FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Thursday 03rd SEPTEMBER, 2020) TIME : 3 PM to 6 PM

MATHEMATICS — TEST PAPER WITH SOLUTION

1. If the surface area of a cube is increasing at a rate of 3.6 cm^2/sec, retaining its shape; then the rate of change of its volume (in cm^3/sec), when the length of a side of the cube is 10 cm, is :

(1) 9 (2) 18

(3) 10 (4) 20

Official Ans. by NTA (1)

Sol. $\frac{d}{dt}(6a^2) = 3.6 \Rightarrow 12a\frac{da}{dt} = 3.6$

$a\frac{da}{dt} = 0.3$

$\frac{dv}{dt} = \frac{d}{dt}(a^3) = 3a\left(a\frac{da}{dt}\right)$

$= 3 \times 10 \times 0.3 = 9$

2. If the value of the integral $\int_0^{1/2}\frac{x^2}{(1-x^2)^{3/2}}dx$ is $\frac{k}{6}$, then k is equal to :

(1) $2\sqrt{3}-\pi$ (2) $3\sqrt{2}+\pi$

(3) $3\sqrt{2}-\pi$ (4) $2\sqrt{3}+\pi$

Official Ans. by NTA (1)

Sol. $\int_0^{1/2}\frac{((x^2-1)+1)}{(1-x^2)^{3/2}}dx$

$\int_0^{1/2}\frac{dx}{(1-x^2)^{3/2}} - \int_0^{1/2}\frac{dx}{\sqrt{1-x^2}}$

$\int_0^{1/2}\frac{x^{-3}}{(x^{-2}-1)^{3/2}}dx - (\sin^{-1}x)_0^{1/2}$

Let $x^{-2} - 1 = t^2 \Rightarrow x^{-3}\,dx = -\,t\,dt$

$\int_{\infty}^{\sqrt{3}}\frac{-t\,dt}{t^3} - \frac{\pi}{6} = \int_{\sqrt{3}}^{\infty}\frac{dt}{t^2} - \frac{\pi}{6} = \frac{1}{\sqrt{3}} - \frac{\pi}{6} = \frac{k}{6}$

$\sqrt{}$

3. Let R_1 and R_2 be two relations defined as follows :

$R_1 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$ and

$R_2 = \{(a, b) \in R^2 : a^2 + b^2 \notin Q\}$,

where Q is the set of all rational numbers. Then:

(1) R_2 is transitive but R_1 is not transitive

(2) R_1 is transitive but R_2 is not transitive

(3) R_1 and R_2 are both transitive

(4) Neither R_1 nor R_2 is transitive

Official Ans. by NTA (4)

Sol. Let $a^2 + b^2 \in Q$ & $b^2 + c^2 \in Q$

eg. $a = 2 + \sqrt{3}$ & $b = 2 - \sqrt{3}$

$a^2 + b^2 = 14 \in Q$

Let $c = \left(1+2\sqrt{3}\right)$

$b^2 + c^2 = 20 \in Q$

But $a^2 + c^2 = \left(2+\sqrt{3}\right)^2 + \left(1+2\sqrt{3}\right)^2 \notin Q$

for R_2 Let $a^2 = 1$, $b^2 = \sqrt{3}$ & $c^2 = 2$

$a^2 + b^2 \notin Q$ & $b^2 + c^2 \notin Q$

But $a^2 + c^2 \in Q$

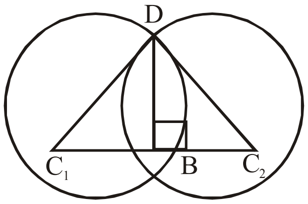
4. Let the latus ractum of the parabola $y^2 = 4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is:

(1) 8 (2) $4\sqrt{5}$

(3) 12 (4) $8\sqrt{5}$

Official Ans. by NTA (1)

