FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Wednesday 08th JANUARY, 2020) TIME : 9 : 30 AM to 12 : 30 PM

PHYSICS

TEST PAPER WITH ANSWER & SOLUTION

1. Consider a solid sphere of radius R and mass density $\rho(r) = \rho_0\left(1 - \frac{r^2}{R^2}\right)$, $0 < r \le R$. The minimum density of a liquid in which it will float is :

(1) $\frac{\rho_0}{5}$ (2) $\frac{\rho_0}{3}$ (3) $\frac{2\rho_0}{3}$ (4) $\frac{2\rho_0}{5}$

NTA Ans. (4)

Sol. In case of minimum density of liqued, sphere will be floating while completely submerged

So mg = B

$$m = \int_0^R \rho(4\pi r^2 dr) = B$$

$$= \rho_0 \int_0^R \left(1 - \frac{r^2}{R^2}\right) 4\pi r^2 dr = \frac{4}{3}\pi R^3 \rho_\ell g$$

On Solving

$$\rho_\ell = \frac{2\rho_0}{5}$$

2. When photon of energy 4.0 eV strikes the surface of a metal A, the ejected photoelectrons have maximum kinetic energy T_A eV end de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photon of energy 4.50 eV is $T_B = (T_A - 1.5)$ eV. If the de-Broglie wavelength of these photoelectrons $\lambda_B = 2\lambda_A$, then the work function of metal B is :

(1) 3eV (2) 2eV (3) 4eV (4) 1.5eV

NTA Ans. (3)

Sol. $\lambda_B = 2\lambda_A$

$$\Rightarrow \frac{h}{\sqrt{2T_B m}} = \frac{2h}{\sqrt{2T_A m}}$$

$T_A = 4T_B$ ....(i)

and $T_B = (T_A - 1.5)$ eV ....(ii)

from (i) and (ii)

$3T_B$ 1.5 eV $\Rightarrow T_B = 0.5$ eV

3. The length of a potentiometer wire is 1200 cm and it carries a current of 60 mA. For a cell of emf 5V and internal resistance of 20Ω, the null point on it is found to be a 1000cm. The resistance of whole wire is :

(1) 120Ω (2) 60Ω

(3) 80Ω (4) 100Ω

NTA Ans. (4)

Sol. $5 = \lambda\ell$

where λ is potential gradient & L is total length of wire.

$$5 = \frac{\Delta V}{L}\ell$$

$$\Delta V = \frac{5 \times L}{\ell} = 5 \times \frac{12}{10} = 6V = 60 \text{ mA} \times R$$

R = 100Ω

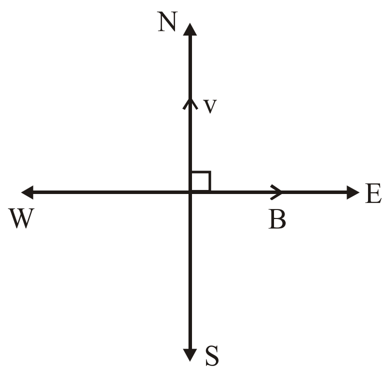
4. Photon with kinetic energy of 1MeV moves from south to north. It gets an acceleration of 10^{12} m/s² by an applied magnetic field (west to east). The value of magnetic field : (Rest mass of proton is 1.6×10^{-27} kg) :

(1) 71mT (2) 7.1mT

(3) 0.071mT (4) 0.71mT

NTA Ans. (4)

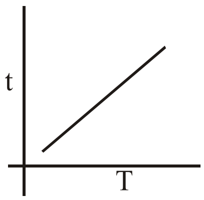
Sol. $a = \frac{qvB}{m}$



$$B = \frac{ma}{qv} = \frac{ma\sqrt{m}}{\sqrt{2k}}$$

$$= \frac{m^{3/2}a}{\sqrt{\ }} = \frac{(1.6 \times 10^{-27})^{3/2} \times 10^{12}}{{}^{-19}\sqrt{\quad 6 \quad {}^{-19}}}$$

