FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Friday 04th SEPTEMBER, 2020) TIME : 9 AM to 12 PM

MATHEMATICS — TEST PAPER WITH SOLUTION

COMPLEX NUMBER

1. If $A=\begin{bmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{bmatrix}$, $\left(\theta=\frac{\pi}{24}\right)$ and $A^5=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, where $i=\sqrt{-1}$, then which one of the following is not true?

(1) $0 \le a^2 + b^2 \le 1$ (2) $a^2 - d^2 = 0$

(3) $a^2 - b^2 = \frac{1}{2}$ (4) $a^2 - c^2 = 1$

1. यदि $A=\begin{bmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{bmatrix}$, $\left(\theta=\frac{\pi}{24}\right)$ तथा $A^5=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, जहाँ $i=\sqrt{-1}$ हो, तो निम्न में से कौनसा एक सत्य नहीं होगा ?

(1) $0 \le a^2 + b^2 \le 1$ (2) $a^2 - d^2 = 0$

(3) $a^2 - b^2 = \frac{1}{2}$ (4) $a^2 - c^2 = 1$

Official Ans. by NTA (3)

Sol. $A^2=\begin{pmatrix}\cos 2\theta & i\sin 2\theta\\ i\sin 2\theta & \cos 2\theta\end{pmatrix}$

Similarly, $A^5=\begin{pmatrix}\cos 5\theta & i\sin 5\theta\\ i\sin 5\theta & \cos 5\theta\end{pmatrix}=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$

(1) $a^2 + b^2 = \cos^2 5\theta - \sin^2 5\theta = \cos 10\theta = \cos 75°$

(2) $a^2 - d^2 = \cos^2 5\theta - \cos^2 5\theta = 0$

(3) $a^2 - b^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

FUNCTION

2. Let [t] denote the greatest integer ≤ t. Then the equation in x, $[x]^2 + 2[x + 2] - 7 = 0$ has :

(1) no integral solution

(2) exactly four integral solutions

(3) exactly two solutions

(4) infinitely many solutions

2. माना [t], t से कम या बराबर महत्तम पूर्णांक फलन को दर्शाता है। तब x में समीकरण $[x]^2 + 2[x + 2] - 7 = 0$

(1) का कोई पूर्णांकीय हल नहीं होगा।

(2) के ठीक चार पूर्णांकीय हल होंगे।

(3) के ठीक दो हल होंगे।

(4) अनंत हल होंगे।

Official Ans. by NTA (4)

Sol. $[x]^2 + 2[x + 2] - 7 = 0$

$\Rightarrow [x]^2 + 2[x] + 4 - 7 = 0$

$\Rightarrow [x] = 1, -3$

$\Rightarrow x \in [1, 2) \cup [-3, -2)$

QUADRATIC EQUATION

3. Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If α, β, γ, δ form a geometric progression. Then ratio (2q + p) : (2q – p) is :

(1) 3 : 1 (2) 33 : 31

(3) 9 : 7 (4) 5 : 3

3. माना $x^2 - 3x + p = 0$ के मूल α तथा β एवं $x^2 - 6x + q = 0$ के मूल γ तथा δ है। यदि α, β, γ, δ

(2q + p) : (2q – p)

(1) 3 : 1 (2) 33 : 31

Official Ans. by NTA (3)

Sol. $x^2 - 3x + p = 0 \begin{matrix} \alpha \\ \beta \end{matrix}$

$\alpha, \beta, \gamma, \delta$ in G.P.

$\alpha + \alpha r = 3$ ....(1)

$x^2 - 6x + q = 0 \begin{matrix} \gamma \\ \delta \end{matrix}$

$\alpha r^2 + \alpha r^3 = 6$ ...(2)

(2) ÷ (1)

$r^2 = 2$

So, $\dfrac{2q+p}{2q-p} = \dfrac{2r^5+r}{2r^5-r} = \dfrac{2r^4+1}{2r^4-1} = \dfrac{9}{7}$

ELLIPSE

4. Let $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ $(a > b)$ be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t) = \dfrac{5}{12} + t - t^2$, then $a^2 + b^2$ is equal to :