Sol. Length of latus rectum = 4



DB = 2

$C_1B = \sqrt{(C_1D)^2 - (DB)^2} = 4$

$C_1C_2 = 8$

5. If $\int \sin^{-1}\left(\sqrt{\frac{x}{1+x}}\right)dx = A(x)\tan^{-1}(\sqrt{x}) + B(x) + C$, where C is a constant of integration, then the ordered pair (A(x), B(x)) can be :

(1) $(x-1, \sqrt{x})$ (2) $(x+1, \sqrt{x})$

(3) $(x+1, -\sqrt{x})$ (4) $(x-1, -\sqrt{x})$

Official Ans. by NTA (3)

Sol. Put $x = \tan^2\theta \Rightarrow dx = 2\tan\theta\sec^2\theta\, d\theta$

$\int \theta.(2\tan\theta\cdot\sec^2\theta)d\theta$

$\downarrow \quad \downarrow$

I II (By parts)

$= \theta.\tan^2\theta - \int \tan^2\theta\, d\theta$

$= \theta.\tan^2\theta - \int (\sec^2\theta - 1)\, d\theta$

$= \theta(1 + \tan^2\theta) - \tan\theta + C$

$= \tan^{-1}(\sqrt{x})(1+x) - \sqrt{x} + C$

6. The probability that a randomly chosen 5-digit number is made from exactly two digits is :

(1) $\frac{121}{10^4}$ (2) $\frac{150}{10^4}$

(3) $\frac{135}{10^4}$ (4) $\frac{134}{10^4}$

Sol. First Case: Choose two non-zero digits ${}^9C_2$

Now, number of 5-digit numbers containing both digits = $2^5 - 2$

Second Case: Choose one non-zero & one zero as digit ${}^9C_1$.

Number of 5-digit numbers containg one non zero and one zero both = $(2^4 - 1)$

Required prob.

$$= \frac{\left({}^9C_2 \times (2^5 - 2) + {}^9C_1 \times (2^4 - 1)\right)}{9 \times 10^4}$$

$$= \frac{36 \times (32-2) + 9 \times (16-1)}{9 \times 10^4}$$

$$= \frac{4 \times 30 + 15}{10^4} = \frac{135}{10^4}$$

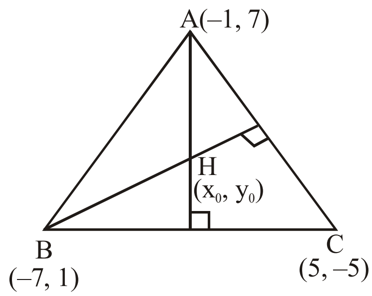
7. If a ΔABC has vertices A(–1, 7), B(–7, 1) and C(5, –5), then its orthocentre has coordinates:

(1) (3, –3) (2) $\left(-\frac{3}{5}, \frac{3}{5}\right)$

(3) (–3, 3) (4) $\left(\frac{3}{5}, -\frac{3}{5}\right)$

Official Ans. by NTA (3)

Sol. Let orthocentre is $H(x_0, y_0)$



$m_{AH}.m_{BC} = -1$

$$\Rightarrow \left(\frac{y_0 - 7}{x_0 + 1}\right)\left(\frac{1+5}{-7-5}\right) = -1$$

$$\Rightarrow 2x_0 - y_0 + 9 = 0 \quad .... (1)$$

$$\Rightarrow \left(\frac{y_0 - 1}{x_0 + 7}\right)\left(\frac{7-(-5)}{-1-5}\right) = -1$$

$$\Rightarrow x_0 - 2y_0 + 9 = 0 \;\;\ldots\ldots\; (2)$$

Solving equation (1) and (2) we get

$$(x_0, y_0) \equiv (-3, 3)$$

8. If z_1, z_2 are complex numbers such that $\text{Re}(z_1) = |z_1 - 1|$, $\text{Re}(z_2) = |z_2 - 1|$ and $\arg(z_1 - z_2) = \frac{\pi}{6}$, then $\text{Im}(z_1 + z_2)$ is equal to:

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{2}{\sqrt{3}}$

(3) $\frac{1}{\sqrt{3}}$ (4) $2\sqrt{3}$

Official Ans. by NTA (4)

