 9 \Rightarrow a = 3\ (a > 0)$

$\Rightarrow$ From (1), b = –10

So, a + b = –7

11. If the tangent to the curve y = x + sin y at a point (a, b) is parallel to the line joining $\left(0, \frac{3}{2}\right)$ and $\left(\frac{1}{2}, 2\right)$, then :

(1) b = a (2) $b = \frac{\pi}{2} + a$

(3) |b – a| = 1 (4) |a+b| = 1

Official Ans. by NTA (3)

Sol. Slope of tangent to the curve $y = x + \sin y$

at (a, b) is $\frac{2 - \frac{3}{2}}{\frac{1}{2} - 0} = 1$

$\Rightarrow \left.\frac{dy}{dx}\right]_{x=a} = 1$

$\frac{dy}{dx} = 1 + \cos y . \frac{dy}{dx}$ (from equation of curve)

$\left.\frac{dy}{dx}\right]_{x=a} = 1 + \cos b . \left.\frac{dy}{dx}\right]_{x=a}$

$\Rightarrow$ cos b = 0

$\Rightarrow$ sin b = ±1

Now, from curve y = x + sin y

b = a + sin b

$\Rightarrow$ |b – a| = |sin b| = 1

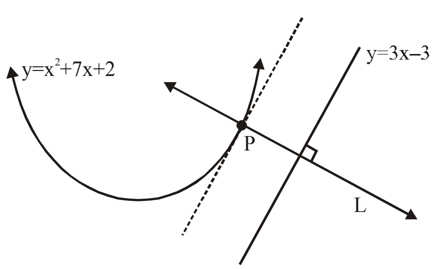
12. Let P(h, k) be a point on the curve y = x^2 + 7x + 2, nearest to the line, y = 3x – 3. Then the equation of the normal to the curve at P is :

(1) x + 3y – 62 = 0 (2) x – 3y – 11 = 0

(3) x – 3y + 22 = 0 (4) x + 3y + 26 = 0

Official Ans. by NTA (4)

Sol.



Let L be the common normal to parabola y = x^2 + 7x + 2 and line y = 3x – 3

$\Rightarrow$ slope of tangent of y = x^2 + 7x + 2 at P = 3

$\Rightarrow \left.\frac{dy}{dx}\right]_{\text{For P}} = 3$

$\Rightarrow$ 2x + 7 = 3 $\Rightarrow$ x = –2 $\Rightarrow$ y = –8

So P(–2, –8)

Normal at P : x + 3y + C = 0

$\Rightarrow$ C = 26 (P satisfies the line)

$\boxed{\text{Normal} : x + 3y + 26 = 0}$

13. The plane passing through the points (1, 2, 1), (2, 1, 2) and parallel to the line, 2x = 3y, z = 1 also passes through the point :

(1) (0, 6, –2) (2) (–2, 0, 1)

(3) (0, –6, 2) (4) (2, 0, –1)

Official Ans. by NTA (2)

Sol. Two points on the line (L say) $\frac{x}{3} = \frac{y}{2}, z = 1$ are (0, 0, 1) & (3, 2, 1)

So dr's of the line is < 3, 2, 0 >

Line passing through (1, 2, 1), parallel to L and coplanar with given plane is

$\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + t(3\hat{i} + 2\hat{j}), t \in R$ (–2, 0, 1) satisfies the line (for t = –1)

$\Rightarrow$ (–2, 0, 1) lies on given plane.

Answer of the question is (2)

We can check other options by finding eqution of plane

Equation plane : $\begin{vmatrix} x-1 & y-2 & z-1 \\ 1+2 & 2-0 & 1-1 \\ 2+2 & 1-0 & 2-1 \end{vmatrix} = 0$

$\Rightarrow$ 2(x – 1) –3(y – 2) –5(z–1) = 0

CollegeDekho

14. Let α and β be the roots of the equation $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$, $n = 1,2,3....$, then :

(1) $5S_6 + 6S_5 = 2S_4$

(2) $5S_6 + 6S_5 + 2S_4 = 0$

(3) $6S_6 + 5S_5 + 2S_4 = 0$

(4) $6S_6 + 5S_5 = 2S_4$

Official Ans. by NTA (1)