(1) 126 (2) 135

(3) 145 (4) 116

4. माना $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ $(a > b)$ एक दीर्घवृत्त दिया गया है जिसके नाभिलम्ब की लम्बाई 10 है। यदि इसकी उत्केन्द्रता, फलन $\phi(t) = \dfrac{5}{12} + t - t^2$ का अधिकतम मान हो, तो $a^2 + b^2$ का मान होगा

(1) 126 (2) 135

(3) 145 (4) 116

Official Ans. by NTA (1)

Sol. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ $(a > b)$; $\dfrac{2b^2}{a} = 10 \Rightarrow b^2 = 5a$ ...(i)

Now, $\phi(t) = \dfrac{5}{12} + t - t^2 = \dfrac{8}{12} - \left(t - \dfrac{1}{2}\right)^2$

$\phi(t)_{max} = \dfrac{8}{12} = \dfrac{2}{3} = e \Rightarrow e^2 = 1 - \dfrac{b^2}{a^2} = \dfrac{4}{9}$ ... (ii)

2

STRAIGHT LINE

5. A triangle ABC lying in the first quadrant has two vertices as A(1, 2) and B(3, 1). If $\angle BAC = 90°$, and $ar(\Delta ABC) = 5\sqrt{5}$ sq. units, then the abscissa of the vertex C is :

(1) $2+\sqrt{5}$ (2) $1+\sqrt{5}$

(3) $1+2\sqrt{5}$ (4) $2\sqrt{5}-1$

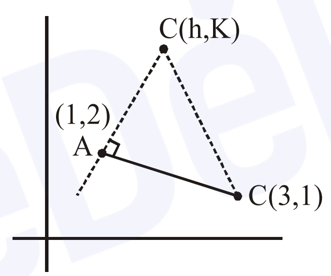
5. एक त्रिभुज ABC प्रथम चतुर्थांश में स्थित है जिसके दो शीर्ष A(1, 2) तथा B(3, 1) है। यदि $\angle BAC = 90°$ तथा $ar(\Delta ABC) = 5\sqrt{5}$ वर्ग इकाई हो, तो शीर्ष C का भुज होगा

(1) $2+\sqrt{5}$ (2) $1+\sqrt{5}$

(3) $1+2\sqrt{5}$ (4) $2\sqrt{5}-1$

Official Ans. by NTA (3)

Sol.



$\left(\dfrac{K-2}{h-1}\right)\left(\dfrac{1-2}{3-1}\right) = -1 \Rightarrow K = 2h$ ...(1)

$\sqrt{5}\,|h-1| = 10$

$\because [\Delta ABC] = 5\sqrt{5}$

$\Rightarrow \dfrac{1}{2}(\sqrt{5})\sqrt{(h-1)^2 + (K-2)^2} = 5\sqrt{5}$ ....(2)

$\Rightarrow h = 2\sqrt{5}+1$ $(h > 0)$

DEFINITE INTEGRATION

6. Let $f(x) = |x - 2|$ and $g(x) = f(f(x))$, $x \in [0, 4]$.

Then $\int_0^3 (g(x) - f(x))dx$ is equal to :

3 1

6. माना $f(x) = |x-2|$ तथा $g(x) = f(f(x)), x \in [0, 4]$ है।

तब $\int_0^3 (g(x)-f(x))dx$ का मान होगा

(1) $\frac{3}{2}$ (2) 0

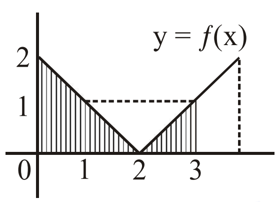
(3) $\frac{1}{2}$ (4) 1

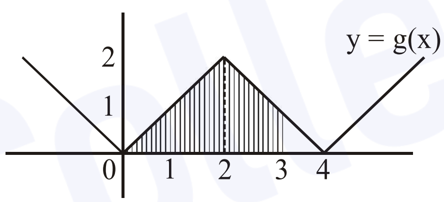
Official Ans. by NTA (4)

Sol. $\int_0^3 g(x)-f(x) = \int_0^3 ||x-2|-2|dx - \int_0^3 |x-2|dx$

$= \left(\frac{1}{2}\times2\times2+1+\frac{1}{2}\times1\times1\right)-\left(\frac{1}{2}\times2\times2+\frac{1}{2}\times1\times1\right)$

$= \left(2+1+\frac{1}{2}\right)-\left(2+\frac{1}{2}\right) = 1$





MATHEMATICAL REASONING / LOGIC

7. Given the following two statements :

$(S_1) : (q \vee p) \rightarrow (p \leftrightarrow \sim q)$ is a tautology.

