FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Wednesday 06th SEPTEMBER, 2020) TIME : 9 AM to 12 PM

MATHEMATICS — TEST PAPER WITH SOLUTION

ELLIPSE-XI

1. Which of the following points lies on the locus of the foot of perpendicular drawn upon any tangent to the ellipse, $\frac{x^2}{4}+\frac{y^2}{2}=1$ from any of its foci ?

(1) $\left(-1,\sqrt{3}\right)$ (2) $\left(-1,\sqrt{2}\right)$

(3) $\left(-2,\sqrt{3}\right)$ (4) (1,2)

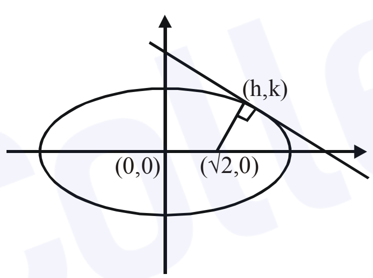
1. निम्न में से कौन सा बिंदु, दीर्घवृत्त $\frac{x^2}{4}+\frac{y^2}{2}=1$ की किसी भी स्पर्श रेखा पर इसकी किसी एक नाभि से खींचे गए लंब के पाद के बिंदु पथ पर स्थित है ?

(1) $\left(-1,\sqrt{3}\right)$ (2) $\left(-1,\sqrt{2}\right)$

(3) $\left(-2,\sqrt{3}\right)$ (4) (1,2)

Official Ans. by NTA (1)

Sol. Let foot of perpendicular is (h,k)



$\frac{x^2}{4}+\frac{y^2}{2}=1$ (Given)

$a=2$, $b=\sqrt{2}$, $e=\sqrt{1-\frac{2}{4}}=\frac{1}{\sqrt{2}}$

$\therefore$ Focus (ae,0) $=\left(\sqrt{2},0\right)$

Equation of tangent

$y=mx+\sqrt{a^2m^2+b^2}$

$y=mx+\sqrt{4m^2+2}$

Passes throguh (h,k)

$(k-mh)^2=4m^2+2$ ...(1)

line perpendicular to tangent will have slope $-\frac{1}{m}$

$y-0=-\frac{1}{m}\left(x-\sqrt{2}\right)$

$my=-x+\sqrt{2}$

$(h+mk)^2=2$ ...(2)

Add equaiton (1) and (2)

$k^2(1+m^2)+h^2(1+m^2)=4(1+m^2)$

$h^2+k^2=4$

$x^2+y^2=4$ (Auxilary circle)

$\therefore$ $\left(-1,\sqrt{3}\right)$ lies on the locus.

PROBABILITY-XII

2. Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated ?

(1) 2!3!4! (2) $(3!)^3.(4!)$

(3) $(3!)^2.(4!)$ (4) $3!(4!)^3$

2. तीन तीन सदस्यों वाले दो परिवारों तथा चार सदस्यों वाले एक परिवार के सदस्यों को एक पंक्ति में बिठाना है। उन्हें कितने तरीकों से बिठाया जा सकता है जबकि एक ही परिवार के सदस्य अलग न हों ?

(1) 2!3!4! (2) $(3!)^3.(4!)$

(3) $(3!)^2.(4!)$ (4) $3!(4!)^3$

Official Ans. by NTA (2)

Sol.

Family 1	Family 2	Family 3
3	3	4

$= \underset{\text{Arrangement of 3 Families}}{3!} \quad \underset{\text{Interval Arrangement of families members}}{3!\times3!\times4!}$

so option(2) is correct.

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LIMIT-XII

3. $\lim_{x\to1}\left(\dfrac{\int_0^{(x-1)^2} t\cos(t^2)\,dt}{(x-1)\sin(x-1)}\right)$

(1) does not exist (2) is equal to $\frac{1}{2}$

(3) is equal to 1 (4) is equal to $-\frac{1}{2}$

3. $\lim_{x\to1}\left(\dfrac{\int_0^{(x-1)^2} t\cos(t^2)\,dt}{(x-1)\sin(x-1)}\right)$

(1) का अस्तित्व नहीं है। (2) $\frac{1}{2}$ के बराबर है।

(3) 1 के बराबर (4) $-\frac{1}{2}$ के बराबर है।

BONUS

Sol. $\lim_{x\to1}\dfrac{\int_0^{(x-1)^2} t\cos(t^2)\,dt}{(x-1)\sin(x-1)}\left(\dfrac{0}{0}\right)$

