FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Tuesday 07th JANUARY, 2020) TIME : 2 : 30 PM to 5 : 30 PM

PHYSICS — TEST PAPER WITH ANSWER & SOLUTION

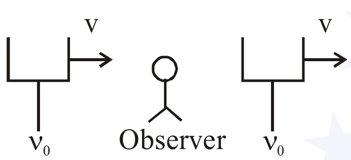
1. A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much less than the speed of sound). The observer hears 2 beats/sec. The oscillation frequency of each tuning fork is ν_0 = 1400 Hz and the velocity of sound in air is 350 m/s. The speed of each tuning fork is close to :

(1) $\frac{1}{8}$ m/s (2) $\frac{1}{2}$ m/s

(3) 1 m/s (4) $\frac{1}{4}$ m/s

NTA Ans. (4)

Sol.



$$\nu_1 = \left(\frac{c}{c-v}\right)\nu_0$$

$$\nu_2 = \left(\frac{c}{c+v}\right)\nu_0$$

beat frequency = $\nu_1 - \nu_2$

$$= c\nu_0 \left(\frac{1}{c-v} - \frac{1}{c+v}\right)$$

$$= c\nu_0\left(\frac{c+v-c+v}{c^2-v^2}\right) = \frac{2c\nu_0^2 v}{c^2-v^2}$$

$$\approx \frac{2c\nu_0 v}{c^2} = \frac{2\nu_0 v}{c} = 2$$

$$\Rightarrow \frac{2\times1400\times v}{350} = 2$$

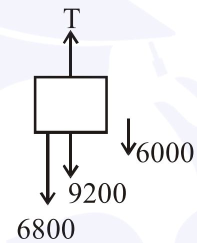
2. An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed 3 m/s. The frictional force opposing the motion is 6000 N. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator (g = 10 m/s^2) must be at least :

(1) 56300 W (2) 48000 W

(3) 66000 W (4) 62360 W

NTA Ans. (3)

Sol.



elevater moving with constant speed hence

T = 6800 + 9200 + 6000

T = 22000 N

power = T·v = 22000 × 3

= 66000 W

3. The activity of a radioactive sample falls from 700 s^{-1} to 500 s^{-1} in 30 minutes. Its half life is close to :

(1) 66 min (2) 52 min

(3) 72 min (4) 62 min

NTA Ans. (4)

Sol. $A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$

$$500 = 700 \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$$0.7 \approx \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$$\left(\frac{1}{2}\right)^{1/2} \approx \frac{t}{T_{1/2}}$$

30 1

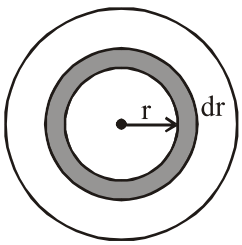
4. Mass per unit area of a circular disc of radius a depends on the distance r from its centre as $\sigma(r) = A + Br$. The moment of inertia of the disc about the the axis, perpendicular to the plane and passing through its centre is :

(1) $2\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$ (2) $\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$

(3) $2\pi a^4\left(\frac{aA}{4}+\frac{B}{5}\right)$ (4) $2\pi a^4\left(\frac{A}{4}+\frac{B}{5}\right)$

NTA Ans. (1)

Sol.



$dI = dm r^2$

$dI = \sigma 2\pi r \, dr \, r^2$

$dI = 2\pi(A + Br) r^3 \, dr$

$$\int dI = 2\pi\int_0^a \left(Ar^3 + Br^4\right)dr$$

$$I = 2\pi a^4\left(\frac{A}{4}+\frac{B9}{5}\right)$$

5. The electric field of a plane electromagnetic wave is given by $\vec{E} = E_0\frac{\hat{i}+\hat{j}}{\sqrt{2}}\cos(kz + \omega t)$

At t = 0, a positively charged particle is at the point (x, y, z) = $\left(0,0,\frac{\pi}{k}\right)$. If its instantaneous velocity at (t = 0) is $\upsilon_0\,\hat{k}$, the force acting on it due to the wave is :

(1) zero (2) parallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

(3) antiparallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (4) parallel to $\hat{k}$

