FINAL JEE-MAIN EXAMINATION – JANUARY, 2020 (Held On Thursday 09th JANUARY, 2020) TIME : 2 : 30 PM to 5 : 30 PM PHYSICS TEST PAPER WITH ANSWER & SOLUTION

- 1. A spring mass system (mass m, spring constant k and natural length *l*) rest in equilibrium on a horizontal disc. The free end of the spring is fixed at the centre of the disc. If the disc together with spring mass system, rotates about it's axis with an angular velocity ω , (k >> m ω^2) the relative change in the length of the spring is best given by the option :
 - (1) $\frac{2m\omega^2}{k}$
 - (2) $\frac{m\omega^2}{3k}$
 - (3) $\sqrt{\frac{2}{3}} \left(\frac{m\omega^2}{k} \right)$

(4)
$$\frac{m\omega^2}{k}$$

NTA Ans. (4)

Sol.

FBD of m in frame of disc/-

$$k\Delta \ell$$
 m $m\omega^2(\ell_0 + \Delta \ell)$

$$\mathbf{k}\Delta\ell\ =\mathbf{m}\omega^2(\,\ell_0\,+\,\Delta\,\ell\,)$$

$$\Delta \ell = \frac{m\omega^2 \ell_0}{k - m\omega^2} \approx \frac{m\omega \ell_0}{k}$$

$$\frac{\Delta \ell}{\ell_0} = \text{Relative change} = \frac{m\omega^2}{k}$$

2. An electron gun is placed inside a long solenoid of radius R on its axis. The solenoid has n turns/length and carries a current I. The electron gun shoots an electron along the radius of the solenoid with speed v. If the electron does not hit the surface of the solenoid, maximum possible value of v is (all symbols have their standard meaning) :



(1)
$$\frac{e\mu_0 nIR}{m}$$
 (2) $\frac{e\mu_0 nIR}{2m}$
(3) $\frac{2e\mu_0 nIR}{m}$ (4) $\frac{e\mu_0 nIR}{4m}$

NTA Ans. (2) Sol. Top view of solenoid



Maximum possible radius of electron = $\frac{R}{2}$

$$\therefore \frac{R}{2} = \frac{mv}{qB} = \frac{mv_{max}}{e(\mu_0 ni)}$$

$$T_{\rm max} = \frac{R}{2} \frac{e\mu_0 ni}{m}$$

ν



3.

A rod of length L has non-uniform linear mass

density given by
$$p(x) = a + b \left(\frac{x}{L}\right)^2$$
, where a

and b are constants and $0 \le x \le L$. The value of x for the centre of mass of the rod is at :

(1)
$$\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$$

(2) $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$

$$(3) \ \frac{3}{2} \left(\frac{2a+b}{3a+b} \right) L$$

$$(4) \ \frac{3}{4} \left(\frac{2a+b}{3a+b}\right) I$$

NTA Ans. (4)



A plane electromagnetic wave is propagating

along the direction $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$, with its polarization

along the direction \hat{k} . The correct form of the magnetic field of the wave would be (here B_0 is an appropriate constant) :

(1)
$$B_{0}\frac{\hat{i}-\hat{j}}{\sqrt{2}}\cos\left(\omega t-k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$$

(2)
$$B_{0}\frac{\hat{i}+\hat{j}}{\sqrt{2}}\cos\left(\omega t-k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$$

(3)
$$B_{0}\hat{k}\cos\left(\omega t-k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$$

(4)
$$B_0 \frac{\hat{j} - \hat{i}}{\sqrt{2}} \cos\left(\omega t + k \frac{\hat{i} + \hat{j}}{\sqrt{2}}\right)$$

NTA Ans. (1)

4.

Sol. Direction of polarisation $= \hat{E} = \hat{k}$

Direction of propagation
$$= \hat{E} \times \hat{B} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$$

$$\therefore \hat{\mathbf{E}} \times \hat{\mathbf{B}} = \frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}$$

$$\hat{\mathbf{B}} = \frac{\hat{\mathbf{i}} - \hat{\mathbf{j}}}{\sqrt{2}}$$

Correct answer (1)

5. The current i in the network is :



(1) 0 A	(2) 0.6 A
(3) 0.3 A	(4) 0.2 A





6. A small spherical droplet of density d is floating exactly half immersed in a liquid of density ρ and surface tension T. The radius of the droplet is (take note that the surface tension applies an upward force on the droplet) :