Apply L Hopital Rule

$$=\lim_{x\to1}\frac{2(x-1).(x-1)^2\cos(x-1)^4-0}{(x-1).\cos(x-1)+\sin(x-1)}\left(\frac{0}{0}\right)$$

$$=\lim_{x\to1}\frac{2(x-1)^3.\cos(x-1)^4}{(x-1)\left[\cos(x-1)+\dfrac{\sin(x-1)}{(x-1)}\right]}$$

$$=\lim_{x\to1}\frac{2(x-1)^2\cos(x-1)^4}{\left[\cos(x-1)+\dfrac{\sin(x-1)}{(x-1)}\right]}$$

$$=\lim_{x\to1}\frac{2(x-1)^2\cos(x-1)^4}{\cos(x-1)+\dfrac{\sin(x-1)}{(x-1)}}$$

on taking limit

$$=\frac{0}{1+1}=0$$

BINOMIAL THEOREM-XI

4. If $\{p\}$ denotes the fractional part of the number p, then $\left\{\dfrac{3^{200}}{8}\right\}$, is equal to

(1) $\frac{1}{8}$ (2) $\frac{5}{8}$ (3) $\frac{3}{8}$ (4) $\frac{7}{8}$

4. यदि $\{p\}$, संख्या p के भिन्नात्मक भाग (fractional part) को दर्शाता है, तो $\left\{\dfrac{3^{200}}{8}\right\}$, बराबर है :

(1) $\frac{1}{8}$ (2) $\frac{5}{8}$

(3) $\frac{3}{8}$ (4) $\frac{7}{8}$

Official Ans. by NTA (1)

Sol. $\left\{\dfrac{3^{200}}{8}\right\}=\left\{\dfrac{(3^2)^{100}}{8}\right\}$

$$=\left\{\frac{(1+8)^{100}}{8}\right\}$$

$$=\left\{\frac{1+{}^{100}C_1.8+{}^{100}C_2.8^2+...+{}^{100}C_{100}8^{100}}{8}\right\}$$

$$=\left\{\frac{1+8m}{8}\right\}$$

$$=\frac{1}{8}$$

DETERMINANT-XI

5. The values of λ and μ for which the system of linear equations

$x + y + z = 2$

$x + 2y + 3z = 5$

$x + 3y + \lambda z = \mu$

has infinitely many solutions are, respectively

(1) 5 and 7 (2) 6 and 8

5. λ तथा μ के क्रमशः मान, जिनके लिए समीकरण निकाय

$x + y + z = 2$

$x + 2y + 3z = 5$

$x + 3y + \lambda z = \mu$

के असंख्य हल हैं, हैं

(1) 5 तथा 7 (2) 6 तथा 8

(3) 4 तथा 9 (4) 5 तथा 8

Official Ans. by NTA (4)

Sol. For infinite many solutions

$D = D_1 = D_2 = D_3 = 0$

$$\text{Now } D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

$1.(2\lambda - 9) - 1.(\lambda - 3) + 1.(3 - 2) = 0$

$\therefore \lambda = 5$

$$\text{Now } D_1 = \begin{vmatrix} 2 & 1 & 1 \\ 5 & 2 & 3 \\ \mu & 3 & 5 \end{vmatrix} = 0$$

$2(10 - 9) - 1(25 - 3\mu) + 1(15 - 2\mu) = 0$

$\mu = 8$

AUC-XII

6. The area (in sq. units) of the region $A = \{(x,y) : |x| + |y| \le 1,\ 2y^2 \ge |x|\}$ is :

(1) $\frac{1}{6}$ (2) $\frac{1}{3}$ (3) $\frac{7}{6}$ (4) $\frac{5}{6}$

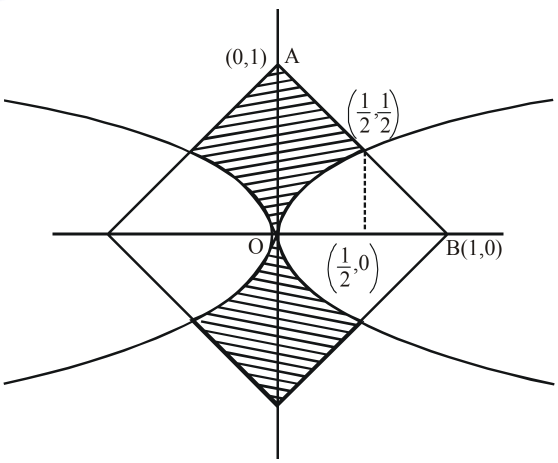
6. क्षेत्र $A = \{(x,y) : |x| + |y| \le 1,\ 2y^2 \ge |x|\}$ का क्षेत्रफल (वर्ग इकाइयों में)है :

(1) $\frac{1}{6}$ (2) $\frac{1}{3}$ (3) $\frac{7}{6}$ (4) $\frac{5}{6}$

Official Ans. by NTA (4)

