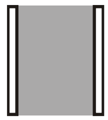
FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Tuesday 07th JANUARY, 2020) TIME : 9 : 30 AM to 12 : 30 PM

PHYSICS

TEST PAPER WITH ANSWER & SOLUTION

1. A parallel plate capacitor has plates of area A separated by distance 'd' between them. It is filled with a dielectric which has a dielectric constant that varies as $k(x) = K(1 + \alpha x)$ where 'x' is the distance measured from one of the plates. If $(\alpha d) << 1$, the total capacitance of the system is best given by the expression :



(1) $\frac{AK\varepsilon_0}{d}\left(1+\frac{\alpha d}{2}\right)$ (2) $\frac{A\varepsilon_0 K}{d}\left(1+\left(\frac{\alpha d}{2}\right)^2\right)$

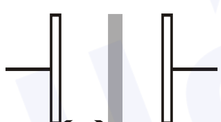
(3) $\frac{A\varepsilon_0 K}{d}\left(1+\frac{\alpha^2 d^2}{2}\right)$ (4) $\frac{AK\varepsilon_0}{d}(1+\alpha d)$

NTA Ans. (1)

Sol. As K is variable we take a plate element of Area A and thickness dx at distance x

Capacitance of element

$$dC = \frac{(A)K(1+\alpha x)\varepsilon_0}{dx}$$



Now all such elements are is series so equivalent capacitance

$$\frac{1}{C} = \int \frac{1}{dC} = \int_0^d \frac{dx}{AK\varepsilon_0(1+\alpha x)}$$

$$\frac{1}{C} = \frac{1}{\alpha AK\varepsilon_0} \ln\left(\frac{1+\alpha d}{1}\right)$$

$$= \frac{1}{C} = \frac{1}{\alpha AK\varepsilon_0}\left(\alpha d - \frac{(\alpha d)^2}{2} + \frac{(\alpha d)^3}{3} + ...\right)$$

$$\Rightarrow \frac{1}{C} = \frac{\alpha d}{\alpha AK\varepsilon_0}\left(1 - \frac{\alpha d}{2} + \frac{(\alpha d)^2}{3} + ...\right)$$

$$\frac{1}{C} = \frac{d}{AK\varepsilon_0}\left(1 - \frac{\alpha d}{2}\right)$$

2. The time period of revolution of electron in its ground state orbit in a hydrogen atom is 1.6×10^{-16} s. The frequency of revolution of the electron in its first excited state (in s^{-1}) is:

(1) 6.2×10^{15} (2) 5.6×10^{12}

(3) 7.8×10^{14} (4) 1.6×10^{14}

NTA Ans. (3)

Sol. Time period of revolution of electron in n^{th} orbit

$$T = \frac{2\pi r}{V} = \frac{2\pi a_0\left(\frac{n^2}{Z}\right)}{V_0\left(\frac{Z}{n}\right)}$$

$$\Rightarrow T \propto \frac{n^3}{Z^2}$$

$$\frac{T_2}{T_1} = \frac{(2)^3}{(1)^3} = 8 \Rightarrow T_2 = 8 \times 1.6 \times 10^{-16}$$

Now frequency $f_2 = \frac{1}{T_2} = \frac{10^{16}}{8 \times 1.6} \approx 7.8 \times 10^{14}$ Hz.

3. A long solenoid of radius R carries a time (t)-dependent current $I(t) = I_0 t(1 - t)$. A ring of radius 2R is placed coaxially near its middle. During the time interval $0 \leq t \leq 1$, the induced current (I_R) and the induced EMF(V_R) in the ring change as :

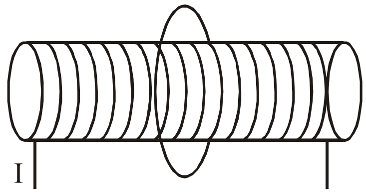
(1) At t = 0.5 direction of I_R reverses and V_R is zero

(2) Direction of I_R remains unchanged and V_R is zero at t = 0.25

(3) Direction of I_R remains unchanged and V_R is maximum at t = 0.5

(4) At t = 0.25 direction of I_R reverses and V_R is maximum

Sol.