5. The plot that depicts the behavior of the mean free time t (time between two successive collisions) for the molecules of an ideal gas, as a function of temperature (T), qualitatively, is: (Graphs are schematic and not drawn to scale)

(1)


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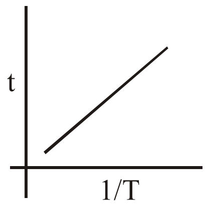

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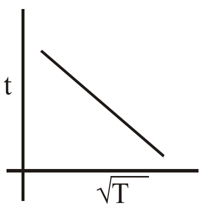

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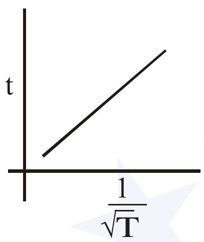


NTA Ans. (4)

Sol. Mean free time = $\dfrac{\text{Mean free path}}{\text{Average speed}}$

$$= \frac{\dfrac{1}{\sqrt{2}\pi D^2 n}}{\sqrt{\dfrac{8RT}{\pi M_w}}}$$

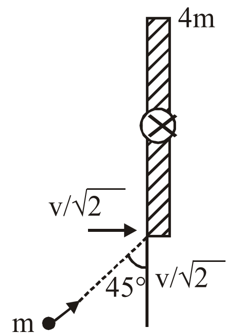
$$t \propto \frac{1}{\sqrt{T}}$$

6. Consider a uniform rod of mass M = 4m and length ℓ pivoted about its centre. A mass m moving with velocity v making angle $\theta = \frac{\pi}{4}$ to the rod's long axis collides with one end of the rod and sticks to it. The angular speed of the rod-mass system just after the collision is :

(1) $\dfrac{3}{7\sqrt{2}}\dfrac{v}{\ell}$ (2) $\dfrac{3\sqrt{2}}{7}\dfrac{v}{\ell}$

(3) $\dfrac{4}{7}\dfrac{v}{\ell}$ (4) $\dfrac{3}{7}\dfrac{v}{\ell}$

Sol.



Let angular velocity of the system after collision be ω.

By conservation of angular momentum about the hinge :

$$m\left(\frac{v}{\sqrt{2}}\right)\left(\frac{\ell}{2}\right) = \left[\frac{4m\ell^2}{12} + \frac{m\ell^2}{4}\right]\omega$$

On solving

$$\omega = \frac{3\sqrt{2}}{7}\left(\frac{v}{\ell}\right)$$

7. The dimension of stopping potential V_0 in photoelectric effect in units of Planck's constant 'h', speed of light 'c' and Gravitational constant 'G' and ampere A is :

(1) $h^2\ G^{3/2}\ c^{1/3}\ A^{-1}$ (2) $h^{-2/3}\ c^{-1/3}\ G^{4/3}\ A^{-1}$

(3) $h^{1/3}\ G^{2/3}\ c^{1/3}\ A^{-1}$ (4) $h^{2/3}\ c^{5/3}\ G^{1/3}\ A^{-1}$

NTA Ans. (BONUS)

Sol. $v_0 = h^x c^y G^z A^w$

$$\frac{ML^2T^{-2}}{AT} = (ML^2T^{-1})^x (LT^{-1})^y (M^{-1}L^3T^{-2})^z A^w$$

$\Rightarrow w = -1$

$(x - z = 1)$

$2x + y + 3x = 2$

$- x - y - 2z = -3$

____

$2x = 0$

$x = 0$

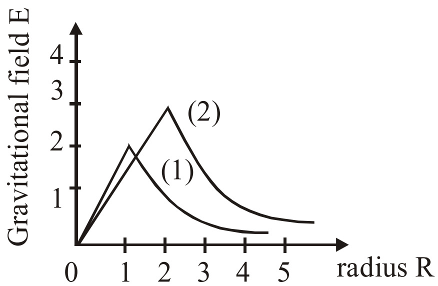
$z = -1$

$2 \times 0 + y + 3x - 1 = 2$

$y = 5$

$\Rightarrow v_0 = h^0 c^5 G^{-1} A^{-1}$

8. Consider two solid spheres of radii R_1 = 1m, R_2 = 2m and masses M_1 and M_2, respectively. The gravitational field due to sphere (1) and (2) are shown. The value of $\frac{M_1}{M_2}$ is :