Sol. $|x| + |y| \le 1$

$2y^2 \ge |x|$



For point of intersection

$x + y = 1 \Rightarrow x = 1 - y$

$y^2 = \frac{x}{2} \Rightarrow 2y^2 = x$

$2y^2 = 1 - y \Rightarrow 2y^2 + y - 1 = 0$

$(2y - 1)(y + 1) = 0$

$y = \frac{1}{2}$ or -1

Now Area of $\Delta OAB = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

Area of Region $R_1 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

Area of Region $R_2 = \frac{1}{\sqrt{2}} \int_0^{\frac{1}{2}} \sqrt{x}\, dx = \frac{1}{6}$

Now area of shaded region in first quadrant

$= \text{Area of } \Delta OAB - R_1 - R_2$

$= \frac{1}{2} - \left(\frac{1}{6}\right) - \left(\frac{1}{8}\right) = \frac{5}{24}$

So required area $= 4\left(\frac{5}{24}\right) = \frac{5}{6}$

so option (4) is correct.

PROABABILITY-XII

7. Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is :

(1) $\frac{15}{101}$ (2) $\frac{5}{101}$ (3) $\frac{5}{33}$ (4) $\frac{10}{99}$

7. 11 क्रमागत प्राकृत संख्याओं में से यदि तीन संख्याएँ यादृच्छया बिना प्रतिस्थापना के निकाली जाती हैं तो इन तीन संख्याओं के समांतर श्रेढ़ी, जिनका सार्वअन्तर धनात्मक है, में होने की प्रायिकता है :

(1) $\frac{15}{101}$ (2) $\frac{5}{101}$ (3) $\frac{5}{33}$ (4) $\frac{10}{99}$

Official Ans. by NTA (3)

Sol. Out of 11 consecutive natural numbers either 6 even and 5 odd numbers or 5 even and 6 odd numbers

when 3 numbers are selected at random then total cases = ${}^{11}C_3$

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Since these 3 numbers are in A.P. Let no's are a,b,c

2b ⇒ even number

$a + c \Rightarrow \begin{pmatrix} \text{even} + \text{even} \\ \text{odd} + \text{odd} \end{pmatrix}$

so favourable cases = ${}^6C_2 + {}^5C_2$
= 15 + 10 = 25

P(3 numbers are in A.P. $= \frac{25}{{}^{11}C_3} = \frac{25}{165} = \frac{5}{33}$)

STATISTICS-XII

8. If $\sum_{i=1}^{n}(x_i - a) = n$ and $\sum_{i=1}^{n}(x_i - a)^2 = na$, (n,a > 1) then the standard deviation of n observations $x_1, x_2, \ldots, x_n$ is

(1) $n\sqrt{a-1}$ (2) $\sqrt{a-1}$

(3) a – 1 (4) $\sqrt{n(a-1)}$

8. यदि $\sum_{i=1}^{n}(x_i - a) = n$ तथा $\sum_{i=1}^{n}(x_i - a)^2 = na$, (n,a > 1) हैं, तो n प्रेक्षणों $x_1, x_2, \ldots, x_n$ का मानक विचलन है :

(1) $n\sqrt{a-1}$ (2) $\sqrt{a-1}$

(3) a – 1 (4) $\sqrt{n(a-1)}$

Official Ans. by NTA (2)

Sol. S.D $= \sqrt{\frac{\sum_{i=1}^{n}(x_i - a)}{n} - \left(\frac{\sum_{i=1}^{n}(x_i - a)}{n}\right)^2}$

$= \sqrt{\frac{na}{n} - \left(\frac{n}{n}\right)^2}$

{Given $\sum_{i=1}^{n}(x_i - a) = n$ $\sum_{i=1}^{n}(x_i - a)^2 = na$ }

$\sqrt{\quad}$

PARABOLA-XI

9. Let L_1 be a tangent to the parabola $y^2 = 4(x + 1)$ and L_2 be a tangent to the parabola $y^2 = 8(x + 2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the straight line :

(1) x + 3 = 0 (2) x + 2y = 0

(3) 2x + 1 = 0 (4) x + 2 = 0

9. माना L_1, परवलय $y^2 = 4(x + 1)$ की एक स्पर्श रेखा है, तथा L_2, परवलय $y^2 = 8(x + 2)$ की एक स्पर्श रेखा है। यदि L_1 तथा L_2 परस्पर लंबवत प्रतिच्छेदन करती हैं, तो वे निम्न में से जिस रेखा पर मिलती हैं, वह है :

(1) x + 3 = 0 (2) x + 2y = 0

(3) 2x + 1 = 0 (4) x + 2 = 0

Official Ans. by NTA (1)