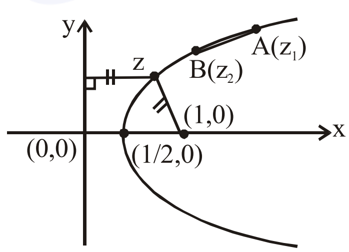
Sol. $\text{Re}(z) = |z - 1|$

$$\Rightarrow x = \sqrt{(x-1)^2 + (y-0)^2} \qquad (x > 0)$$

$$\Rightarrow y^2 = 2x - 1 = 4.\frac{1}{2}\left(x - \frac{1}{2}\right)$$

$\Rightarrow$ a parabola with focus (1, 0) & directrix as imaginary axis.

$$\therefore \text{Vertex} = \left(\frac{1}{2}, 0\right)$$



$A(z_1)$ & $B(z_2)$ are two points on it such that

slope of AB = $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$\left(\arg(z_1 - z_2) = \frac{\pi}{6}\right)$

for $y^2 = 4ax$

Let $A(at_1^2, 2at_1)$ & $B(at_2^2, 2at_2)$

$$m_{AB} = \frac{2}{t_1 + t_2} = \frac{4a}{y_1 + y_2} = \frac{1}{\sqrt{3}}$$

$$\left(\text{Here } a = \frac{1}{2}\right)$$

$$\Rightarrow y_1 + y_2 = 4a\sqrt{3} = 2\sqrt{3}$$

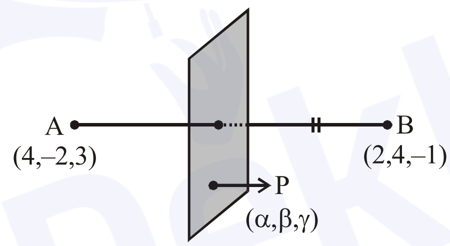
9. The plane which bisects the line joining the points (4, –2, 3) and (2, 4, –1) at right angles also passes through the point :

(1) (4, 0, –1) (2) (4, 0, 1)

(3) (0, 1, –1) (4) (0, –1, 1)

Official Ans. by NTA (1)

Sol.



$$PA = PB$$

$$\Rightarrow PA^2 = PB^2$$

$$\Rightarrow (\alpha - 4)^2 + (\beta + 2)^2 + (\gamma - 3)^2$$

$$= (\alpha - 2)^2 + (\beta - 4)^2 + (\gamma + 1)^2$$

$$\Rightarrow -4\alpha + 12\beta - 8\gamma = -8$$

$$\Rightarrow 2x - 6y + 4z = 4$$

10. $\lim\limits_{x \to a} \frac{(a+2x)^{\frac{1}{3}} - (3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}} - (4x)^{\frac{1}{3}}} (a \neq 0)$ is equal to :

(1) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$ (2) $\left(\frac{2}{3}\right)^{\frac{4}{3}}$

(3) $\left(\frac{2}{9}\right)^{\frac{4}{3}}$ (4) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

Sol. Required limit

$$L = \lim_{h\to 0}\frac{(a+2(a+h))^{1/3}-(3(a+h))^{1/3}}{(3a+a+h)^{1/3}-(4(a+h))^{1/3}}$$

$$= \lim_{h\to 0}\frac{(3a)^{1/3}\left(1+\frac{2h}{3a}\right)^{1/3}-(3a)^{1/3}\left(1+\frac{h}{a}\right)^{1/3}}{(4a)^{1/3}\left(1+\frac{h}{4a}\right)^{1/3}-(4a)^{1/3}\left(1+\frac{h}{a}\right)^{1/3}}$$

$$= \lim_{h\to 0}\left(\frac{3^{1/3}}{4^{1/3}}\right)\left[\frac{\left(1+\frac{2h}{9a}\right)-\left(1+\frac{h}{3a}\right)}{\left(1+\frac{h}{12a}\right)-\left(1+\frac{h}{3a}\right)}\right]$$