Sol. $\vec{F} = q(\vec{E}+\vec{v}\times\vec{B})$

$$\vec{E} = E_0\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)\cos\pi$$

$$= -E_0\,\frac{\hat{i}+\hat{j}}{\sqrt{2}}$$

as $\vec{E}\times\vec{B} = \vec{c}$

$$+E_0\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)\times\vec{B} = c\hat{k}$$

$$\Rightarrow \vec{B} = -\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)\frac{E_0}{c}$$

$$\vec{F} = q\left(-E_0\frac{(\hat{i}+\hat{j})}{\sqrt{2}} - \frac{v_0\hat{k}}{c}\times\left(\hat{i}-\hat{j}\right)E_0\right)$$

since $\frac{v_0}{c} << 1$

$\Rightarrow$ F is antiparallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

6. A particle of mass m and charge q has an initial velocity $\vec{\upsilon} = \upsilon_0\hat{j}$. If an electric field $\vec{E} = E_0\hat{i}$ and magnetic field $\vec{B} = B_0\hat{i}$ act on the particle, its speed will double after a time:

(1) $\frac{2m\upsilon_0}{qE_0}$ (2) $\frac{3m\upsilon_0}{qE_0}$

(3) $\frac{\sqrt{3}m\upsilon_0}{qE_0}$ (4) $\frac{\sqrt{2}m\upsilon_0}{qE_0}$

NTA Ans. (3)

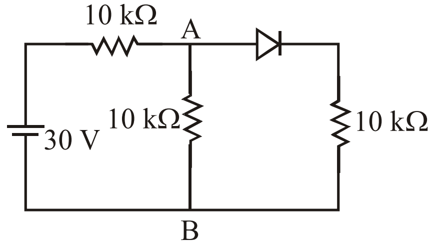
Sol. $(2V_0)^2 = v_0^2 + \upsilon_x^2$

$v_x = \sqrt{3}\, v_0$

$$\sqrt{3}\, v_0 = 0 + \frac{qE_0}{m}t$$

$\sqrt{3}v$ m

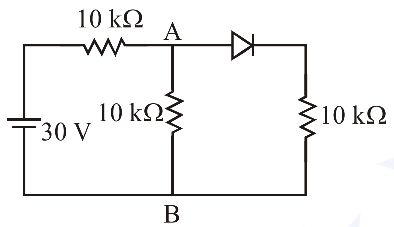
7. In the figure, potential difference between A and B is :

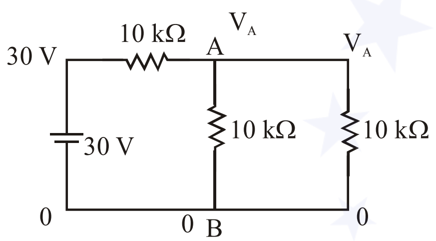


(1) 5V (2) 10 V
(3) zero (4) 15 V

NTA Ans. (2)

Sol.





$$\frac{30-V_A}{10}+\frac{0-V_A}{10}+\frac{0-V_A}{10}=0$$

$$3=\frac{3V_A}{10}$$

$V_A = 10$ V

8. The dimension of $\frac{B^2}{2\mu_0}$, where B is magnetic field and μ_0 is the magnetic permeability of vacuum, is:

(1) $ML^{-1}T^{-2}$ (2) ML^2T^{-1}
(3) MLT^{-2} (4) ML^2T^{-2}

NTA Ans. (1)

Sol. Magnetic energy stored per unit volume is

$$\frac{B^2}{2\mu_0}$$

9. In a building there are 15 bulbs of 45 W, 15 bulbs of 100 W, 15 small fans of 10 W and 2 heaters of 1 kW. The voltage of electric main is 220 V. The minimum fuse capacity (rated value) of the building will be:

(1) 10 A (2) 25 A (3) 15 A (4) 20 A

NTA Ans. (4)

Sol. $220\,I = P = 15\times45+15\times100+15\times10+2\times10^3$

$$I=\frac{4325}{220}=19.66$$

$I \simeq 20$ A

10. An emf of 20 V is applied at time t=0 to a circuit containing in series 10 mH inductor and 5 Ω resistor. The ratio of the currents at time $t=\infty$ and at t = 40 s is close to : (Take $e^2 = 7.389$)

(1) 1.06 (2) 1.15
(3) 1.46 (4) 0.84

NTA Ans. (1)