Sol. $y^2 = 4(x + 1)$

equation of tangent $y = m(x + 1) + \frac{1}{m}$

$y = mx + m + \frac{1}{m}$

$y^2 = 8(x + 2)$

equation of tangent $y = m'(x+2) + \frac{2}{m'}$

$y = m'x + 2\left(m' + \frac{1}{m'}\right)$

since lines intersect at right angles
∴ mm' = –1

Now $y = mx + m + \frac{1}{m}$ ...(1)

$y = m'x + 2\left(m' + \frac{1}{m'}\right)$

$y = -\frac{1}{m}x + 2\left(-\frac{1}{m} - m\right)$

$y = -\frac{1}{m}x - 2\left(m + \frac{1}{m}\right)$ ...(2)

From equation (1) and (2)

$$mx + m + \frac{1}{m} = -\frac{1}{m}x - 2\left(m + \frac{1}{m}\right)$$

$$\left(m + \frac{1}{m}\right)x + 3\left(m + \frac{1}{m}\right) = 0$$

$\therefore$ x + 3 = 0

LOGICAL REASONING-XII

10. The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to :

(1) $\sim p \vee \sim q$ (2) $\sim p \vee q$

(3) $\sim p \wedge \sim q$ (4) $p \wedge \sim q$

10. बूले के व्यंजक (Boolean expression) $p \vee (\sim p \wedge q)$ का निषेधन (Negation) निम्न में से किसके तुल्य है ?

(1) $\sim p \vee \sim q$ (2) $\sim p \vee q$

(3) $\sim p \wedge \sim q$ (4) $p \wedge \sim q$

Official Ans. by NTA (3)

Sol. Negation of $\phi \vee (\sim p \wedge q)$

$p \vee (\sim p \wedge q) = (p \vee \sim p) \wedge (p \vee q)$

$= (T) \wedge (p \vee q)$

$= (p \vee q)$

now negation of $(p \vee q)$ is

$\sim (p \vee q) = \sim p \wedge \sim q$

S.S.-XI

11. If $f(x + y) = f(x)\, f(y)$ and $\sum_{x=1}^{\infty} f(x) = 2, x, y \in N$, where N is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is

(1) $\frac{1}{9}$ (2) $\frac{4}{9}$

(3) $\frac{1}{3}$ (4) $\frac{2}{3}$

11. यदि $f(x + y) = f(x)\ f(y)$ तथा $\sum_{x=1}^{\infty} f(x) = 2, x, y \in N$, हैं, जहाँ N, सभी प्राकृत संख्याओं का समुच्चय है, तो $\frac{f(4)}{f(2)}$ का मान है :

(1) $\frac{1}{9}$ (2) $\frac{4}{9}$ (3) $\frac{1}{3}$ (4) $\frac{2}{3}$

Official Ans. by NTA (2)

Sol. $f(x + y) = f(x).\, f(y)$

$\sum_{x=1}^{\infty} f(x) = 2$ where x,y $\in$ N

$f(1) + f(2) + f(3) + ... \infty = 2$ ...(1) (Given)

Now for $f(2)$ put x = y = 1

$f(2) = f(1 + 1) = f(1).\, f(1) = (f(1))^2$

$f(3) = f(2 + 1) = f(2).\, f(1) = (f(1))^3$

Now put these values in equation (1)

$f(1) + (f(1))^2 + [f(1)^2 + ...\infty = 2]$

$$\frac{f(1)}{1 - f(1)} = 2$$

$$\Rightarrow f(1) = \frac{2}{3}$$

Now $f(2) = \left(\frac{2}{3}\right)^2$

$f(4) = \left(\frac{2}{3}\right)^4$

then the value of $\frac{f(4)}{f(2)} = \frac{\left(\frac{2}{3}\right)^4}{\left(\frac{2}{3}\right)^2} = \frac{4}{9}$

D.E.-XII

12. The general solution of the differential equation $\sqrt{1 + x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0$ is :

(where C is a constant of integration)

(1) $\sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C$

(2) $\sqrt{1+y^2}-\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C$

(3) $\sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right)+C$

(4) $\sqrt{1+y^2}-\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right)+C$

12. अवकल समीकरण

$\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$ का व्यापक हल है :

(जहाँ C एक समाकलन अचर है)

(1) $\sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C$

(2) $\sqrt{1+y^2}-\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C$

(3) $\sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right)+C$

(4) $\sqrt{1+y^2}-\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right)+C$

Official Ans. by NTA (1)

Sol. $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$

$\Rightarrow\sqrt{(1+x)^2(1+y^2)}+xy\frac{dy}{dx}=0$

$\Rightarrow\sqrt{1+x^2}\sqrt{1+y^2}=-xy\frac{dy}{dx}$

$\Rightarrow\int\frac{ydy}{\sqrt{1+y^2}}=-\int\frac{\sqrt{1+x^2}}{x}dx$ ...(1)