$$= \left(\frac{3}{4}\right)^{1/3}\frac{\left(\frac{2}{9}-\frac{1}{3}\right)}{\left(\frac{1}{12}-\frac{1}{3}\right)} = \left(\frac{3}{4}\right)^{1/3}\left(\frac{8-12}{3-12}\right)$$

$$= \left(\frac{3}{4}\right)^{1/3}\left(\frac{-4}{-9}\right) = \frac{4^{1-\frac{1}{3}}}{3^{2-\frac{1}{3}}} = \frac{4^{2/3}}{3^{5/3}}$$

$$= \frac{(8\times 2)^{1/3}}{(27\times 9)^{1/3}} = \frac{2}{3}\left(\frac{2}{9}\right)^{1/3}$$

11. Let A be a 3 × 3 matrix such that

$$\text{adj } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix} \text{ and}$$

B = adj (adj A).

If $|A| = \lambda$ and $|(B^{-1})^T| = \mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to :

(1) $\left(9, \frac{1}{9}\right)$ (2) $\left(9, \frac{1}{81}\right)$

$\left(\quad 1 \quad \right)$

Official Ans. by NTA (3)

Sol. $C = \text{adj } A = \begin{vmatrix} +2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{vmatrix}$

$|C| = |\text{adj } A| = +2(0 + 4) + 1.(1 - 2) + 1.(2, 4)$

$= +8 - 1 + 2$

$|\text{adj } A| = |A|^2 = 9 = 9$

$\lambda = |A| = \pm 3$

$|\lambda| = 3$

B = adj C

$|B| = |\text{adj } C| = |C|^2 = 81$

$|(B^{-1})^T| = |B|^{-1} = \frac{1}{81}$

$(|\lambda|, \mu) = \left(3, \frac{1}{81}\right)$

12. Suppose f(x) is a polynomial of degree four, having critical points at –1, 0, 1. If $T = \{x \in R |\, f(x) = f(0)\}$, then the sum of squares of all the elements of T is :

(1) 6 (2) 8

(3) 4 (4) 2

Official Ans. by NTA (3)

Sol. $f'(x) = x(x + 1)(x - 1) = x^3 - x$

$$\int d f(x) = \int x^3 - x \, dx$$

$$f(x) = \frac{x^4}{4} - \frac{x^2}{2} + C$$

$f(x) = f(0)$

$$\frac{x^4}{4} - \frac{x^2}{2} = 0$$

$x^2 (x^2 - 2) = 0$

$x = 0, 0, \sqrt{2}, -\sqrt{2}$

13. Let a, b, c ∈ R be such that $a^2 + b^2 + c^2 = 1$.

If $a\cos\theta = b\cos\left(\theta+\frac{2\pi}{3}\right) = c\cos\left(\theta+\frac{4\pi}{3}\right)$,

where $\theta=\frac{\pi}{9}$, then the angle between the vectors $a\hat{i}+b\hat{j}+c\hat{k}$ and $b\hat{i}+c\hat{j}+a\hat{k}$ is :

(1) $\frac{\pi}{2}$ (2) 0

(3) $\frac{\pi}{9}$ (4) $\frac{2\pi}{3}$

Official Ans. by NTA (1)

Sol. $\cos\phi=\frac{\overline{p}.\overline{q}}{|\overline{p}||\overline{q}|}=\frac{ab+bc+ca}{a^2+b^2+c^2}=\frac{\Sigma ab}{1}$

$=abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

$=\frac{abc}{\lambda}\left(\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)+\cos\left(\theta+\frac{4\pi}{3}\right)\right)$

$=\frac{abc}{\lambda}\left(\cos+2\cos(\theta+\pi)\cos\frac{\pi}{3}\right)$

$=\frac{abc}{\lambda}(\cos\theta-\cos\theta)=0$

$\phi=\frac{\pi}{2}$

14. If the sum of the series

$20+19\frac{3}{5}+19\frac{1}{5}+18\frac{4}{5}+....$ upto n^{th} term is 488

and the n^{th} term is negative, then :