$(S_2) : \sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy.

Then :

(1) only (S_1) is correct.

(2) both (S_1) and (S_2) are correct.

(3) both (S_1) and (S_2) are not correct.

7. दिये गये निम्न दो कथन :

$(S_1) : (q \vee p) \rightarrow (p \leftrightarrow \sim q)$ पुनरूक्ति है।

$(S_2) : \sim q \wedge (\sim p \leftrightarrow q)$ व्याघात है।

तब

(1) केवल (S_1) सही होगा।

(2) दोनों (S_1) तथा (S_2) सही होंगे।

(3) दोनों (S_1) तथा (S_2) सही नहीं होंगे।

(4) केवल (S_2) सही होगा।

Official Ans. by NTA (3)

Sol. Let TV(r) denotes truth value of a statement r.

Now, if TV(p) = TV(q) = T

$\Rightarrow TV(S_1) = F$

Also, if TV(p) = T & TV(q) = F

$\Rightarrow TV(S_2) = T$

HYPERBOLA

8. Let P(3, 3) be a point on the hyperbola,

$\frac{x^2}{a^2}-\frac{y^2}{b^2} = 1$. If the normal to it at P intesects the x-axis at (9, 0) and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to :

(1) $\left(\frac{9}{2}, 3\right)$ (2) $\left(\frac{9}{2}, 2\right)$

(3) $\left(\frac{3}{2}, 2\right)$ (4) (9, 3)

8. माना अतिपरवलय $\frac{x^2}{a^2}-\frac{y^2}{b^2} = 1$ पर एक बिन्दु P(3, 3) है। यदि बिन्दु P पर इसका अभिलम्ब x-अक्ष को बिन्दु (9, 0) पर प्रतिच्छेद करता है तथा इसकी उत्केन्द्रता e है, तो क्रमित युग्म (a^2, e^2) होगा

(1) $\left(\frac{9}{2}, 3\right)$ (2) $\left(\frac{9}{2}, 2\right)$

$\left(\frac{3}{\ }\right.$

Official Ans. by NTA (1)

Sol. Since, (3, 3) lies on $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\frac{9}{a^2}-\frac{9}{b^2}=1$ ....(1)

Now, normal at (3, 3) is $y-3=-\frac{a^2}{b^2}(x-3)$,

which passes through (9, 0) $\Rightarrow$ $b^2 = 2a^2$ ....(2)

So, $e^2 = 1+\frac{b^2}{a^2} = 3$

Also, $a^2 = \frac{9}{2}$ (from (i) & (ii))

Thus, $(a^2, e^2) = \left(\frac{9}{2}, 3\right)$

DEFINITE INTEGRATION

9. Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2}dx\ (x \ge 0)$. Then f(3) – f(1) is equal to :

(1) $-\frac{\pi}{6}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ (2) $\frac{\pi}{6}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

(3) $-\frac{\pi}{12}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

9. माना $f(x) = \int \frac{\sqrt{x}}{(1+x)^2}dx\ (x \ge 0)$ है। तब f(3) – f(1) का मान होगा

(1) $-\frac{\pi}{6}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ (2) $\frac{\pi}{6}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

$\frac{\pi}{\ }\ \frac{1}{\ }\ \frac{\sqrt{3}}{\ }$ $\frac{\pi}{\ }\ \frac{1}{\ }\ \frac{\sqrt{3}}{\ }$

Official Ans. by NTA (4)

Sol. $f(x) = \int_1^3 \frac{\sqrt{x}\,dx}{(1+x)^2} = \int_1^{\sqrt{3}} \frac{t.2t\,dt}{(1+t^2)^2}$ $\left(\text{put } \sqrt{x}=t\right)$

$= \left(-\frac{t}{1+t^2}\right)_1^{\sqrt{3}} + (\tan^{-1}t)_1^{\sqrt{3}}$ [Appling by parts]

$= -\left(\frac{\sqrt{3}}{4}-\frac{1}{2}\right)+\frac{\pi}{3}-\frac{\pi}{4}$

$= \frac{1}{2}-\frac{\sqrt{3}}{4}+\frac{\pi}{12}$

SET

10. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be:

(1) 65 (2) 37

(3) 29 (4) 55

10. एक सर्वेक्षण से पता चलता है कि शहर के 63% व्यक्ति अखबार A पढ़ते है जबकि 76% व्यक्ति अखबार B पढ़ते है। यदि x% व्यक्ति दोनों अखबार पढ़ते है, तो x का संभव मान हो सकता है :

(1) 65 (2) 37

(3) 29 (4) 55

Official Ans. by NTA (4)

Sol. $n(B) \le n(A \cup B) \le n(U)$

$\Rightarrow 76 \le 76 + 63 - x \le 100$

$\Rightarrow -63 \le -x \le -39$

$\Rightarrow 63 \ge x \ge 39$

COMPLEX NUMBER

11. Let $u = \frac{2z+i}{z-ki}$, z = x + iy and k > 0. If the curve represented by Re(u) + Im(u) = 1 intersects the y-axis at the points P and Q where PQ = 5, then the value of k is :

(1) 3/2 (2) 4

11. माना $u=\frac{2z+i}{z-ki}$, $z = x + iy$ तथा $k > 0$ है। Re(u) + Im(u) = 1 द्वारा प्रदर्शित वक्र y-अक्ष को बिन्दु P तथा Q पर काटता हैं जहाँ PQ = 5 हो, तो k का मान होगा :

(1) 3/2 (2) 4

(3) 2 (4) 1/2

Official Ans. by NTA (3)

Sol. $u = \frac{2z+i}{z-ki}$

$= \frac{2x^2+(2y+1)(y-k)}{x^2+(y-k)^2} + i\frac{(x(2y+1)-2x(y-k))}{x^2+(y-k)^2}$

Since Re(u) + Im(u) = 1

$\Rightarrow 2x^2 + (2y+1)(y-k)+x(2y+1)-2x(y-k)$

$= x^2 + (y-k)^2$

$\left.\begin{matrix}P(0,y_1)\\Q(0,y_2)\end{matrix}\right\rangle \Rightarrow y^2+y-k-k^2=0 \left\langle\begin{matrix}y_1+y_2=-1\\y_1y_2=-k-k^2\end{matrix}\right.$

$\because$ PQ = 5

$\Rightarrow |y_1-y_2|=5 \Rightarrow k^2+k-6=0$

$\Rightarrow k = -3, 2$

So, k = 2 (k > 0)

VECTOR

12. Let x_0 be the point of local maxima of $f(x)=\vec{a}.(\vec{b}\times\vec{c})$, where $\vec{a}=x\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=-2\hat{i}+x\hat{j}-\hat{k}$ and $\vec{c}=7\hat{i}-2\hat{j}+x\hat{k}$. Then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ at $x = x_0$ is :

(1) –30 (2) 14

(3) –4 (4) –22

12. माना $f(x)=\vec{a}.(\vec{b}\times\vec{c})$ का स्थानीय उच्चिष्ठ x_0 है, जहाँ

$\vec{a}=x\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=-2\hat{i}+x\hat{j}-\hat{k}$ तथा

$\vec{c}=7\hat{i}-2\hat{j}+x\hat{k}$ $x=x_0$ $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$

:

(1) –30 (2) 14

Official Ans. by NTA (4)

Sol. $f(x)=\vec{a}.(\vec{b}\times\vec{c})=\begin{vmatrix}x & -2 & 3\\-2 & x & -1\\7 & -2 & x\end{vmatrix} = x^3 - 27x + 26$

$f'(x) = 3x^2 - 27 = 0 \Rightarrow x = \pm 3$

and $f''(-3) < 0$

$\Rightarrow$ local maxima at $x = x_0 = -3$

Thus, $\vec{a}=-3\hat{i}-2\hat{j}+3\hat{k}$,

$\vec{b}=-2\hat{i}-3\hat{j}-\hat{k}$,

and $\vec{c}=7\hat{i}-2\hat{j}-3\hat{k}$

$\Rightarrow \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=9-5-26=-22$

HEIGHT AND DISTANCE (SOT)