Now put $1+x^2=u^2$ and $1+y^2=v^2$

$2xdx=2udu$ and $2ydy=2vdv$

$\Rightarrow xdx=udu$ and $ydy=vdv$

substitude these values in equation (1)

$\int\frac{vdv}{v}=-\int\frac{u^2.du}{u^2-1}$

$\Rightarrow\int dv=-\int\frac{u^2-1+1}{u^2-1}du$

$\Rightarrow v=-\int\left(1+\frac{1}{u^2-1}\right)du$

$\Rightarrow v=-u-\frac{1}{2}\log_e\left|\frac{u-1}{u+1}\right|+c$

$\Rightarrow\sqrt{1+y^2}=-\sqrt{1+x^2}+\frac{1}{2}\log_e\left|\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right|+c$

$\Rightarrow\sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left|\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right|+c$

STRAIGHT LINE-XII

13. A ray of light coming from the point $(2,2\sqrt{3})$ is incident at an angle 30° on the line x=l at the point A. The ray gets reflected on the line x = 1 and meets x-axis at the point B. Then, the line AB passes through the point:

(1) $\left(3,-\frac{1}{\sqrt{3}}\right)$ (2) $(3,-\sqrt{3})$

(3) $\left(4,-\frac{\sqrt{3}}{2}\right)$ (4) $(4,-\sqrt{3})$

13. $(2,2\sqrt{3})$ से होकर आती हुई प्रकाश की एक किरण रेखा x = 1 पर 30° के कोण पर बिन्दु A पर आपतित (incident) होती है तथा रेखा x = 1 से प्रावर्तित हो कर x-अक्ष को बिंदु B पर मिलती है, तो रेखा AB निम्न में से किस बिन्दु से होकर जाती है :

(1) $\left(3,-\frac{1}{\sqrt{3}}\right)$ (2) $(3,-\sqrt{3})$

(3) $\left(4,-\frac{\sqrt{3}}{2}\right)$ (4) $(4,-\sqrt{3})$

Official Ans. by NTA (2)

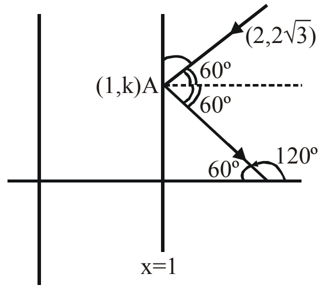
Sol. For point A

$$\tan 60° = \frac{2\sqrt{3}-k}{2-1}$$

$$\sqrt{3} = 2\sqrt{3} - k$$

$$\therefore \; k = \sqrt{3}$$

so point $A\left(1,\sqrt{3}\right)$



Now slope of line AB is m_{AB} = tan120°

$$m\, m_{AB} = -\sqrt{3}$$

Now equation of line AB is

$$y - \sqrt{3} = -\sqrt{3}\left(x-1\right)$$

$$\sqrt{3}x + y = 2\sqrt{3}$$

Now satisfy options

S.S.-XI

14. Let a,b,c,d and p be any non zero distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$. Then :

(1) a,c,p are in G.P. (2) a,c,p are in A.P.

(3) a,b,c,d are in G.P. (4) a,b,c,d are in A.P.

14. यदि a,b,c,d तथा p कोई भी अशून्य वास्तविक संख्याएँ हैं, कि $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$, है, तो :

(1) a,c,p समांतर श्रेढ़ी में हैं।

(2) a,c,p गुणोत्तर श्रेढ़ी में हैं।

(3) a,b,c,d समांतर श्रेढ़ी में हैं।

(4) a,b,c,d गुणोत्तर श्रेढ़ी में हैं।

Official Ans. by NTA (3)

Sol. $(a^2 + b^2 + c^2)p^2 + 2(ab + bc + cd)p + b^2 + c^2 + d^2 = 0$

$\Rightarrow (a^2p^2 + 2abp + b^2) + (b^2p^2 + 2bcp + c^2) + (c^2p^2 + 2cdp + d^2) = 0$

$\Rightarrow (ab + b)^2 + (bp + c)^2 + (cp + d)^2 = 0$

This is possible only when

ap + b = 0 and bp + c = 0 and cp + d = 0

$$p = -\frac{b}{a} = -\frac{c}{b} = -\frac{d}{c}$$

$$\text{or } \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

$\therefore$ a,b,c,d are in G.P.