Magnetic flux (ϕ) through ring is $\phi = \pi(R)^2.B$

$\phi = (\pi R^2)(\mu_0 nI) = (\pi R^2\mu_0 nI_0)(t - t^2)$

Induced e.m.f. of $V_R = \frac{-d\phi}{dt}$

$= \left(\pi R^2\mu_0 nI_0\right)\left(2t-1\right)$

and induced current $I_R = \frac{\pi R^2\mu_0 nI_0\left(2t-1\right)}{R_R}$

($R_R \to$ Resistance of Ring)

Clearly V_R and I_R are zero at $t = \frac{1}{2}$ = 0.5 sec.

and their sign also changes at t = 0.5 sec.

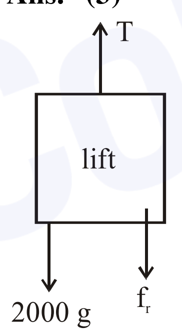
4. A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to: (1 HP = 746 W, g = 10 ms^{-2})

(1) 1.7 ms^{-1} (2) 2.0 ms^{-1}

(3) 1.9 ms^{-1} (4) 1.5 ms^{-1}

NTA Ans. (3)

Sol.



Let elevator is moving upward with constant speed V.

Tension in cable

$T = 2000\ g + f_r = 2000 + 4000$

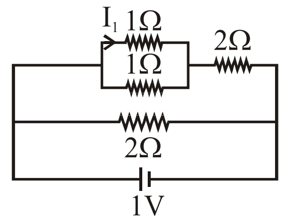
T = 24000 N

Power P = TV

$\Rightarrow$ 60 × 746 = (24000) V

60×746

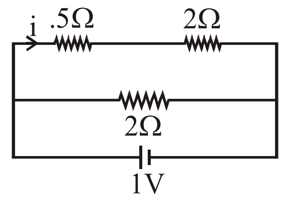
5. The current I_1 (in A) flowing through 1 Ω resistor in the following circuit is :



(1) 0.5 (2) 0.2 (3) 0.25 (4) 0.4

NTA Ans. (2)

Sol. Equivalent resistance of upper branch of circuit R = 2.5 Ω



Voltage across upper branch = 1 V

$\Rightarrow\ i = \frac{1}{2.5} = .4$ A

$\Rightarrow\ I_1 = 0.2$ A

6. A litre of dry air at STP expands adiabatically to a volume of 3 litres. If γ = 1.40, the work done by air is : ($3^{1.4} = 4.6555$) [Take air to be an ideal gas]

(1) 90.5 J (2) 48 J

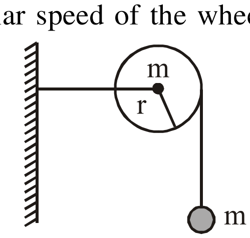
(3) 60.7 J (4) 100.8 J

NTA Ans. (1)

Sol. $w = \frac{nR\left(T_1 - T_2\right)}{\gamma - 1} = \frac{P_1V_1 - P_2V_2}{0.4}$

$= \frac{100 - \frac{100}{4.6555}\times 3}{0.4} = 88.90\cdot$

7. As shown in the figure, a bob of mass m is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. When released from rest the bob starts falling vertically. When it has covered a distance of h, the angular speed of the wheel will be :



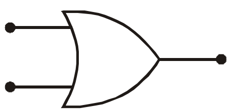
(1) $\frac{1}{r}\sqrt{\frac{2gh}{\ }}$ (2) $r\sqrt{\frac{3}{\ }}$ (3) $\frac{1}{r}\sqrt{\frac{4gh}{\ }}$ (4) $r\sqrt{\frac{3}{\ }}$

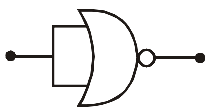
Sol. $mgh = \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{1}{2}mr^2\times\frac{v^2}{r^2} = \frac{3}{4}mv^2$