13. Two vertical poles AB = 15 m and CD = 10 m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is :

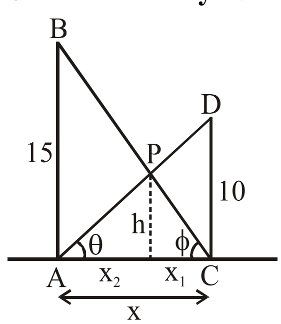
(1) 20/3 (2) 5 (3) 10/3 (4) 6

13. दो ऊर्ध्वाधर खंभे AB = 15 मीटर CD = 10 मीटर जमीन पर A तथा C के साथ क्षैतिज जमीन पर अलग खड़े है। यदि भुजा BC तथा AD का प्रतिच्छेद बिन्दु P है, तो P की ऊँचाई (मीटर में) रेखा AC के ऊपर है

(1) 20/3 (2) 5 (3) 10/3 (4) 6

Official Ans. by NTA (4)

Sol.



$\tan\theta=\frac{10}{x}=\frac{h}{x_2}\Rightarrow x_2=\frac{hx}{10}$

$\tan\phi=\frac{15}{x}=\frac{h}{x_1}\Rightarrow x_1=\frac{hx}{15}$

Now, $x_1 + x_2 = x = \frac{hx}{15}+\frac{hx}{10}$

h h

STATISTICS

14. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is :

(1) 7 (2) 3
(3) 5 (4) 9

14. 8 प्रेक्षणों का माध्य तथा प्रसरण क्रमशः 10 तथा 13.5 है। यदि इनमें से 6 प्रेक्षण 5, 7, 10, 12, 14, 15 हैं, तो शेष दो प्रेक्षणों का निरपेक्ष अन्तर होगा :

(1) 7 (2) 3
(3) 5 (4) 9

Official Ans. by NTA (1)

Sol. $\bar{x} = 10$

$$\Rightarrow \bar{x} = \frac{63+a+b}{8} = 10 \Rightarrow a + b = 17 \quad ...(1)$$

Since, variance is independent of origin.
So, we subtract 10 from each observation.

So, $\sigma^2 = 13.5 = \frac{79+(a-10)^2+(b-10)^2}{8} - (10-10)^2$

$\Rightarrow a^2 + b^2 - 20(a + b) = -171$

$\Rightarrow a^2 + b^2 = 169 \quad ...(2)$

From (i) & (ii) ; a = 12 & b = 5

INDEFINITE INTEGRATION

15. The integral $\int \left(\frac{x}{x\sin x + \cos x}\right)^2 dx$ is equal to :

(where C is a constant of integration)

(1) $\sec x + \frac{x\tan x}{x\sin x + \cos x} + C$

(2) $\sec x - \frac{x\tan x}{x\sin x + \cos x} + C$

(3) $\tan x + \frac{x\sec x}{x\sin x + \cos x} + C$

(4) $\tan x - \frac{x\sec x}{x\sin x + \cos x} + C$

15. समाकलन $\int \left(\frac{x}{x\sin x + \cos x}\right)^2 dx$ होगा :

(जहाँ C, समाकलन अचर है)

(1) $\sec x + \frac{x\tan x}{x\sin x + \cos x} + C$

(2) $\sec x - \frac{x\tan x}{x\sin x + \cos x} + C$

(3) $\tan x + \frac{x\sec x}{x\sin x + \cos x} + C$

(4) $\tan x - \frac{x\sec x}{x\sin x + \cos x} + C$

Official Ans. by NTA (4)

Sol. $\int \left(\frac{x}{x\sin x + \cos x}\right)^2 dx = \int \left(\frac{x}{\cos x}\right) \cdot \frac{x\cos x\, dx}{(x\sin x + \cos x)^2}$

$$= \frac{x}{\cos x}\left(-\frac{1}{x\sin x + \cos x}\right)$$

$$+ \int \left(\frac{\cos x + x\sin x}{\cos^2 x}\right)\left(\frac{1}{x\sin x + \cos x}\right) dx$$

$$= -\frac{x\sec x}{x\sin x + \cos x} + \int \sec^2 x\, dx$$

$$= -\frac{x\sec x}{x\sin x + \cos x} + \tan x + C$$

SEQUENCE AND SERIES

16. If $1+(1-2^2.1)+(1-4^2.3)+(1-6^2.5)+.....+(1-20^2.19) = \alpha - 220\beta$, then an ordered pair (α, β) is equal to :

(1) (10, 97) (2) (11, 103)
(3) (10, 103) (4) (11, 97)