D.I.-XII

15. If $I_1 = \int_0^1 \left(1-x^{50}\right)^{100} dx$ and $I_2 = \int_0^1 \left(1-x^{50}\right)^{101} dx$ such that $I_2 = \alpha I_1$ then α equals to

(1) $\frac{5050}{5051}$ (2) $\frac{5050}{5049}$

(3) $\frac{5049}{5050}$ (4) $\frac{5051}{5050}$

15. यदि $I_1 = \int_0^1 \left(1-x^{50}\right)^{100} dx$ तथा $I_2 = \int_0^1 \left(1-x^{50}\right)^{101} dx$ है जिन के लिए $I_2 = \alpha I_1$ है, तो α बराबर है :

(1) $\frac{5050}{5051}$ (2) $\frac{5050}{5049}$ (3) $\frac{5049}{5050}$ (4) $\frac{5051}{5050}$

Official Ans. by NTA (1)

Sol. $I_1 = \int_0^1 \left(1-x^{50}\right)^{100} dx$ and $I_2 = \int_0^1 \left(1-x^{50}\right)^{101} dx$

and $I_1 = \lambda I_2$

$$I_2 = \int_0^1 \left(1-x^{50}\right)^{101} dx$$

$$I_2 = \int_0^1 \left(1-x^{50}\right)\left(1-x^{50}\right)^{100} dx$$

$$I_2 = \int_0^1 \left(1-x^{50}\right)dx - \int_0^1 x^{50}.\left(1-x^{50}\right)^{100} dx$$

$$I_2 = I_1 - \int_0^1 \underbrace{x}_{I}.\underbrace{x^{49}.\left(1-x^{50}\right)^{100} dx}_{II}$$

Now apply IBP

$$I_2 = I_1 - \left[x\int x^{49}.(1-x^{50})^{100}dx - \int \frac{d(x)}{dx}.\int \frac{d(x)}{dx}.\int x^{49}.(1-x^{50})^{100}dx\right]$$

Let $(1 - x^{50}) = t$

$-50x^{49}dx = dt$

$$I_2 = I_1 - \left[x.\left(-\frac{1}{50}\right)\frac{(1-x^{50})^{101}}{101}\Bigg|_{x=0}^{x=1} - \int_0^1 \left(-\frac{1}{50}\right)\frac{(1-x^{50})^{101}}{101}dx\right]$$

$$I_2 = I_1 - 0 - \frac{1}{50}.\frac{1}{101}.I_2 = I_1 - \frac{1}{5050}I_2$$

$$I_2 + \frac{1}{5050}I_2 = I_1 \Rightarrow \frac{5051}{5050}I_2 = I_1$$

$$\therefore\ \alpha = \frac{5050}{5051}$$

$$I_2 = \frac{5050}{5051}I_1$$

$\because\ I_2 = \alpha.I_1$

AOD-XII

16. The position of a moving car at time t is given by $f(t) = at^2 + bt + c$, $t > 0$, where a, b and c are real numbers greater than 1. Then the average speed of the car over the time interval $[t_1,t_2]$ is attained at the point :

(1) $a(t_2 - t_1) + b$ (2) $(t_2 - t_1)/2$

(3) $2a(t_1 + t_2) + b$ (4) $(t_1 + t_2)/2$

16. एक गतिशील कार की t समय पर स्थिति (position) $f(t) = at^2 + bt + c$, $t > 0$ द्वारा दी गई है, जहाँ $a > 1$, $b > 1$ तथा $c > 1$ वास्तविक संख्याएँ हैं, तो समय अंतराल $[t_1,t_2]$ में कार की औसत गति निम्न में से किस बिन्दु पर प्राप्त होती है ?

(1) $a(t_2 - t_1) + b$ (2) $(t_2 - t_1)/2$

(3) $2a(t_1 + t_2) + b$ (4) $(t_1 + t_2)/2$

Official Ans. by NTA (4)

Sol. $$\frac{f(t_2) - f(t_1)}{t_2 - t_1} = 2at + b$$

$$\frac{a(t_2^2 - t_1^2) + b(t_2 - t_1)}{} = 2at + b$$

$\Rightarrow a(t_2 + t_1) + b = 2at + b$

$$\Rightarrow\ t = \frac{t_1 + t_2}{2}$$

COMPLEX NUMBER-XII

17. The region represented by $\{z = x + iy \in C : |z| - Re(z) \le 1\}$ is also given by the inequality :

(1) $y^2 \ge x + 1$ (2) $y^2 \ge 2(x + 1)$

(3) $y^2 \le x + \frac{1}{2}$ (4) $y^2 \le 2\left(x + \frac{1}{2}\right)$

17. $\{z = x + iy \in C : |z| - Re(z) \le 1\}$ द्वारा निरुपित क्षेत्र निम्न में से किस असमता द्वारा भी दिया जाता है :

(1) $y^2 \ge x + 1$ (2) $y^2 \ge 2(x + 1)$

(3) $y^2 \le x + \frac{1}{2}$ (4) $y^2 \le 2\left(x + \frac{1}{2}\right)$

Official Ans. by NTA (4)

Sol. $z = x + iy$

$|z| - ke(z) \le 1$

$$\Rightarrow \sqrt{x^2 + y^2} - x \le 1$$

$$\Rightarrow \sqrt{x^2 + y^2} \le 1 + x$$

$$\Rightarrow x^2 + y^2 \le 1 + 2x + x^2$$

$$\Rightarrow y^2 \le 2x + 1$$

$$\Rightarrow y^2 \le 2\left(x + \frac{1}{2}\right)$$

QUADRATIC EQUATION-XII

18. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$.

Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is

(1) 1 (2) 3 (3) 4 (4) 2

18. यदि α तथा β, समीकरण $x^2 - 64x + 256 = 0$ के दो मूल

$\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$:

(1) 1 (2) 3 (3) 4 (4) 2

Official Ans. by NTA (4)

Sol. $x^2 - 64x + 256 = 0$

$\alpha + \beta = 64,\ \alpha\beta = 256$

$$\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$$

$$= \frac{\alpha^{3/8}}{\beta^{5/8}} + \frac{\beta^{3/8}}{\alpha^{5/8}}$$

$$= \frac{\alpha+\beta}{(\alpha\beta)^{5/8}}$$

$$= \frac{64}{(256)^{5/8}}$$

$= 2$

3D-XII

19. The shortest distance between the lines $\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$ and $x + y + z + 1 = 0$, $2x - y + z + 3 = 0$ is :

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{\sqrt{3}}$

19. रेखाओं $\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$ तथा $x + y + z + 1 = 0$, $2x - y + z + 3 = 0$ के बीच की न्यूनतम दूरी है:

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{\sqrt{3}}$

Official Ans. by NTA (4)

Sol. Line of intersection of planes

$x + y + z + 1 = 0$ ...(1)

$2x - y + z + 3 = 0$ ...(2)

eliminate y

$3x + 2z + 4 = 0$

$$x = \frac{-2z-4}{3} \quad ...(3)$$

put in equaiton (1)

$z = -3y + 1$ ...(4)

from (3) and (4)

$$\frac{3x+4}{-2} = -3y + 1 = z$$

$$\frac{x-\left(-\frac{4}{3}\right)}{-\frac{2}{3}} = \frac{y-\frac{1}{3}}{-\frac{1}{3}} = \frac{z-0}{1}$$

now shortest distance between skew lines

$$\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$$

$$\frac{x-\left(-\frac{4}{3}\right)}{-\frac{2}{3}} = \frac{y-\left(\frac{1}{3}\right)}{-\frac{1}{3}} = \frac{z-0}{1}$$

$$S.D. = \left|\frac{(\vec{b}-\vec{a}).(\vec{c}\times\vec{d})}{|\vec{c}\times\vec{d}|}\right|$$

where $\vec{a} = (1,-1,0)$

$\vec{b} = \left(-\frac{4}{3}, \frac{1}{3}, 0\right)$

$\vec{c} = (0,-1,1)$

$\vec{d} = \left(-\frac{2}{3}, -\frac{1}{3}, 1\right)$

$\Rightarrow S.D = \frac{1}{\sqrt{3}}$

DETERMINANT-XI

20. Let m and M be respectively the minimum and maximum values of

$$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}.$$ Then the ordered pair (m,M) is equal to

(1) (–3,–1) (2) (–4,–1)

(3) (1,3) (4) (–3,3)

20. माना m तथा M

$$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$$ के, क्रमश:

(m,M)

:

(1) (–3,–1) (2) (–4,–1)

Official Ans. by NTA (1)

Sol. $\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$

$R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$\begin{vmatrix} -1 & 1 & 0 \\ 1 & 0 & -1 \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$

$= -1\left(\sin^2 x\right) - 1\left(1+\sin 2x+\cos^2 x\right)$

$= -\sin 2x - 2$

$m = -3,\ M = -1$

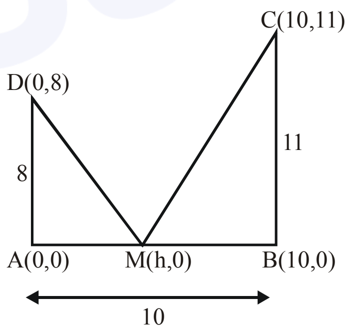
AOD-XII

21. Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If AD = 8 m, BC = 11 m and AB = 10 m; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is_.

21. माना AD तथा BC क्षैतिज समतल भूमि पर क्रमश: A तथा B पर सीधे खड़े दो खम्भे हैं। यदि AD = 8 मी., BC = 11 मी. तथा AB = 10 मी. है, तो AB पर स्थित एक बिंदु M की, बिंदु A से वह दूरी (मीटरों में) जिसके लिए $MD^2 + MC^2$ का मान न्यूनतम है, है _______.