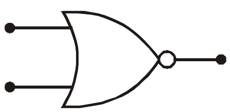
$$u = \sqrt{\frac{4}{3}gh}$$

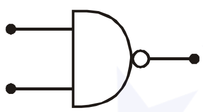
$$\omega = \frac{v}{r}$$

8. Which of the following gives a reversible operation?

(1) 

(2) 

(3) 

(4) 

NTA Ans. (2)

9. If we need a magnification of 375 from a compound microscope of tube length 150 mm and an objective of focal length 5 mm, the focal length of the eye-piece, should be close to :

(1) 22 mm (2) 12 mm

(3) 33 mm (4) 2 mm

NTA Ans. (1)

Sol. $m = \frac{LD}{f_e \times f_0} = \frac{150\times 250}{f_e \times 25} = 375$

$f_e = 20$ mm.

10. The radius of gyration of a uniform rod of length l, about an axis passing through a point $\frac{l}{4}$ away from the centre of the rod, and perpendicular to it, is :

(1) $\frac{1}{8}l$ (2) $\sqrt{\frac{7}{48}}l$ (3) $\sqrt{\frac{3}{8}}l$ (4) $\frac{1}{4}l$

NTA Ans. (2)

Sol. $m\frac{l^2}{12} + m\frac{l^2}{16} = mk^2$

2

11. If the magnetic field in a plane electromagnetic wave is given by $\vec{B} = 3\times10^{-8}\sin\left(1.6\times10^3 x + 48\times10^{10} t\right)\hat{j}\,T$, then what will be expression for electric field?

(1) $\vec{E} = \left(9\sin\left(1.6\times10^3 x + 48\times10^{10} t\right)\hat{k}\ V/m\right)$

(2) $\vec{E} = \left(3\times10^{-8}\sin\left(1.6\times10^3 x + 48\times10^{10} t\right)\hat{i}\ V/m\right)$

(3) $\vec{E} = \left(60\sin\left(1.6\times10^3 x + 48\times10^{10} t\right)\hat{k}\ V/m\right)$

(4) $\vec{E} = \left(3\times10^{-8}\sin\left(1.6\times10^3 x + 48\times10^{10} t\right)\hat{j}\ V/m\right)$

NTA Ans. (1)

Sol. $\vec{E}\times\vec{B} = \vec{C} = -\hat{i}$

where $\vec{B}$ is along $\hat{j}$

$\frac{E}{B} = C$

$E = 3\times10^{-8}\times 3\times10^{8} = 9$ V/m.

12. Consider a circular coil of wire carrying constant current I, forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given by ϕ_i. The magnetic flux through the area of the circular coil area is given by ϕ_0. Which of the following option is correct ?

(1) $\phi_i = -\phi_0$ (2) $\phi_i = \phi_0$

(3) $\phi_i < \phi_0$ (4) $\phi_i > \phi_0$

NTA Ans. (1)

13. Speed of a transverse wave on a straight wire (mass 6.0 g, length 60 cm and area of cross-section 1.0 mm^2) is 90 ms^{-1}. If the Young's modulus of wire is 16×10^{11} Nm^{-2}, the extension of wire over its natural length is :

(1) 0.02 mm (2) 0.04 mm

(3) 0.03 mm (4) 0.01 mm

NTA Ans. (3)

Sol. $v = \sqrt{\frac{T}{\mu}}$

$$90 = \sqrt{\frac{\frac{YA}{l}\Delta l}{\frac{m}{l}}} = \sqrt{\frac{16\times10^{11}\times10^{-6}\times\Delta l}{6\times10^{-3}}}$$

8100×3 $\quad$ -8

14. Visible light of wavelength 6000×10^{-8} cm falls normally on a single slit and produces a diffraction pattern. It is found that the second diffraction minimum is at 60° from the central maximum. If the first minimum is produced at θ_1, then θ_1 is close to :

(1) 20° (2) 45° (3) 30° (4) 25°

NTA Ans. (4)