16. यदि $1+(1-2^2.1)+(1-4^2.3)+(1-6^2.5)+.....+(1-20^2.19) = \alpha - 220\beta$

(α, β) :

(1) (10, 97) (2) (11, 103)
(3) (10, 103) (4) (11, 97)

Official Ans. by NTA (2)

Sol. $1 + (1 - 2^2.1) + (1 - 4^2.3) + ... + (1 - 20^2.19)$

$= \alpha - 220\ \beta$

$= 11 - (2^2.1 + 4^2.3 + ... + 20^2.19)$

$= 11 - 2^2.\sum_{r=1}^{10} r^2(2r-1) = 11 - 4\left(\frac{110^2}{2} - 35\times 11\right)$

$= 11 - 220(103)$

$\Rightarrow \alpha = 11, \beta = 103$

DIFFERENTIAL EQUATION

17. Let y = y(x) be the solution of the differential equation, $xy' - y = x^2(x\cos x + \sin x)$, $x > 0$.

If $y(\pi) = \pi$, then $y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ is equal to :

(1) $2+\frac{\pi}{2}$ (2) $1+\frac{\pi}{2}$

(3) $1+\frac{\pi}{2}+\frac{\pi^2}{4}$ (4) $2+\frac{\pi}{2}+\frac{\pi^2}{4}$

17. माना अवकल समीकरण $xy' - y = x^2(x\cos x + \sin x)$, $x > 0$ का हल $y = y(x)$ हैं। यदि $y(\pi) = \pi$ हो, तो $y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ होगा

(1) $2+\frac{\pi}{2}$ (2) $1+\frac{\pi}{2}$

(3) $1+\frac{\pi}{2}+\frac{\pi^2}{4}$ (4) $2+\frac{\pi}{2}+\frac{\pi^2}{4}$

Official Ans. by NTA (1)

Sol. $x\frac{dy}{dx} - y = x^2(x\cos x + \sin x),\ x > 0$

$\frac{dy}{dx} - \frac{y}{x} = x(x\cos x + \sin x) \Rightarrow \frac{dy}{dx} + Py = Q$

so, I.F. $= e^{\int -\frac{1}{x}dx} = \frac{1}{|x|} = \frac{1}{x}$ $(x > 0)$

Thus, $\frac{y}{x} = \int \frac{1}{x}\left(x(x\cos x + \sin x)\right)dx$

$\Rightarrow \frac{y}{x} = x\sin x + C$

so, $y = x^2\sin x + x \Rightarrow (y)_{\pi/2} = \frac{\pi^2}{4} + \frac{\pi}{2}$

Also, $\frac{dy}{dx} = x^2\cos x + 2x\sin x + 1$

$\Rightarrow \frac{d^2y}{dx^2} = -x^2\sin x + 4x\cos x + 2\sin x$

$\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{\frac{\pi}{2}} = -\frac{\pi^2}{4} + 2$

Thus, $y_{\left(\frac{\pi}{2}\right)} + \frac{d^2y}{dx^2}_{\left(\frac{\pi}{2}\right)} = \frac{\pi}{2} + 2$

BINOMIAL THEOREM

18. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to :

(1) ${}^{51}C_7 + {}^{30}C_7$ (2) ${}^{51}C_7 - {}^{30}C_7$

(3) ${}^{50}C_7 - {}^{30}C_7$ (4) ${}^{50}C_6 - {}^{30}C_6$

18. $\sum_{r=0}^{20} {}^{50-r}C_6$ का मान होगा

(1) ${}^{51}C_7 + {}^{30}C_7$ (2) ${}^{51}C_7 - {}^{30}C_7$

(3) ${}^{50}C_7 - {}^{30}C_7$ (4) ${}^{50}C_6 - {}^{30}C_6$

Official Ans. by NTA (2)

Sol. $\sum_{r=0}^{20} {}^{50-r}C_6 = {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + ... + {}^{30}C_6$

$=$

${}^{50}C_6 + {}^{49}C_6 + ... + {}^{31}C_6 + \left({}^{30}C_6 + {}^{30}C_7\right) - {}^{30}C_7$

$= {}^{50}C_6 + {}^{49}C_6 + ... + \left({}^{31}C_6 + {}^{31}C_7\right) - {}^{30}C_7$