Sol. α and β are roots of $5x^2 + 6x - 2 = 0$

$\Rightarrow 5\alpha^2 + 6\alpha - 2 = 0$

$\Rightarrow 5\alpha^{n+2} + 6\alpha^{n+1} - 2\alpha^n = 0$ ...(1)

(By multiplying α^n)

Similarly $5\beta^{n+2} + 6\beta^{n+1} - 2\beta^n = 0$ ...(2)

By adding (1) & (2)

$5S_{n+2} + 6S_{n+1} - 2S_n = 0$

For $n = 4$

$$\boxed{5S_6 + 6S_5 = 2S_4}$$

15. If $R = \{(x,y) : x,y \in Z, x^2 + 3y^2 \le 8\}$ is a relation on the set of integers Z, then the domain of R^{-1} is :

(1) $\{-2, -1, 1, 2\}$

(2) $\{-1, 0, 1\}$

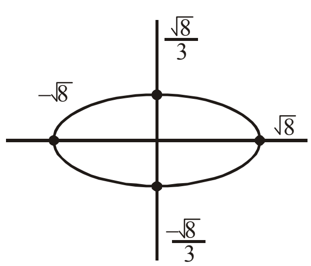
(3) $\{-2, -1, 0, 1, 2\}$

(4) $\{0, 1\}$

Official Ans. by NTA (2)

Sol. $R = \{(x, y) : x, y \in z, x^2 + 3y^2 \le 8\}$

For domain of R^{-1}



Collection of all integral of y's

For $x = 0$, $3y^2 \le 8$

16. The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in :

(1) $[-3, \infty)$

(2) $(-\infty, 9]$

(3) $(-\infty, -9] \cup [3, \infty)$

(4) $(-\infty, -3] \cup [9, \infty)$

Official Ans. by NTA (4)

Sol. Let three terms of G.P. are $\frac{a}{r}$, a, ar

product = 27

$\Rightarrow a^3 = 27 \Rightarrow a = 3$

$$S = \frac{3}{r} + 3r + 3$$

For $r > 0$

$$\frac{\frac{3}{r} + 3r}{2} \ge \sqrt{3^2} \quad \text{(By AM} \ge \text{GM)}$$

$$\Rightarrow \frac{3}{r} + 3r \ge 6 \quad ...(1)$$

For $r < 0$ $\frac{3}{r} + 3r \le -6$ ...(2)

From (1) & (2)

$S \in (-\infty - 3] \cup [9, \infty]$

17. A line parallel to the straight line $2x - y = 0$ is tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point (x_1, y_1). Then $x_1^2 + 5y_1^2$ is equal to :

(1) 5

(2) 6

(3) 8

(4) 10

Official Ans. by NTA (2)

Sol. Slope of tangent is 2, Tangent of hyperbola

$\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point (x_1, y_1) is

$$\frac{xx_1}{4} - \frac{yy_1}{2} = 1 \quad (T = 0)$$

$$\text{Slope} : \frac{1}{2}\frac{x_1}{y_1} = 2 \Rightarrow \boxed{x_1 = 4y_1} \quad ...(1)$$

CollegeDekho

$\Rightarrow \boxed{\frac{x_1^2}{4} - \frac{y_1^2}{2} = 1}$ ...(2)

From (1) & (2)

$\frac{(4y_1)^2}{4} - \frac{y_1^2}{2} = 1 \Rightarrow 4y_1^2 - \frac{y_1^2}{2} = 1$

$\Rightarrow 7y_1^2 = 2 \Rightarrow \boxed{y_1^2 = 2/7}$

Now $x_1^2 + 5y_1^2 = (4y_1)^2 + 5y_1^2$

$= (21)y_1^2 = 21 \times \frac{2}{7} = 6$

18. The domain of the function $f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is $(-\infty, -a] \cup [a, \infty)$. Then a is equal to :

(1) $\frac{1+\sqrt{17}}{2}$ (2) $\frac{\sqrt{17}-1}{2}$

(3) $\frac{\sqrt{17}}{2}+1$ (4) $\frac{\sqrt{17}}{2}$

Official Ans. by NTA (1)

Sol. $f(x) = \sin\left(\frac{|x|+5}{x^2+1}\right)$

For domain :