(1)
$$r = \sqrt{\frac{2T}{3(d+\rho)g}}$$
 (2) $r = \sqrt{\frac{3T}{(2d-\rho)g}}$
(3) $r = \sqrt{\frac{T}{(d-\rho)g}}$ (4) $r = \sqrt{\frac{T}{(d+\rho)g}}$

NTA Ans. (2) Sol. FBD of droplet



A small circular loop of conducting wire has radius a and carries current I. It is placed in a uniform magnetic field B perpendicular to its plane such that when rotated slightly about its diameter and released, it starts performing simple harmonic motion of time period T. If the mass of the loop is m then :

(1)
$$T = \sqrt{\frac{\pi m}{2IB}}$$
 (2) $T = \sqrt{\frac{2\pi m}{IB}}$

(3)
$$T = \sqrt{\frac{\pi m}{IB}}$$
 (4) $T = \sqrt{\frac{2m}{IB}}$

NTA Ans. (2)

7.



 $\vec{T} = \vec{M} \times \vec{B} = -MB\sin\theta$ I $\alpha = -MB\sin\theta$ for small θ ,

$$\alpha = -\frac{MB}{I}\theta$$

$$\omega = \sqrt{\frac{\text{MB}}{\text{I}}} = \sqrt{\frac{(i)(\pi R^2)B}{\left(\frac{mR^2}{2}\right)}}$$

$$\omega = \sqrt{\frac{2i\pi B}{m}}$$

$$\therefore = \mathrm{T} = \frac{2\pi}{\omega} = \sqrt{\frac{2\pi\mathrm{m}}{\mathrm{iB}}}$$

: Correct answer (2)

8. A wire of length L and mass per unit length $6.0 \times 10^{-3} \text{ kgm}^{-1}$ is put under tension of 540 N. Two consecutive frequencies that it resonates at are : 420 Hz and 490 Hz. Then L in meters is :

 $(1) \ 8.1 \ m \quad (2) \ 5.1 \ m \quad (3) \ 1.1 \ m \quad (4) \ 2.1 \ m$



Sol.
$$\frac{nv}{2\ell} = 420$$

 $\frac{(n+1)v}{2\ell} = 490$
 $\frac{v}{2\ell} = 70$
 $\ell = \frac{v}{140} = \frac{1}{140}\sqrt{\frac{540}{6 \times 10^{-3}}} = \frac{1}{140}\sqrt{90 \times 10^{3}}$
 $\ell = \frac{300}{140} = 2.142$
 \therefore Correct answer (4)
9. In LC circuit the inductance L = 40 mH and capacitance C = 100 µF. If a voltage V(t) = 10sin(314 t) is applied to the circuit, the current in the circuit is given as :
(1) 0.52 cos 314 t (2) 0.52 sin 314 t
(3) 10 cos 314 t (4) 5.2 cos 314 t
(3) 10 cos 314 t (4) 5.2 cos 314 t
NTA Ans. (1)
L = 40mH C = 100µF
 $\sqrt{000000}$ 9V
Sol. $\sqrt{v} = 10sin(314t)$
 $X_{L} = \omega L = 314 \times 40 \times 10^{-3} = 12.56\Omega$
 $x_{C} = \frac{1}{\omega C} = \frac{1}{314 \times 100 \times 10^{-6}}$
 $= \frac{10^{4}}{314} = 31.84\Omega$
Phasor
 $\sqrt{1X_{L}}$
 $V_{m} = I_{m}(X_{C} - X_{L})$
 $10 = I_{m}(31.84 - 12.56)$
 $I_{m} = \frac{10}{19.28} = 0.52A$
 $I = 0.52 sin $\left(314t + \frac{\pi}{2}\right)$$

10. There is a small source of light at some depth below the surface of water (refractive index = $\frac{4}{3}$) in a tank of large cross sectional surface area. Neglecting any reflection from the bottom and absorption by water, percentage of light that emerges out of surface is (nearly) : [Use the fact that surface area of a spherical cap of height h and radius of curvature r is 2π rh]: (1) 17% (2) 21% (3) 34% (4) 50%

NTA Ans. (1)



$$\sin\theta = \frac{3}{4}$$

Area of sphere in which light spread = $4\pi R^2$ $\Omega = 2\pi (1 - \cos \theta)$

$$\Omega = 2\pi \left(1 - \frac{\sqrt{7}}{4}\right)$$

 $P \to 4\pi$ steradians

$$P' \rightarrow \frac{P}{4\pi} (1 - \cos\theta)$$

Ratio =
$$\frac{P'}{P} = \frac{2\pi(1 - \cos\theta)}{4\pi} = \frac{(1 - \cos\theta)}{2} = \frac{1 - \frac{\sqrt{7}}{4}}{2}$$