(1) n^{th} term is $-4\frac{2}{5}$ (2) n = 41

(3) n^{th} term is –4 (4) n = 60

Official Ans. by NTA (3)

Sol. $S=\frac{100}{}+\frac{98}{}+\frac{96}{}+\frac{94}{}+.....n$

$S_n=\frac{n}{2}\left(2\times\frac{100}{5}+(n-1)\left(-\frac{2}{5}\right)\right)=188$

$n(100 - n + 1) = 488 \times 5$

$n^2 - 101n + 488 \times 5 = 0$

n = 61, 40

$T_n = a + (n - 1)d = \frac{100}{5}-\frac{2}{5}\times 60$

= 20 – 24 = –4

15. Let x_i $(1 \le i \le 10)$ be ten observations of a random variable X. If $\sum_{i=1}^{10}(x_i-p)=3$ and $\sum_{i=1}^{10}(x_i-p)^2=9$ where $0 \ne p \in R$, then the standard deviation of these observations is :

(1) $\sqrt{\frac{3}{5}}$ (2) $\frac{7}{10}$

(3) $\frac{9}{10}$ (4) $\frac{4}{5}$

Official Ans. by NTA (3)

Sol. $\text{Variance}=\frac{\Sigma(x_i-p)^2}{n}-\left(\frac{\Sigma(x_i-p)}{n}\right)^2$

$=\frac{9}{10}-\left(\frac{3}{10}\right)^2=\frac{81}{100}$

S.D. = $\frac{9}{10}$

16. If $x^3dy + xy\,dx = x^2\,dy + 2y\,dx$; y(2) = e and x > 1, then y(4) is equal to :

(1) $\frac{3}{2}+\sqrt{e}$ (2) $\frac{3}{2}\sqrt{e}$

(3) $\frac{1}{2}+\sqrt{e}$ (4) $\frac{\sqrt{e}}{2}$

Sol. $x^3dy + xy\,dx = x^2\,dy + 2y\,dx$

$\Rightarrow$ $dy(x^3 - x^2) = dx\ (2y - xy)$

$\Rightarrow$ $-\int \frac{1}{y}dy = \int \frac{x-2}{x^2(x-1)}dx$

$\Rightarrow$ $-\ell ny = \int \left(\frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-1)}\right)dx$

Where A = 1, B = +2, C = –1

$\Rightarrow$ $-\ell ny = \ell n\,x - \frac{2}{x} - \ell n(x-1) + \lambda$

$\Rightarrow$ $y(2) = e$

$\Rightarrow$ $-1 = \ell n\ 2 - 1 - 0 + \lambda$

$\therefore$ $\lambda = -\ \ell n\,2$

$\Rightarrow$ $\ell n\,y = -\ \ell nx + \frac{2}{x} + \ell n(x-1) + \ell n 2$

Now put x = 4 in equation

$\Rightarrow$ $\ell n\,y = -\ell n 4 + \frac{1}{2} + \ell n 3 + \ell n 2$

$\Rightarrow$ $\ell n\,y = \ell n\left(\frac{3}{2}\right) + \frac{1}{2}\ell n e$

$\Rightarrow$ $y = \frac{3}{2}\sqrt{e}$

17. Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25} + \frac{y^2}{b^2} = 1 (b < 5)$ and the hyperbola, $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$ respectively satisfying $e_1e_2 = 1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α, β) is equal to :

(1) (8, 10) (2) (8, 12)

(3) $\left(\frac{20}{3}, 12\right)$ (4) $\left(\frac{24}{5}, 10\right)$

Sol. For ellipse $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$ (b < 5)

Let e_1 is eccentricity of ellipse

$\therefore$ $b^2 = 25\ (1 - e_1^2)$ .... (1)

Again for hyperbola

$$\frac{x^2}{16} - \frac{y^2}{b^2} = 1$$

Let e_2 is eccentricity of hyperbola.