$-1 \le \frac{|x|+5}{x^2+1} \le 1$

Since $|x| + 5$ & $x^2 + 1$ is always positive

So $\frac{|x|+5}{x^2+1} \ge 0 \ \forall x \in R$

So for domain :

$\frac{|x|+5}{x^2+1} \le 1$

$\Rightarrow |x| + 5 \le x^2 + 1$

$\Rightarrow 0 \le x^2 - |x| - 4$

$\Big(\quad \frac{1+\sqrt{17}}{} \Big)\Big(\quad \frac{1-\sqrt{17}}{} \Big)$

$\Rightarrow |x| \ge \frac{1+\sqrt{17}}{2}$ or $|x| \le \frac{1-\sqrt{17}}{2}$ (Rejected)

$\Rightarrow x \in \left(-\infty, -\frac{1+\sqrt{17}}{2}\right] \cup \left[\frac{1+\sqrt{17}}{2}, \infty\right)$

So, $a = \frac{1+\sqrt{17}}{2}$

19. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is :

(1) $\frac{1}{2}\left(\sqrt{3}-i\right)$ (2) $-\frac{1}{2}\left(\sqrt{3}-i\right)$

(3) $-\frac{1}{2}\left(1-i\sqrt{3}\right)$ (4) $\frac{1}{2}\left(1-i\sqrt{3}\right)$

Official Ans. by NTA (2)

Sol. The value of $\left(\frac{1+\sin 2\pi/9+i\cos 2\pi/9}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)$

$$= \left(\frac{1+\sin\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)+i\cos\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)}{1+\sin\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)-i\cos\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)}\right)^3$$

$$= \left(\frac{1+\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}}{1+\cos\frac{5\pi}{18}-i\sin\frac{5\pi}{18}}\right)^3$$

$$= \left(\frac{2\cos^2\frac{5\pi}{36}+2i\sin\frac{5\pi}{36}\cos\frac{5\pi}{36}}{{}_2\,5\pi \quad 5\pi \quad 5\pi}\right)^3$$

$$= \left(\frac{\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}}{\cos\frac{5\pi}{36}-i\sin\frac{5\pi}{36}}\right)^3$$

$$= \left(\frac{e^{i5\pi/36}}{e^{-i5\pi/36}}\right)^3 = \left(e^{i5\pi/18}\right)^3$$

$$= \cos\frac{5\pi}{6}+i\sin 5\pi/6$$

$$= -\frac{\sqrt{3}}{2}+i/2$$

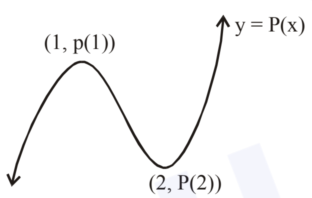
20. If p(x) be a polynomial of degree three that has a local maximum value 8 at x = 1 and a local minimum value 4 at x = 2; then p(0) is equal to:

(1) 12 (2) –24

(3) 6 (4) –12

Official Ans. by NTA (4)

Sol.



Since p(x) has realtive extreme at

x = 1 & 2

so p'(x) = 0 at x = 1 & 2

$\Rightarrow$ p'(x) = A(x – 1) (x – 2)

$\Rightarrow$ p(x) = $\int A(x^2-3x+2)dx$

$$p(x) = A\left(\frac{x^3}{3}-\frac{3x^2}{2}+2x\right)+C \quad \ldots(1)$$

P(1) = 8

From (1)

$$8 = A\left(\frac{1}{3}-\frac{3}{2}+2\right)+C$$

$$\Rightarrow 8 = \frac{5A}{6}+C \Rightarrow \boxed{48 = 5A+6C} \quad \ldots(3)$$

P(2) = 4

$$\Rightarrow 4 = A\left(\frac{8}{3}-6+4\right)+C$$

$$\Rightarrow 4 = \frac{2A}{3}+C \Rightarrow \boxed{12 = 2A+3C} \quad \ldots(4)$$

From 3 & 4, C = –12

So P(0) = C = $\boxed{-12}$

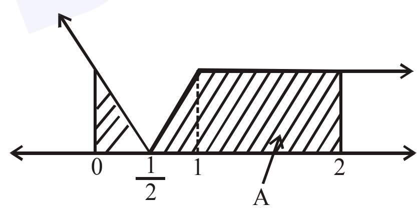
21. The integral $\int_0^2 ||x-1|-x|\,dx$ is equal to______.