= $\frac{0.33}{2} = 0.17$

11. Two gases-argon (atomic radius 0.07 nm, 13. A particle of mass m is projected with a speed atomic weight 40) and xenon (atomic radius u from the ground at an angle $\theta = \frac{\pi}{2}$ w.r.t. 0.1 nm, atomic weight 140) have the same number density and are at the same horizontal (x-axis). When it has reached its temperature. The raito of their respective mean maximum height, it collides completely free times is closest to : inelastically with another particle of the same (2) 4.67 (1) 3.67mass and velocity $u\hat{i}$. The horizontal distance (3) 1.83 (4) 2.3 NTA Ans. (1) covered by the combined mass before reaching Ans. (3) the ground is: **Sol.** $\lambda = \frac{1}{\sqrt{2}\pi n_u d^2}$ (1) $\frac{3\sqrt{2}}{4}\frac{u^2}{\sigma}$ (2) $2\sqrt{2}\frac{u^2}{g}$ $(3) \ \frac{3\sqrt{3}}{8} \frac{u^2}{\sigma}$ $\tau = \frac{\lambda}{v} = \frac{1}{\sqrt{2}\pi n_v d^2 v} = \frac{1}{\sqrt{2}\pi n_v d^2} \sqrt{\frac{M}{3RT}}$ (4) $\frac{5}{8} \frac{u^2}{g}$ NTA Ans. (3) $\frac{\tau_1}{\tau_2} = \sqrt{\frac{M_1}{M_2}\frac{d_2^2}{d_1^2}}$ $\longrightarrow ucos60^{\circ} \xrightarrow{m} \frac{u}{2} \xrightarrow{m} u$ $=\sqrt{\frac{40}{140}}\frac{\left(0.1\right)^2}{\left(0.07\right)^2}$ Sol. $\theta = 60^{\circ}$ = 1.09By momentum conservation, \therefore Nearest possible answer (3) $\frac{mu}{2} + mu = 2mv'$ 12. A particle starts from the origin at t = 0 with an initial velocity of $3.0\hat{i}$ m/s and moves in the $v' = \frac{3v}{4}$ x-y plane with a constant acceleration Range after collision = $\frac{3v}{4}\sqrt{\frac{2H}{g}}$ $(6.0\hat{i}+4.0\hat{j})m/s^2$. The x-coordinate of the particle at the instant when its y-coordinate is $=\frac{3v}{4}\sqrt{\frac{2\cdot u^2\sin^2 60^\circ}{g2g}}$ 32 m is D meters. The value of D is :-(1) 50(2) 32 (3) 60 (4) 40 $=\frac{3}{4}\frac{\sqrt{3}}{2}\cdot\frac{u^2}{g}=\frac{3\sqrt{3}u^2}{8g}$ NTA Ans. (3) **Sol.** $x = u_x t + \frac{1}{2}a_x t^2$ \therefore Correct answer (3) 14. The energy required to ionise a hydrogen like $y = u_y t + \frac{1}{2}a_y t^2$ ion in its ground state is 9 Rydbergs. What is the wavelength of the radiation emitted when $32 = 0 \times t + \frac{1}{2}(4)(t)^{2}$ the electron in this ion jumps from the second excited state to the ground state ? $t^2 = 16$ (1) 35.8 nm t = 4 sec(2) 24.2 nm $\mathbf{x} = 3 \times 4 + \frac{1}{2} \times 6 \times 4^2$ (3) 8.6 nm (4) 11.4 nm



Sol. 1 Rydberg energy = 13.6 eV

So, ionisation energy =
$$(13.6 \text{ Z}^2)\text{eV}$$

= 9 × 13.6eV
Z = 3
 $\frac{1}{\lambda} = \text{RZ}^2 \left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 1.09 \times 10^7 \times 9 \times \frac{8}{9}$
 $\lambda = 11.4 \text{ nm}$

15. Two identical capacitors A and B, charged to the same potential 5V are connected in two different circuits as shown below at time t = 0. If the charge on capacitors A and B at time t = CR is Q_A and Q_B respectively, then (Here e is the base of natural logarithm)





No current flows Hence $Q_A = CV$ For (B)



$$i = \frac{V}{R}e^{-\frac{t}{RC}}$$

$$q = CVe^{-\frac{t}{RC}}$$
at t = CR
$$Q_{B} = CVe^{-1} = \frac{CV}{e}$$

: Correct answer (1)