$\therefore$ $b^2 = 16(e_2^2 - 1)$ .... (2)

by (1) & (2)

$25(1 - e_1^2) = 16(e_2^2 - 1)$

Now $e_1.e_2 = 1$ (given)

$\therefore$ $25(1 - e_1^2) = 16\left(\frac{1-e_1^2}{e_1^2}\right)$

or $e_1 = \frac{4}{5}$ $\therefore$ $e_2 = \frac{5}{4}$

Now distance between foci is 2ae

$\therefore$ distance for ellipse = $2 \times 5 \times \frac{4}{5} = 8 = \alpha$

distance for hyperbola = $2 \times 4 \times \frac{5}{4} = 10 = \beta$

$\therefore$ $(\alpha, \beta) \equiv (8, 10)$

18. The set of all real values of λ for which the quadratic equations,

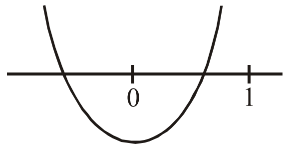
$(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval (0, 1) is :

(1) (–3, –1) (2) (1, 3]

(3) (0, 2) (4) (2, 4]

Official Ans. by NTA (2)

Sol. If exactly one root in (0, 1) then



$\Rightarrow \quad f(0).f(1) < 0$

$\Rightarrow \quad 2(\lambda^2 - 4\lambda + 3) < 0$

$\Rightarrow \quad 1 < \lambda < 3$

Now for $\lambda = 1$, $2x^2 - 4x + 2 = 0$

$(x - 1)^2 = 0$, $x = 1, 1$

So both roots doesn't lie between (0, 1)

$\therefore \lambda \neq 1$

Again for $\lambda = 3$

$10x^2 - 12x + 2 = 0$

$\Rightarrow \quad x = 1, \frac{1}{5}$

so if one root is 1 then second root lie between (0, 1)

so $\lambda = 3$ is correct.

$\therefore \quad \lambda \in (1, 3]$.

19. If the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is k, then 18 k is equal to :

(1) 9 (2) 11

(3) 5 (4) 7

Official Ans. by NTA (4)

Sol. $T_{r+1} = {}^9C_r\left(\frac{3}{2}x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r$

$$T_{r+1} = {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{18-3r}$$

For independent of x

$18 - 3r = 0$, $r = 6$

$$\therefore \quad T_7 = {}^9C_6\left(\frac{3}{2}\right)^3\left(-\frac{1}{3}\right)^6 = \frac{21}{54} = k$$

$$\therefore \quad 18k = \frac{21}{54} \times 18 = 7$$

20. Let p, q, r be three statements such that the truth value of $(p \wedge q) \rightarrow (\sim q \vee r)$ is F. Then the truth values of p, q, r are respectively :

(1) T, F, T (2) F, T, F

(3) T, T, F (4) T, T, T

Official Ans. by NTA (3)

Sol. $(p \wedge q) \rightarrow (\sim q \vee r)$ = false

when $(p \wedge q)$ = T

and $(\sim q \vee r)$ = F

So $(p \wedge q)$ = T is possible when p = q = true

$\therefore$ ~q = False (q = true)

So $(\sim q \vee r)$ = False is possible if r is false

$\therefore$ p = T, q = T, r = F

21. If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4[th] A.M. is equal to 2[nd] G.M., then m is equal to _______.

Official Ans. by NTA (39)

Sol. $3, A_1, A_2 \ldots\ldots\ldots A_m, 243$

$$d = \frac{243 - 3}{m + 1} = \frac{240}{m + 1}$$

Now $3, G_1, G_2, G_3, 243$

$$r = \left(\frac{243}{3}\right)^{\frac{1}{3+1}} = 3$$

$\therefore \quad A_4 = G_2$

$\Rightarrow \quad a + 4d = ar^2$

$$3 + 4\left(\frac{240}{m + 1}\right) = 3(3)^2$$

$m = 39$

22. If the tangent of the curve, $y = e^x$ at a point (c, e^c) and the normal to the parabola, $y^2 = 4x$ at the point (1, 2) intersect at the same point on the x-axis, then the value of c is _______.