Sol. $\sin\theta = \frac{2\lambda}{\omega}$

$\sin 60° = \frac{2\lambda}{\omega}$

$\sin\theta_1 = \frac{\lambda}{\omega} = \frac{\sqrt{3}}{4}$

$\theta_1 = 25°$

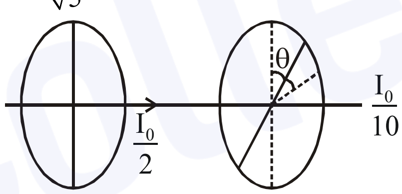
15. A polarizer - analyser set is adjusted such that the intensity of light coming out of the analyser is just 10% of the original intensity. Assuming that the polarizer - analyser set does not absorb any light, the angle by which the analyser need to be rotated further to reduce the output intensity to be zero, is :

(1) 18.4° (2) 71.6° (3) 90° (4) 45°

NTA Ans. (1)

Sol. $\frac{I_0}{10} = I = \frac{I_0}{2} \times \cos^2\theta$

$\cos\theta = \frac{1}{\sqrt{5}}$



$\theta = 63.44°$

angle rotated = 90 – 63.44° = 26.56°

Closest is 1.

16. A satellite of mass m is launched vertically upwards with an initial speed u from the surface of the earth. After it reaches height R (R = radius of the earth), it ejects a rocket of mass $\frac{m}{10}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant; M is the mass

(1) $\frac{m}{20}\left(u - \sqrt{\frac{2GM}{3R}}\right)^2$

(2) $5m\left(u^2 - \frac{119}{200}\frac{GM}{R}\right)$

(3) $\frac{3m}{8}\left(u + \sqrt{\frac{5GM}{6R}}\right)^2$

(4) $\frac{m}{20}\left(u^2 + \frac{113}{200}\frac{GM}{R}\right)$

NTA Ans. (2)

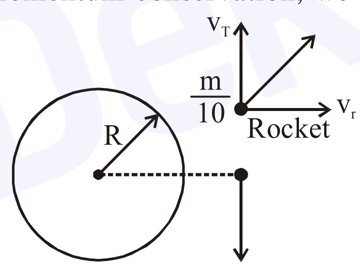
Sol. Applying energy conservation

$K_i + U_i = K_f + U_f$

$$\frac{1}{2}mu^2 + \left(-\frac{GMm}{R}\right) = \frac{1}{2}mv^2 - \frac{GMm}{2R}$$

$$v = \sqrt{u^2 - \frac{GM}{R}} \quad \text{....(i)}$$

By momentum conservation, we have



$$\frac{m}{10}v_T = \frac{9m}{10}\sqrt{\frac{GM}{2R}} \quad \text{....(ii)}$$

& $\frac{m}{10}v_r = mv$

$$\Rightarrow \frac{m}{10}v_r = m\sqrt{u^2 - \frac{GM}{R}} \quad \text{....(iii)}$$

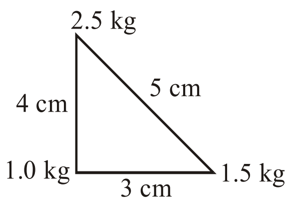
Kinetic energy of rocket

$$= \frac{1}{2}m\left(v_T^2 + v_r^2\right)$$

$$= \frac{m}{20}\left(81\frac{GM}{2R} + 100u^2 - 100\frac{GM}{R}\right)$$

$$= \frac{m}{20}\left(100u^2 - \frac{119GM}{2R}\right)$$

17. Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point:



(1) 1.5 cm right and 1.2 cm above 1 kg mass

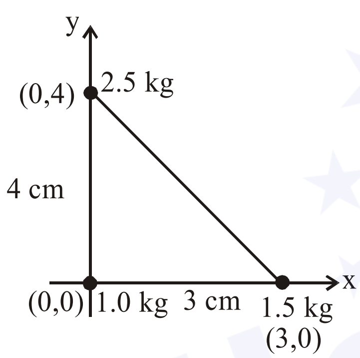
(2) 0.9 cm right and 2.0 cm above 1 kg mass

(3) 0.6 cm right and 2.0 cm above 1 kg mass

(4) 2.0 cm right and 0.9 cm above 1 kg mass

NTA Ans. (2)

Sol.