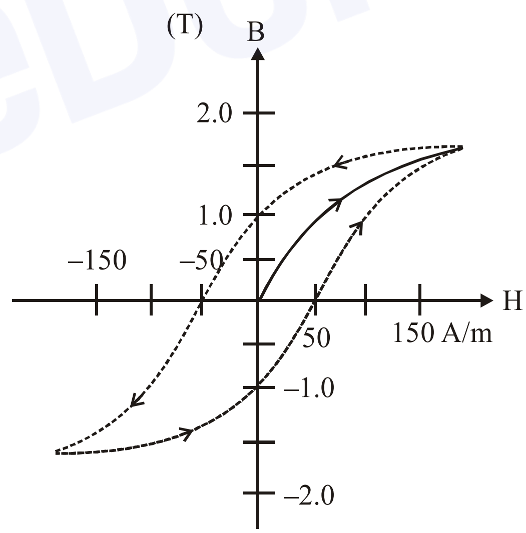
Ans. (2)

Sol. $i = i_0\,(1-e^{-Rt/L})$

$$\frac{i_0}{i}=\frac{1}{1-e^{-2\times10^4}}$$

$$\frac{i_0}{i}\simeq 1$$

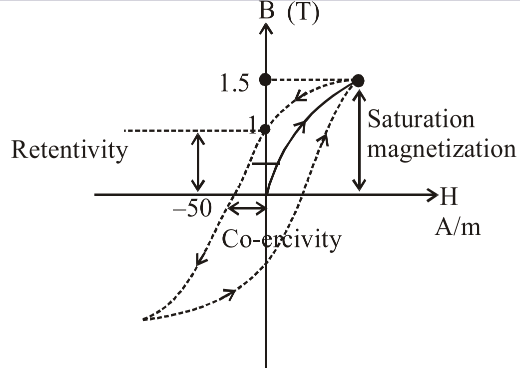
11.



The figure gives experimentally measured B vs. H variation in a ferromagnetic material. The retentivity, co-ercivity and saturation, respectively, of the material are:

(1) 150 A/m, 1.0 T and 1.5 T
(2) 1.0 T, 50 A/m and 1.5 T
(3) 1.5 T, 50 A/m and 1.0 T

Sol.



Retentivity = 1.0 T

Co-ercivity = 50 A/m

Saturation = 1.5 T

12. In a Young's double slit experiment, the separation between the slits is 0.15 mm. In the experiment, a source of light of wavelength 589 nm is used and the interference pattern is observed on a screen kept 1.5 m away. The separation between the successive bright fringes on the screen is:

(1) 6.9 mm (2) 5.9 mm

(3) 4.9 mm (4) 3.9 mm

NTA Ans. (2)

Sol. Finge width, $\beta = \frac{D\lambda}{d} = \frac{1.5 \times 589 \times 10^{-9}}{0.15 \times 10^{-3}}$

$= 5.9 \times 10^{-3}$ m

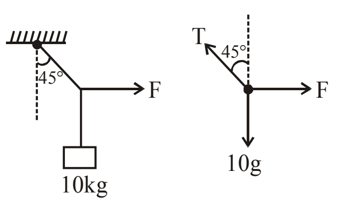
= 5.9 mm

13. A mass of 10 kg is suspended by a rope of length 4 m, from the ceiling. A force F is applied horizontally at the mid-point of the rope such that the top half of the rope makes an angle of 45° with the vertical. Then F equals: (Take g= 10 ms^{-2} and the rope to be massless)

(1) 100 N (2) 90 N (3) 75 N (4) 70 N

NTA Ans. (1)

Sol.