(1) $\frac{1}{2}$ (2) $\frac{2}{3}$ (3) $\frac{1}{3}$ (4) $\frac{1}{6}$

NTA Ans. (4)

Sol. Gravitational field on the surface of a solid sphere $I_g = \frac{GM}{R^2}$

By the graph

$$\frac{GM_1}{(1)^2} = 2$$

and $$\frac{GM_2}{(2)^2} = 3$$

On solving

$$\frac{M_1}{M_2} = \frac{1}{6}$$

9. In finding the electric field using Gauss Law the formula $|\vec{E}| = \frac{q_{enc}}{\varepsilon_0 |A|}$ is applicable. In the formula ε_0 is permittivity of free space, A is the area of Gaussian surface and q_{enc} is charge enclosed by the Gaussian surface. The equation can be used in which of the following situation?

(1) Only when the Gaussian surface is an equipotential surface.

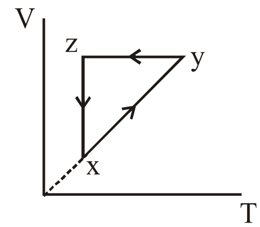
(2) Only when $|\vec{E}|$ = constant on the surface.

(3) For any choice of Gaussian surface.

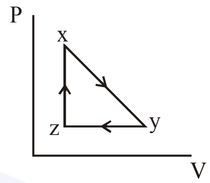
(4) Only when the Gaussian surface is an equipotential surface and $|\vec{E}|$ is constant on the surface.

NTA Ans. (4)

10. A thermodynamic cycle xyzx is shown on a V-T diagram.



The P-V diagram that best describes this cycle is : (Diagrams are schematic and not to scale)

(1)


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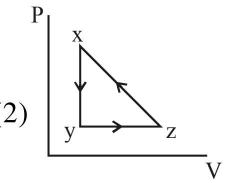
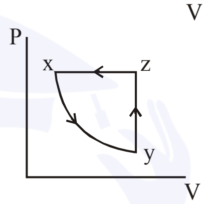
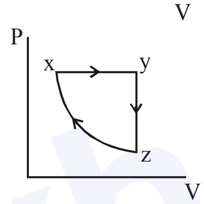


(3)


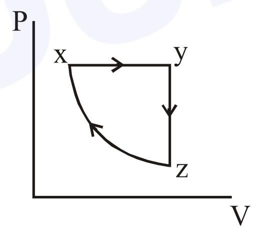
(4)


NTA Ans. (4)

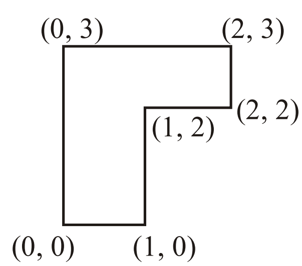
Sol. x → y ⇒ Isobaric

y → z ⇒ Isochoric

z → x ⇒ Isothermal



11. The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4kg. (The coordinates of the same are shown in figure) are :

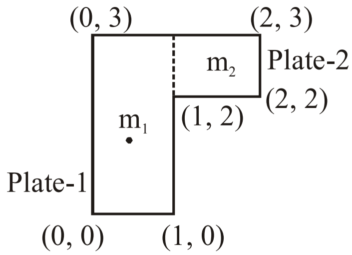


(1) (1.25m, 1.50m) (2) (1m, 1.75m)

(3) (0.75m, 0.75m) (4) (0.75m, 1.75m)

Sol. $m_1 = 3kg$

$m_2 = 1kg$



Mass of plate-1 is assumed to be concentrated at (0.5, 1.5)

Mass of plate-2 is assumed to be concentrated at (1.5, 2.5).