Let 1 kg as origin and x-y axis as shown

$$x_{cm} = \frac{1(0)+1.5(3)+2.5(0)}{5} = 0.9 \text{ cm}$$

$$y_{cm} = \frac{1(0)+1.5(0)+2.5(4)}{5} = 2 \text{ cm}$$

18. Two moles of an ideal gas with $\frac{C_P}{C_V} = \frac{5}{3}$ are mixed with 3 moles of another ideal gas with $\frac{C_P}{C_V} = \frac{4}{3}$. The value of $\frac{C_P}{C_V}$ for the mixture is:

(1) 1.50 (2) 1.42

(3) 1.45 (4) 1.47

Sol. $C_{P_{eq}} = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 + n_2}$

$$C_{V_{eq}} = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$$

$$\gamma_{eq} = \frac{C_{P_{eq}}}{C_{V_{eq}}} = \frac{2 \times \frac{5R}{2} + 3 \times \frac{8R}{2}}{2 \times \frac{3R}{2} + 3 \times \frac{6R}{2}}$$

$$= \frac{5+12}{3+9} = \frac{17}{12} \simeq 1.42$$

Correct Answer : 2

19. A LCR circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring-mass damped oscillator having damping constant 'b', the correct equivalence would be:

(1) $L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$

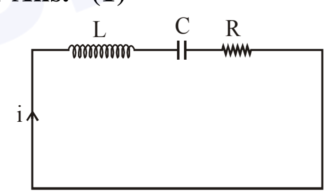
(2) $L \leftrightarrow \frac{1}{b}, C \leftrightarrow \frac{1}{m}, R \leftrightarrow \frac{1}{k}$

(3) $L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b$

(4) $L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m$

NTA Ans. (1)

Sol.



By kVL

$$-L\frac{di}{dt} - \frac{q}{C} - iR = 0$$

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q + R\frac{dq}{dt} = 0$$

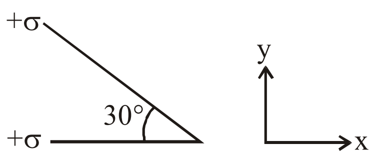
for damped oscillator

net force = –kx – bv = ma

$$\frac{md^2x}{dt^2} + kx + \frac{bdx}{dt} = 0$$

by comparing ; Equivalence is

20. Two infinite planes each with uniform surface charge density + σ are kept in such a way that the angle between them is 30°. The electric field in the region shown between them is given by:



(1) $\frac{\sigma}{\varepsilon_0}\left[\left(1+\frac{\sqrt{3}}{2}\right)\hat{y}+\frac{\hat{x}}{2}\right]$

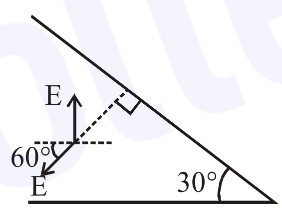
(2) $\frac{\sigma}{2\varepsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y}-\frac{\hat{x}}{2}\right]$

(3) $\frac{\sigma}{2\varepsilon_0}\left[\left(1+\sqrt{3}\right)\hat{y}+\frac{\hat{x}}{2}\right]$

(4) $\frac{\sigma}{2\varepsilon_0}\left[\left(1+\sqrt{3}\right)\hat{y}-\frac{\hat{x}}{2}\right]$

NTA Ans. (2)

Sol. Electric field due to each sheet is uniform and equal to $E=\frac{\sigma}{2\varepsilon_0}$

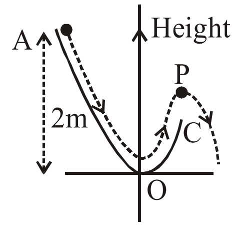


Now net electric field between plates

$$\vec{E}_{net} = E\cos 60°(-\hat{x}) + (E - E\sin 60°)(\hat{y})$$

$$= \frac{\sigma}{2\varepsilon_0}\left[-\frac{\hat{x}}{2}+\left(1-\frac{\sqrt{3}}{2}\right)\hat{y}\right].$$

21. A particle (m = 1 kg) slides down a frictionless track (AOC) starting from rest at a point A (height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it reaching its highest point P (height 1 m), the kinetic energy of the particle (in J) is : (Figure drawn is schematic and not to scale; take g=10 ms^{-2})_____.