For equilibrium,

T sin 45° = F ....(1)

and T cos 45° = 10g ....(2)

14. A thin lens made of glass (refractive index = 1.5) of focal length f = 16 cm is immersed in a liquid of refractive index 1.42. If its focal length in liquid is f_1, then the ratio f_1/f is closest to the integer :

(1) 1 (2) 5 (3) 9 (4) 17

NTA Ans. (3)

Sol. Using $\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\frac{1}{f} = \left(\frac{1.5}{1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad ....(1)$$

and $$\frac{1}{f_1} = \left(\frac{1.5}{1.42} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad ....(2)$$

equation (1)/(2),

we get $\frac{f_1}{f} = \frac{0.5}{0.056}$

$= 8.93 \approx 9$

15. A planar loop of wire rotates in a uniform magnetic field. Initially, at t = 0, the plane of the loop is perpendicular to the magnetic field. If it rotates with a period of 10 s about an axis in its plane then the magnitude of induced emf will be maximum and minimum, respectively at :

(1) 2.5 s and 7.5 s (2) 5.0 s and 7.5s

(3) 5.0 s and 10.0 s (4) 2.5s and 5.0 s

NTA Ans. (4)

Sol. Flux $\phi = \vec{B}\cdot\vec{A} = BA\cos\theta = BA\cos\omega t$

$|\text{Induced emf}| = |e| = \left|\frac{d\phi}{dt}\right| = |BA\omega \sin \omega t|$

|e| will be maximum at $\omega t = \frac{\pi}{2}$

$$\left(\frac{2\pi}{T}\right)t = \frac{\pi}{2}$$

$$\left(\frac{2\pi}{10}\right)t = \frac{\pi}{2} \Rightarrow t = 2.5 \text{ sec}$$

|e| will be minimum at $\omega t = \pi$

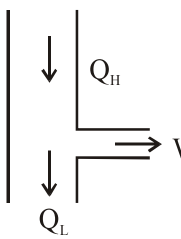
16. Two ideal Carnot engines operate in cascade (all heat given up by one engine is used by the other engine to produce work) between temperatures, T_1 and T_2. The temperature of the hot reservoir of the first engine is T_1 and the temperature of the cold reservoir of the second engine is T_2. T is temperature of the sink of first engine which is also the source for the second engine. How is T related to T_1 and T_2, if both the engines perform equal amount of work ?

(1) $T = \frac{2T_1T_2}{T_1 + T_2}$ (2) $T = \sqrt{T_1T_2}$

(3) $T = \frac{T_1 + T_2}{2}$ (4) $T = 0$

NTA Ans. (3)

Sol.



$\frac{Q_H}{Q_L} = \frac{T_1}{T}$ and $W = Q_H - Q_L$ ....(1)

$\frac{Q_L}{Q'_L} = \frac{T}{T_2}$ and $W = Q_L - Q'_L$ ....(2)

17. A box weighs 196 N on a spring balance at the north pole. Its weight recorded on the same balance if it is shifted to the equator is close to (Take $g = 10\ ms^{-2}$ at the north pole and the radius of the earth = 6400 km):

(1) 195.66 N (2) 194.66 N
(3) 194.32 N (4) 195.32 N

NTA Ans. (4)

Sol. $W = 196 - m\omega^2R$

18. Under an adiabatic process, the volume of an ideal gas gets doubled. Consequently the mean collision time between the gas molecule changes from τ_1 to τ_2. If $\frac{C_p}{C_v} = \gamma$ for this gas then a good estimate for $\frac{\tau_2}{\tau_1}$ is given by :

(1) $\left(\frac{1}{\ }\right)^{\frac{\gamma+1}{2}}$ (2) 2 (3) $\frac{1}{\ }$ (4) $\left(\frac{1}{\ }\right)^{\gamma}$

Sol. $t \propto \frac{V}{\sqrt{T}}$ ....(1)

$TV^{\gamma-1}$ = constant ....(2)

$\therefore t \propto V^{\frac{\gamma+1}{2}}$

19. An ideal fluid flows (laminar flow) through a pipe of non-uniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and the maximum velocities of fluid in this pipe is:

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{3}{4}$ (3) $\frac{81}{256}$ (4) $\frac{9}{16}$

NTA Ans. (4)

Sol. $A_1v_1 = A_2v_2$

$\frac{v_{min}}{v_{max}} = \frac{A_{min}}{A_{max}}$

$\frac{v_{min}}{v_{max}} = \left(\frac{4.8}{6.4}\right)^2$

$\frac{v_{min}}{v_{max}} = \frac{9}{16}$

20. An electron (of mass m) and a photon have the same energy E in the range of a few eV. The ratio of the de-Broglie wavelength associated with the electron and the wavelength of the photon is (c = speed of light in vacuum)

(1) $\left(\frac{E}{2m}\right)^{1/2}$ (2) $\frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$