Official Ans. by NTA (5.00)

Sol.



$(MD)^2 + (MC)^2 = h^2 + 64 + (h - 10)^2 + 121$

$= 2h^2 - 20h + 64 + 100 + 121$

$= 2(h^2 - 10h) + 285$

$= 2(h - 5)^2 + 235$

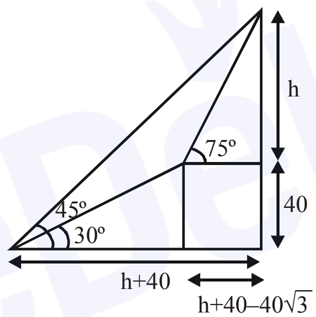
HEIGHT & DISTANCE-XI

22. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is_.

22. एक पहाड़ की चोटी का इसके पाद से हो कर जाने वाले क्षैतिज समतल पर स्थित एक बिंदु पर उन्नयन कोण 45° पाया गया। इस बिंदु से क्षैतिज तल से 30° का कोण बनाते हुए तल पर पहाड़ की चोटी की ओर 80 मीटर चलने के बाद चोटी का उन्नयन कोण 75° हो जाता है, तो पहाड़ की ऊँचाई (मीटरों में) है_________.

Official Ans. by NTA (80.00)

Sol.



$\tan 75^\circ = \dfrac{h}{h+40-40\sqrt{3}}$

$\dfrac{2+\sqrt{3}}{1} = \dfrac{h}{h+40-40\sqrt{3}}$

$\Rightarrow 2h + 80 - 80\sqrt{3} + \sqrt{3}h + 40\sqrt{3} - 120 = h$

$\Rightarrow h\left(\sqrt{3}+1\right) = 40 + 40\sqrt{3}$

$\Rightarrow h = 40$

$\therefore$ Height of hill = 40 + 40 = 80m

SET-XI

23. Set A has m elements and Set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of m.n is ___________.

23. समुच्चय A में m अवयव हैं तथा समुच्चय B में n अवयव हैं। यदि A के सभी उपसमुच्चयों की संख्या, B के सभी उपसमुच्चयों की संख्या से 112 अधिक है, तो m.n का मान है________.

Official Ans. by NTA (28.00)

Sol. $2^m - 2^n = 112$

$m = 7, n = 4$

$(2^7 - 2^4 = 112)$

$m \times n = 7 \times 4 = 28$

VECTOR-XII

24. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|$ is _.

24. यदि $\vec{a}$ तथा $\vec{b}$ एकक सदिश है तो $\sqrt{3}|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|$ का अधिकतम मान है _____.

Official Ans. by NTA (4.00)

Sol. $\sqrt{3}|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|$

$= \sqrt{3}\left(\sqrt{2+2\cos\theta}\right)+\sqrt{2-2\cos\theta}$

$= \sqrt{6}\left(\sqrt{1+\cos\theta}\right)+\sqrt{2}\left(\sqrt{1-\cos\theta}\right)$

$= 2\sqrt{3}\left|\cos\frac{\theta}{2}\right|+2\left|\sin\frac{\theta}{2}\right|$

$\leq \sqrt{\left(2\sqrt{3}\right)^2+(2)^2} = 4$

CONTINUITY & DIFFERENTIABILITY-XII

25. Let $f : R \to R$ be defined as

$$f(x)=\begin{cases} x^5\sin\left(\frac{1}{x}\right)+5x^2 & , \quad x<0 \\ 0 & , \quad x=0 \\ x^5\cos\left(\frac{1}{x}\right)+\lambda x^2 & , \quad x>0 \end{cases}$$

. The value of λ for which $f''(0)$ exists, is _.

25. माना $f : R \to R$

$$f(x)=\begin{cases} x^5\sin\left(\frac{1}{x}\right)+5x^2 & , \quad x<0 \\ 0 & , \quad x=0 \\ x^5\cos\left(\frac{1}{x}\right)+\lambda x^2 & , \quad x>0 \end{cases}$$

द्वारा परिभाषित है। λ का मान जिसके लिए $f''(0)$ का अस्तित्व है, है__________.

Official Ans. by NTA (5.00)

Sol. $f(x)=x^5.\sin\frac{1}{x}+5x^2 \quad$ if $x<0$

$f(x)=0 \quad$ if $x=0$

$f(x)=x^5.\cos\frac{1}{x}+\lambda x^2 \quad$ if $x>0$

LHD of $f'(x)$ at $x = 0$ is 10

RHD of $f'(x)$ at $x = 0$ is 2λ

if $f''(0)$ exists then

$2\lambda = 10$

$\Rightarrow \lambda = 5$