$= {}^{50}C_6 + {}^{50}C_7 - {}^{30}C_7$

$= {}^{51}C_7 - {}^{30}C_7$

$\boxed{{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r}$

MONOTONICITY (AOD)

19. Let f be a twice differentiable function on (1, 6). If f(2) = 8, f′(2) = 5, f′(x) ≥ 1 and f″(x) ≥ 4, for all x ∈ (1, 6), then :

(1) f(5) ≤ 10 (2) f′(5) + f″(5) ≤ 20

(3) f(5) + f′(5) ≥ 28 (4) f(5) + f′(5) ≤ 26

19. माना अन्तराल (1, 6) में *f* दो बार अवकलनीय फलन है। यदि f(2) = 8, f′(2) = 5, f′(x) ≥ 1 तथा f″(x) ≥ 4, ∀x ∈ (1, 6) हो, तो

(1) f(5) ≤ 10 (2) f′(5) + f″(5) ≤ 20

(3) f(5) + f′(5) ≥ 28 (4) f(5) + f′(5) ≤ 26

Official Ans. by NTA (3)

Sol. f(2) = 8, f′(2) = 5, f′(x)≥1, f″(x) ≥ 4, ∀x∈(1,6)

$$f''(x) = \frac{f'(5)-f'(2)}{5-2} \geq 4 \Rightarrow f'(5) \geq 17 \quad ...(1)$$

$$f'(x) = \frac{f(5)-f(2)}{5-2} \geq 1 \Rightarrow f(5) \geq 11 \quad ...(2)$$

$$\overline{f'(5)+f(5) \geq 28}$$

MOD

20. If $\left(a+\sqrt{2}\,b\cos x\right)\left(a-\sqrt{2}\,b\cos y\right)=a^2-b^2$, where a > b > 0, then $\frac{dx}{dy}$ at $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$ is :

(1) $\frac{a-b}{a+b}$ (2) $\frac{a+b}{a-b}$

(3) $\frac{2a+b}{2a-b}$ (4) $\frac{a-2b}{a+2b}$

20. यदि $\left(a+\sqrt{2}\,b\cos x\right)\left(a-\sqrt{2}\,b\cos y\right)=a^2-b^2$, जहाँ a > b > 0 हो, तो $\frac{dx}{dy}$ पर $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$ होगा।

(1) $\frac{a-b}{a+b}$ (2) $\frac{a+b}{a-b}$

(3) $\frac{2a+b}{2a-b}$ (4) $\frac{a-2b}{a+2b}$

Official Ans. by NTA (2)

Sol. $\left(a+\sqrt{2}b\cos x\right)\left(a-\sqrt{2}b\cos y\right)=a^2-b^2$

$$\Rightarrow a^2-\sqrt{2}ab\cos y+\sqrt{2}ab\cos x - 2b^2\cos x\cos y = a^2-b^2$$

Differentiating both sides :

$$0-\sqrt{2}ab\left(-\sin y\frac{dy}{dx}\right) + \sqrt{2}ab(-\sin x)$$

$$-2b^2\left[\cos x\left(-\sin y\frac{dy}{dx}\right)+\cos y\left(-\sin x\right)\right] = 0$$

At $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$:

$$ab\frac{dy}{dx} - ab - 2b^2\left(-\frac{1}{2}\frac{dy}{dx}-\frac{1}{2}\right) = 0$$

$$\Rightarrow \frac{dx}{dy} = \frac{ab+b^2}{ab-b^2} = \frac{a+b}{a-b} \ ; \quad a, b > 0$$

DETERMINANT

21. If the system of equations

x – 2y + 3z = 9

2x + y + z = b

x – 7y + az = 24,

has infinitely many solutions, then a – b is equal to ________ .