(3) $c(2mE)^{1/2}$ (4) $\frac{1}{c}\left(\frac{2E}{m}\right)^{1/2}$

NTA Ans. (2)

20. $\frac{\lambda_{electron}}{\lambda_{photon}} = ?$

$E = \frac{hc}{\lambda_{photon}}$ ....(1)

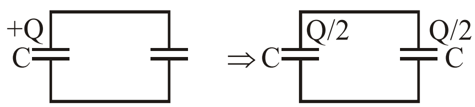
$\lambda_{electron} = \frac{h}{\sqrt{2mE}}$ ....(2)

from (1) and (2)

λ $^{1/2}$

21. A 60 pF capacitor is fully charged by a 20 V supply. It is then disconnected from the supply and is connected to another uncharged 60 pF capaction is parallel. The electrostatic energy that is lost in this process by the time the charge is redistributed between them is (in nJ) ____.

NTA Ans. (6)

Sol. 

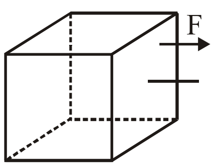
$Q = CV$

$$\Delta Q_L = \frac{Q^2}{2C} - \left[\frac{(Q/2)^2}{2C} \times 2\right] = \frac{Q^2}{4C}$$

$$= \frac{1}{4}CV^2$$

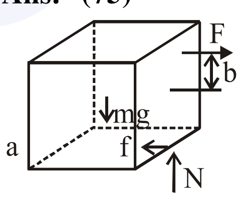
$$= \frac{1}{4} \times 60 \times 10^{-12} \times 4 \times 10^2$$

$$= 6\text{nJ}$$

22. 

Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100 \times \frac{b}{a}$ for a box not to topple before moving is _______ .

NTA Ans. (75)

Sol. 

$F = \mu mg$ ...(1)

$F\left(b + \frac{a}{2}\right) = mg\ \frac{a}{2}$ ...(2)

$$\mu mg \left(b + \frac{a}{2}\right) = mg \times \frac{a}{2}$$

$$\left(b + \frac{a}{2}\right) \mu = \frac{a}{2}$$

$$0.4 = \mu = \frac{a}{2b + a}$$

$$0.8b + 0.4a = a$$

$$0.8\ b = 0.6\ a$$

$$\frac{b}{a} = \frac{3}{4}$$

23. The balancing length for a cell is 560 cm in a potentiometer experiment. When an external resistance of 10 Ω is connected in parallel to the cell, the balancing length changes by 60cm. If the internal resistance of the cell is $\frac{N}{10}\Omega$, where N is an integer then value of N is____ .

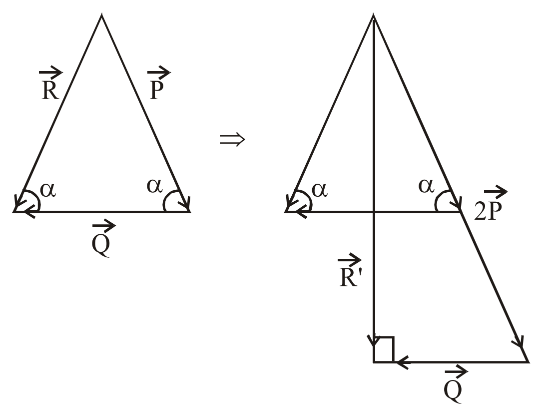
NTA Ans. (12)

Sol. $r = R\left(\frac{x - x'}{x'}\right)$

$$= 10 \times \frac{60}{500} = 12$$

24. The sum of two forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$ such that $|\vec{R}| = |\vec{P}|$. The angle θ (in degrees) that the resultant of $2\vec{P}$ and $\vec{Q}$ will make with $\vec{Q}$ is, ____ .

NTA Ans. (90)

Sol. 

Hence angle 90°

25. M grams of steam at 100°C is mixed with 200 g of ice at its melting point in a thermally insulated container. If it produces liquid water at 40°C [heat of vaporization of water is 540 cal/ g and heat of fusion of ice is 80 cal/g], the value of M is______.

NTA Ans. (40)

Sol. $M \times 540 + M + 60 = 200 \times 80 + 200 \times 1 \times (40 - 0)$

$\Rightarrow M = 40$