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{3\times0.5+1\times1.5}{4} = 0.75$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{3\times1.5+1\times2.5}{4} = 1.75$$

12. The magnifying power of a telescope with tube 60 cm is 5. What is the focal length of its eye piece ?

(1) 30 cm (2) 40 cm (3) 20 cm (4) 10 cm

NTA Ans. (4)

Sol. $L = f_0 + f_e = 60$ cm

$$M = \frac{f_0}{f_e} = 5$$

$\Rightarrow f_0 = 5f_e$

$\therefore\ 6f_e = 60$ cm

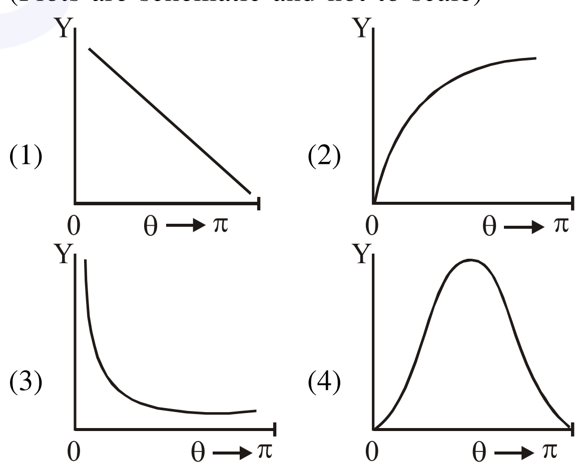
$f_e = 10$ cm

13. The graph which depicts the results of Rutherform gold foil experiment with α-particales is :

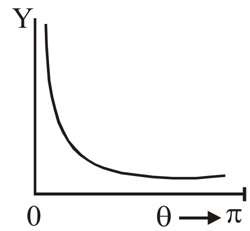
θ : Scattering angle

Y : Number of scattered α-particles detected

(Plots are schematic and not to scale)



Sol.



$$Y \propto \frac{1}{\left(\sin\frac{\theta}{2}\right)^4}$$

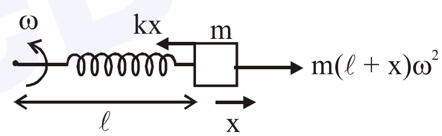
14. A particle of mass m is fixed to one end of a light spring having force constant k and unstretched length ℓ. The other end is fixed. The system is given an angular speed ω about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in the spring is :

(1) $\frac{m\ell\omega^2}{k+m\omega^2}$ (2) $\frac{m\ell\omega^2}{k-m\omega^2}$

(3) $\frac{m\ell\omega^2}{k-\omega m}$ (4) $\frac{m\ell\omega^2}{k+m\omega}$

NTA Ans. (2)

Sol.



$kx = m\ell\omega^2 + mx\omega^2$

$$x = \frac{m\ell\omega^2}{k - m\omega^2}$$

15. The critical angle of a medium for a specific wavelength, if the medium has relative permittivity 3 and relative permeability $\frac{4}{3}$ for this wavelength, will be :

(1) 60° (2) 15° (3) 45° (4) 30°

NTA Ans. (4)

Sol. $\sin\theta_C = \frac{1}{\mu} = \frac{1}{\sqrt{3\times4/3}}$

$\theta_C = 30°$

16. A leak proof cylinder of length 1m, made of a metal which has very low coefficient of expansion is floating vertically in water at 0°C such that its height above the water surface is 20 cm. When the temperature of water is increased to 4°C, the height of the cylinder above the water surface becomes 21 cm. The density of water at T = 4°C, relative to the density at T = 0°C is close to :