Official Ans. by NTA (4)

Sol. $y = e^x \Rightarrow \frac{dy}{dx} = e^x$

$$m = \left(\frac{dy}{dx}\right)_{(c,e^c)} = e^c$$

$\Rightarrow$ Tangent at (c, e^c)

$y - e^c = e^c (x - c)$

it intersect x-axis

Put $y = 0 \Rightarrow x = c - 1$ ......(1)

Now $y^2 = 4x \Rightarrow \frac{dy}{dx} = \frac{2}{y} \Rightarrow \left(\frac{dy}{dx}\right)_{(1,\,2)} = 1$

$\Rightarrow$ Slope of normal = –1

Equation of normal $y - 2 = -1(x - 1)$

$x + y = 3$ it intersect x-axis

Put $y = 0 \Rightarrow x = 3$ ......(2)

Points are same

$\Rightarrow$ $x = c - 1 = 3$

$\Rightarrow$ $c = 4$

23. Let a plane P contain two lines

$\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j}),\ \lambda \in R$ and

$\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k}),\ \mu \in R$

If $Q(\alpha, \beta, \gamma)$ is the foot of the perpendicular drawn from the point M(1, 0, 1) to P, then $3(\alpha + \beta + \gamma)$ equals _______.

Official Ans. by NTA (5)

Sol. Dr's normal to plane

$$= \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = -\hat{i} + \hat{j} + \hat{k}$$

Equation of plane

$-1(x - 1) + 1(y - 0) + 1(z - 0) = 0$

$x - y - z - 1 = 0$ ......(1)

Now $\frac{\alpha - 1}{1} = \frac{\beta - 0}{-1} = \frac{\gamma - 1}{-1} = -\frac{(1-0-1-1)}{3}$

$$\frac{\alpha - 1}{1} = \frac{\beta}{-1} = \frac{\gamma - 1}{-1} = \frac{1}{3}$$

$$\alpha = \frac{4}{3},\ \beta = -\frac{1}{3},\ \gamma = \frac{2}{3}$$

$$3(\alpha + \beta + \gamma) = 3\left(\frac{4}{3} - \frac{1}{3} + \frac{2}{3}\right) = 5$$

24. Let S be the set of all integer solutions, (x, y, z), of the system of equations

$x - 2y + 5z = 0$

$-2x + 4y + z = 0$

$-7x + 14y + 9z = 0$

such that $15 \le x^2 + y^2 + z^2 \le 150$. Then, the number of elements in the set S is equal to _______.

Official Ans. by NTA (8)

CollegeDekho

Sol. $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ -2 & 4 & 1 \\ -7 & 14 & 9 \end{vmatrix} = 0$

Let $x = k$

$\Rightarrow$ Put in (1) & (2)

$k - 2y + 5z = 0$

$-2k + 4y + z = 0$

$z = 0, \; y = \frac{k}{2}$

$\therefore$ x, y, z are integer

$\Rightarrow$ k is even integer

Now $x = k, \; y = \frac{k}{2}, \; z = 0$ put in condition

$$15 \le k^2 + \left(\frac{k}{2}\right)^2 + 0 \le 150$$

$$12 \le k^2 \le 120$$

$\Rightarrow$ $k = \pm 4, \pm 6, \pm 8, \pm 10$

$\Rightarrow$ Number of element in S = 8.

25. The total number of 3-digit numbers, whose sum of digits is 10, is ________.

Official Ans. by NTA (54)

Sol. Let three digit number is xyz

$x + y + z = 10$; $x \ge 1, y \ge 0 \; z \ge 0$ ..... (1)

Let $T = x - 1 \Rightarrow x = T + 1$ where $T \ge 0$

Put in (1)

$T + y + z = 9$; $0 \le T \le 8, 0 \le y, z \le 9$

No. of non negative integral solution

$= {}^{9+3-1}C_{3-1} - 1$ (when T = 9)

$= 55 - 1 = 54$