Official Ans. by NTA (1.50)

Sol. $\int_0^2 |x-1|-x|\,dx$

Let f(x) ||x – 1|–x|

$$= \begin{cases} 1, & x \geq 1 \\ |1-2x|, & x \leq 1 \end{cases}$$



$$A = \frac{1}{2}+1 = \frac{3}{2}$$

or

$$\int_0^{1/2}(1-2x)dx + \int_{1/2}^{1}(2x-1) + \int_0^2 1dx$$

$$= \left[x-x^2\right]_0^{\frac{1}{2}} + \left[x^2-x\right]_{1/2}^{1} + \left[x\right]_1^2$$

$$= \boxed{3/2}$$

22. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that

$$|\vec{a}-\vec{b}|^2 + |\vec{a}-\vec{c}|^2 = 8.$$

Then $|\vec{a}+2\vec{b}|^2 + |\vec{a}+2\vec{c}|^2$ is equal to ________.

Official Ans. by NTA (2.00)

Sol. $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$$\left|\vec{a}-\vec{b}\right|^2 + \left|\vec{a}-\vec{b}\right|^2 = 8$$

$$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b} + |\vec{a}|^2 + |\vec{c}|^2 - 2\vec{a}.\vec{c} = 8$$

$$\Rightarrow \quad 4 - 2(\vec{a}.\vec{b} + \vec{a}.\vec{c}) = 8$$

$$\Rightarrow \quad \boxed{\vec{a}.\vec{b} + \vec{a}.\vec{c} = -2}$$

$$|\vec{a}+2\vec{b}|^2 + |\vec{a}+2\vec{c}|^2$$

$$= |a^2| + 4|\vec{b}|^2 + 4\vec{a}.\vec{b} + |\vec{a}|^2 + 4|\vec{c}|^2 + 4\vec{a}.\vec{c}$$

$$= 10 + 4(\vec{a}.\vec{b} + \vec{a}.\vec{c})$$

$$= 10 - 8$$

$$= \boxed{2}$$

23. If $\lim_{x\to 1} \frac{x + x^2 + x^3 + ... + x^n - n}{x-1} = 820, (n \in N)$ then the value of n is equal to________.

Official Ans. by NTA (40.00)

Sol. $\lim_{x\to 1} \frac{x + x^2 + ... + x^2 - n}{x-1} = 820$

$$\Rightarrow \quad \lim_{x\to 1}\left(\frac{x-1}{x-1} + \frac{x^2-1}{x-1} + ... \frac{x^n-1}{x-1}\right) = 820$$

$$\Rightarrow \quad 1 + 2 + ... + n = 820$$

$$\Rightarrow \quad n(n+1) = 2 \times 820$$

$$\Rightarrow \quad n(n+1) = 40 \times 41$$

24. If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is ______.

Official Ans. by NTA (309.00)

Sol. MOTHER

$1 \rightarrow$ E

$2 \rightarrow$ H

$3 \rightarrow$ M

$4 \rightarrow$ O

$5 \rightarrow$ R

$6 \rightarrow$ T

So position of word MOTHER in dictionary

$$2 \times 5! + 2 \times 4! + 3 \times 3! + 2! + 1$$

$$= 240 + 48 + 18 + 2 + 1$$

$$= \boxed{309}$$

25. The number of integral values of k for which the line, $3x + 4y = k$ intersects the circle, $x^2 + y^2 - 2x - 4y + 4 = 0$ at two distinct points is ______.

Official Ans. by NTA (9.00)

Sol. Circle $x^2 + y^2 - 2x - 4y + 4 = 0$

$\Rightarrow \quad (x-1)^2 + (y-2)^2 = 1$

Centre : (1, 2) radius = 1

line $3x + 4y - k = 0$ intersects the circle at two distinct points.

$\Rightarrow$ distance of centre from the line < radius

$$\Rightarrow \quad \left|\frac{3\times 1 + 4\times 2 - k}{\sqrt{3^2 + 4^2}}\right| < 1$$

$\Rightarrow \quad |11 - k| < 5$

$\Rightarrow \quad 6 < k < 16$

$\Rightarrow \quad k \in \{7, 8, 9, ... 15\}$ since $k \in I$

CollegeDekho