NTA Ans. (10)

Sol. Mechanical energy conservation between A & P

$U_1 + \cancel{K_1} = K_2 + U_2$

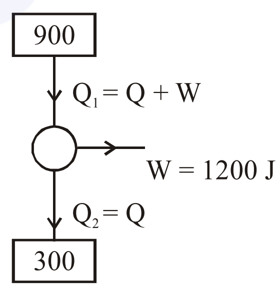
mg × 2 = mg × 1 + K_2

K_2 = mg × 1 = 10 J.

22. A Carnot engine operates between two reservoirs of temperatures 900 K and 300 K. The engine performs 1200 J of work per cycle. The heat energy (in J) delivered by the engine to the low temperature reservoir, in a cycle, is____.

NTA Ans. (600)

Sol.



for carnot engine

$$\frac{Q_1}{Q_2}=\frac{T_1}{T_2}$$

$$\frac{Q+1200}{Q}=\frac{900}{300}$$

Q + 1200 = 3Q

Q = 600 J.

23. A beam of electromagnetic radiation of intensity 6.4×10^{-5} W/cm^2 is comprised of wavelength, λ = 310 nm. It falls normally on a metal (work function φ = 2eV) of surface area of 1 cm^2. If one in 10^3 photons ejects an electron, total number of electrons ejected in 1 s is 10^x. (hc=1240 eVnm, 1eV=1.6×10^{-19} J), then x is____.

NTA Ans. (11)

Sol. Power incident P = I × A

n = no. of photons incident/second

$nE_{ph} = IA$

$$n = \frac{IA}{E_{ph}}$$

$$n = \frac{IA}{\left(\frac{hc}{\lambda}\right)} = \frac{6.4\times10^{-5}\times1}{\frac{1240}{310}\times1.6\times10^{-19}}$$

$n = 10^{+14}$ per second

Since efficiency = 10^{-3}

no. of electrons emitted = 10^{+11} per second.

x = 11.

24. A non-isotropic solid metal cube has coefficients of linear expansion as :

5×10^{-5}/°C along the x-axis and 5×10^{-6}/°C along the y and the z-axis. If the coefficient of volume expansion of the solid is C × 10^{-16}/°C then the value of C is ______.

NTA Ans. (60)

Sol. $\gamma = \alpha_x + \alpha_y + \alpha_z$

$= 5 \times 10^{-5} + 5 \times 10^{-6} + 5 \times 10^{-6}$

$= (50 + 5 + 5) \times 10^{-6}$

$\gamma = 60 \times 10^{-6}$

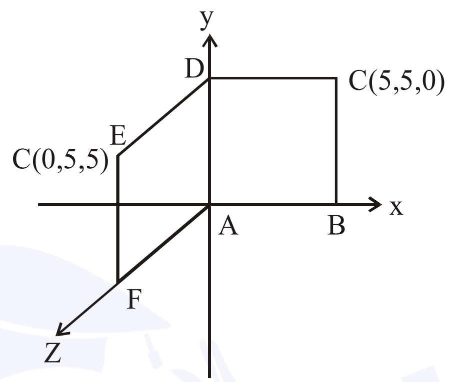
C = 60.

25. A loop ABCDEFA of straight edges has six corner points A(0,0,0), B(5,0,0), C(5,5,0), D(0, 5, 0), E(0, 5, 5) and F(0, 0, 5). The magnetic field in this region is $\vec{B} = \left(3\hat{i} + 4\hat{k}\right)T$.

The quantity of flux through the loop ABCDEFA (in Wb) is ______ .

NTA Ans. (175)

Sol.



$\vec{A}_{ABCD} = 25\hat{k}$

$\vec{A}_{ADEF} = 25\hat{i}$

$\vec{A}_{net} = 25\hat{i} + 25\hat{k}$

$\vec{B} = 3\hat{i} + 4\hat{k}$

$\phi = \vec{B}.\vec{A}$

$= 25 \times 3 + 25 \times 4$

ϕ = 175 $W_{b.}$