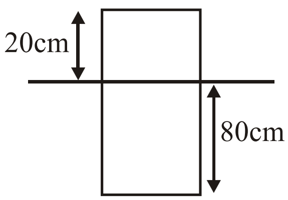
(1) 1.01 (2) 1.04

(3) 1.03 (4) 1.26

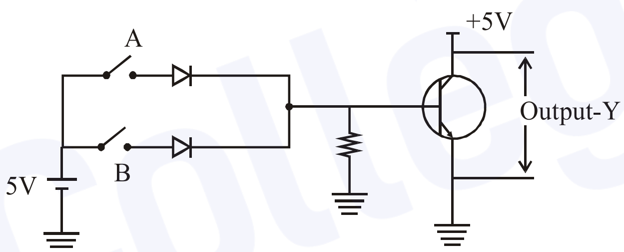
NTA Ans. (1)

Sol. $m = \rho_0 A\ (80)$ ....(i)

$m = \rho A\ (79)$ ....(ii)



17. Boolean relation at the output stage-Y for the following circuit is :



(1) A + B (2) $\overline{A}+\overline{B}$

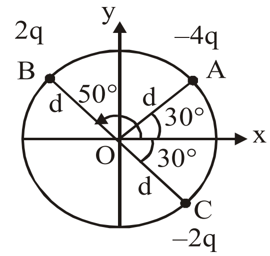
(3) $\overline{A}\cdot\overline{B}$ (4) $A\cdot B$

NTA Ans. (3)

Sol.

A	B	Y
0	0	1
1	0	0
0	1	0
1	1	0

18. Three charged particle A, B and C with charges –4q, 2q and –2q are present on the circumference of a circle of radius d. the charged particles A, C and centre O of the circle formed an equilateral triangle as shown in figure. Electric field at O along x-direction is :



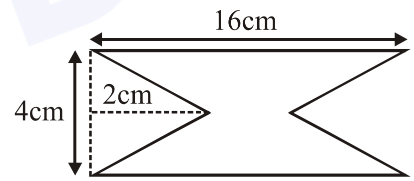
(1) $\dfrac{2\sqrt{3}\,q}{\pi\varepsilon_0 d^2}$ (2) $\dfrac{\sqrt{3}\,q}{4\pi\varepsilon_0 d^2}$

(3) $\dfrac{3\sqrt{3}\,q}{4\pi\varepsilon_0 d^2}$ (4) $\dfrac{\sqrt{3}\,q}{\pi\varepsilon_0 d^2}$

NTA Ans. (4)

Sol. $E_x = \dfrac{K(4q)}{R^2}\cos 30^\circ + \dfrac{K(2q)}{R^2}\cos 30^\circ + \dfrac{K(2q)}{R^2}\cos 30^\circ$

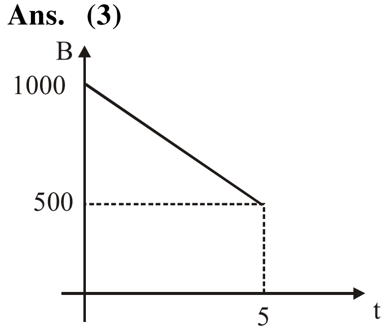
19. At time t = 0 magnetic field of 100 Gauss is passing perpendicularly through the area defined by the closed loop shown in the figure. If the magnetic field reduces linearly to 500 Gauss, in the next 5s, then induced EMF in the loop is :



(1) 36 μV (2) 48 μV (3) 56 μV (4) 28 μV

NTA Ans. (3)

Sol.



$\dfrac{dB}{dt} = 100$

$A = 16 \times 4 - 4 \times 2 = 56\ cm^2$

dB –4 –4

20. Effective capacitance of parallel combination of two capacitors C_1 and C_2 is 10 μF. When these capacitors are individually connected to a voltage source of 1V, the energy stored in the capacitor C_2 is 4 times that of C_1. If these capacitors are connected in series, their effective capacitance will be :

(1) 3.2 μF (2) 8.4 μF
(3) 1.6 μF (4) 4.2 μF

NTA Ans. (3)

Sol. $C_1 + C_2 = 10$ ....(i)

$$\frac{1}{2}C_2V^2 = 4\times\frac{1}{2}C_1V^2$$

$\therefore$ $C_2 = 4C_1$ ....(ii)