21. यदि समीकरण निकाय

x – 2y + 3z = 9

2x + y + z = b

x – 7y + az = 24,

के अनंत हल हो, तो a – b का मान होगा

Official Ans. by NTA (5)

Sol. $D=\begin{vmatrix}1 & -2 & 3\\ 2 & 1 & 1\\ 1 & -7 & a\end{vmatrix}=0 \Rightarrow a=8$

also, $D_1=\begin{vmatrix}9 & -2 & 3\\ b & 1 & 1\\ 24 & -7 & 8\end{vmatrix}=0 \Rightarrow b=3$

PROBABILITY

22. The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater than $\frac{1}{4}$, is ________.

22. एक व्यक्ति के द्वारा किसी लक्ष्य को भेदने की प्रायिकता $\frac{1}{10}$ है। आवश्यक शॉट की न्यूनतम संख्या, ताकि कम से कम एक बार लक्ष्य को मारने की प्रायिकता $\frac{1}{4}$ से अधिक हो, होगी

Official Ans. by NTA (3)

Sol. We have, 1 – (probability of all shots result in failure) > $\frac{1}{4}$

$$\Rightarrow 1-\left(\frac{9}{10}\right)^n > \frac{1}{4}$$

$$\Rightarrow \frac{3}{4} > \left(\frac{9}{10}\right)^n \Rightarrow n \geq 3$$

DIFFERENTIABILITY

23. Suppose a differentiable function f(x) satisfies the identity $f(x + y) = f(x) + f(y) + xy^2 + x^2y$, for all real x and y. If $\lim\limits_{x\to 0}\frac{f(x)}{x}=1$, then f'(3) is equal to ________.

23. माना अवकलनीय फलन f(x) है जो सभी वास्तविक x तथा y के लिये सर्वसमिका $f(x + y) = f(x) + f(y) + xy^2 + x^2y$ को

$$\lim_{x\to 0}\frac{f(x)}{x}=1$$

Official Ans. by NTA (10)

Sol. Since, $\lim\limits_{x\to 0}\frac{f(x)}{x}$ exist $\Rightarrow f(0) = 0$

Now, $f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$=\lim_{h\to 0}\frac{f(h)+xh^2+x^2h}{h} \quad \text{(take y = h)}$$

$$=\lim_{h\to 0}\frac{f(h)}{h}+\lim_{h\to 0}(xh)+x^2$$

$\Rightarrow f'(x) = 1 + 0 + x^2$

$\Rightarrow f'(3) = 10$

BINOMIAL THEOREM

24. Let $(2x^2 + 3x + 4)^{10} = \sum\limits_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is equal to ________.

24. माना $(2x^2 + 3x + 4)^{10} = \sum\limits_{r=0}^{20} a_r x^r$ है। तब $\frac{a_7}{a_{13}}$ का मान होगा

Official Ans. by NTA (8)

Sol. Given $(2x^2 + 3x + 4)^{10} = \sum\limits_{r=0}^{20} a_r x^r$ .... (1)

replace x by $\frac{2}{x}$ in above identity :-

$$\frac{2^{10}(2x^2+3x+4)^{10}}{x^{20}}=\sum_{r=0}^{20}\frac{a_r 2^r}{x^r}$$

$$\Rightarrow 2^{10}\sum_{r=0}^{20} a_r x^r=\sum_{r=0}^{20} a_r 2^r x^{(20-r)} \quad \text{(from (i))}$$

now, comparing coefficient of x^7 from both sides (take r = 7 in L.H.S. & r = 13 in R.H.S.)

$$2^{10} a_7 = a_{13} 2^{13} \Rightarrow \frac{a_7}{a_{13}}=2^3=8$$

3D

25. If the equation of a plane P, passing through the intesection of the planes, $x + 4y - z + 7 = 0$ and $3x + y + 5z = 8$ is $ax + by + 6z = 15$ for some a, b $\in$ R, then the distance of the point (3, 2, –1) from the plane P is _________.

25. यदि समतल P का समीकरण, जो समतलों

$x + 4y - z + 7 = 0$ तथा $3x + y + 5z = 8$ के प्रतिच्छेदन से गुजरता है, किसी a, b $\in$ R केलिये

$ax + by + 6z = 15$ हो, तो समतल P से बिन्दु

(3, 2, –1) की दूरी होगी

Official Ans. by NTA (3)

Sol. $$D_1 = \begin{vmatrix} -7 & 4 & -1 \\ 8 & 1 & 5 \\ 15 & b & 6 \end{vmatrix} = 0 \Rightarrow b = -3$$

$$D = \begin{vmatrix} 1 & 4 & -1 \\ 3 & 1 & 5 \\ a & b & 6 \end{vmatrix} = 0 \Rightarrow 21a - 8b - 66 = 0 \quad .... (1)$$

$P : 2x - 3y + 6z = 15$

so required distance $= \frac{21}{7} = 3$