$\therefore$ $C_1 = 2$ & $C_2 = 8$

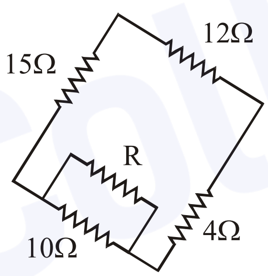
For series combination

$$C_{eq} = \frac{C_1C_2}{C_1+C_2} = 1.6$$

21. Four resistances of 15Ω, 12Ω, 4Ω and 10Ω respectively in cyclic order to form Wheatstone's network. The resistance that is to be connected in parallel with the resistance of 10Ω to balance the network is ________ Ω.

NTA Ans. (10.00)

Sol.



Let the resistance to be connected is R.

For balanced wheatstone bridge,

$$15 \times 4 = 12 \times \frac{10R}{10+R}$$

$\Rightarrow R = 10\Omega$

22. A point object in air is in front of the curved surface of a plano-convex lens. The radius of curvature of the curved surface is 30 cm and the refractive index of the lens material is 1.5, then the focal length of the lens (in cm) is ——.

NTA Ans. (60.00)

Sol. Using Lens-Maker's formula :

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{f} = (1.5-1)\left(\frac{1}{30}-0\right)$$

f = 60 cm

23. A body A, of mass m = 0.1 kg has an initial velocity of $3\hat{i}\ ms^{-1}$. It collides elastically with another body, B of the same mass which has an initial velocity of $5\hat{j}\ ms^{-1}$. After collision, A moves with a velocity $\vec{v} = 4(\hat{i}+\hat{j})$. The energy of B after collision is written as $\frac{x}{10}J$. The value of x is ___________.

NTA Ans. (1.00)

Sol. By conservation of linear momentum :

$$(0.1)(3\hat{i}) + (0.1)(5\hat{j}) = (0.1)(4)(\hat{i}+\hat{j}) + (0.1)\vec{v}$$

$$\Rightarrow \vec{v} = -\hat{i}+\hat{j}$$

$\therefore$ Speed of B after collision $|\vec{v}| = \sqrt{2}$

Now, kinetic energy $= \frac{1}{2}mV^2 = \frac{1}{2}(0.1)(2) = \frac{1}{10}$

$\therefore$ x = 1

24. A particle is moving along the x-axis with its coordinate with the time 't' given be $x(t) = 10 + 8t - 3t^2$. Another particle is moving the y-axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$. At t = 1s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v (in m/s) is __________.

NTA Ans. (580.00)

Sol. $x = 10 + 8t - 3t^2$

$v_x = 8 - 6t$

$(v_x)_{t=1} = 2\hat{i}$

$y = 5 - 8t^3$

$v_y = -24t^2$

$(v_y)_{t=1} = -24\hat{j}$

Now

$$\sqrt{v} = \sqrt{(24)^2 + (2)^2} = \sqrt{580}$$

$\therefore$ v = 580 m^2/s^2

25. A one metre long (both ends open) organ pipe is kept in a gas that has double the density of air at STP. Assuming the speed of sound in air at STP is 300 m/s, the frequency difference between the fundamental and second harmonic of this pipe is ________ Hz.

NTA Ans. (106.00 to 107.20)

Sol. $v_s = \sqrt{\frac{\gamma P}{\rho}}$

$$\frac{v_{gas}}{v_{air}} = \sqrt{\frac{\rho_{air}}{\rho_{gas}}} \qquad \Rightarrow \frac{v_{gas}}{300} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow v_{gas} = \frac{300}{\sqrt{2}}$$

$$\therefore\ v_{gas} = 150\sqrt{2}$$

Now $n_2 - n_1 = \frac{v_{gas}}{2\ell} = \frac{150\sqrt{2}}{2(1)} = 75\sqrt{2}$

$\Rightarrow$ Δn = 106.06 Hz