16. A uniformly thick wheel with moment of inertia I and radius R is free to rotate about its centre of mass (see fig). A massless string is wrapped over its rim and two blocks of masses m_1 and m_2 ($m_1 > m_2$) are attached to the ends of the string. The system is released from rest. The angular speed of the wheel when m_1 descents by a distance h is :



$$(1) \left[\frac{m_1 + m_2}{(m_1 + m_2)R^2 + I} \right]^{\frac{1}{2}} gh$$

$$(2) \left[\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I} \right]^{\frac{1}{2}}$$

$$(3) \left[\frac{2(m_1 + m_2)gh}{(m_1 + m_2)R^2 + I} \right]^{\frac{1}{2}}$$

$$(4) \left[\frac{(m_1 - m_2)}{(m_1 + m_2)R^2 + I} \right]^{\frac{1}{2}} gh$$





 Planet A has mass M and radius R. Planet B has half the mass and half the radius of Planet A. If the escape velocities from the Planets A and

B are v_A and v_B , respectively, then $\frac{v_A}{v_B} = \frac{n}{4}$.

(3) 2

(4) 3

The value of n is :

(2) 1

(1) 4 NTA Ans. (1)

Sol. $V_e = \sqrt{\frac{2GM}{R}}$ (Escape velocity) $V_A = \sqrt{\frac{2GM}{R}}$ $V_B = \sqrt{\frac{2G[M/2]}{R/2}} = \sqrt{\frac{2GM}{R}}$ $\frac{V_A}{V_B} = 1 = \frac{n}{4} \Rightarrow n = 4$ 18. Two steel wires having same length are suspended from a ceiling under the same load. If the ratio of their energy stored per unit volume is 1 : 4, the ratio of their diameters is:

(1) 1:
$$\sqrt{2}$$
 (2) 1:2 (3) 2:1 (4) $\sqrt{2}$:1
Ans (4)

NTA Ans. (4)



$$\Rightarrow \frac{d_1}{d_2} = \sqrt{2} : 1$$

 \therefore Correct answer (4)

19. For the four sets of three measured physical quantities as given below. Which of the following options is correct ? (i) $A_1 = 24.36$, $B_1 = 0.0724$, $C_1 = 256.2$ (ii) $A_2 = 24.44$, $B_2 = 16.082$, $C_2 = 240.2$ (iii) $A_3 = 25.2$, $B_3 = 19.2812$, $C_3 = 236.183$ (iv) $A_4 = 25$, $B_4 = 236.191$, $C_4 = 19.5$ (1) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1 < A_3 + B_3 + C_3$ $< A_2 + B_2 + C_2$ (2) $A_1 + B_1 + C_1 < A_3 + B_3 + C_3 < A_2 + B_2 + C_2$ $< A_4 + B_4 + C_4$ (3) $A_1 + B_1 + C_1 = A_2 + B_2 + C_2 = A_3 + B_3 + C_3$ $= A_4 + B_4 + C_4$ (4) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1 = A_2 + B_2 + C_2$ = A + B + C

Sol. $A_1 + B_1 + C_1 = 24.36 + 0.0724 + 256.2$ = 280.6324= 280.6 (After rounding off) $A_2 + B_2 + C_2 = 24.44 + 16.082 + 240.2$ = 280.722= 280.7 (After rounding off) $A_3 + B_3 + C_3 = 25.2 + 19.2812 + 236.183$ = 280.6642= 280.7 (After rounding off) $A_4 + B_4 + C_4 = 25 + 236.191 + 19.5$ = 280.691= 281 (After rounding off) $A_4 + B_4 + C_4 > A_3 + B_3 + C_3 = A_2 + B_2 +$ $C_2 > A_1 + B_1 + C_1$ No option is matching Question should be (BONUS) Best possible option is (2)

 \therefore Correct answer (2)

20. An electron of mass m and magnitude of charge |e| initially at rest gets accelerated by a constant electric field E. The rate of change of de-Broglie wavelength of this electron at time t ignoring relativistic effects is :

|Et

e Et

(1)
$$\frac{-\mathbf{h}}{|\mathbf{e}| \mathrm{Et}^2}$$
 (2) $\frac{|\mathbf{e}|}{|\mathbf{e}|}$

$$(3) - \frac{h}{|e|E\sqrt{t}}$$

NTA Ans. (1)

Sol.
$$a = \frac{eE}{m}$$

$$v = u + at = \left(\frac{eE}{m}\right)$$

$$\lambda = \frac{n}{mv}$$

$$\frac{d\lambda}{dt} = \frac{-(hm) \cdot \frac{dv}{dt}}{(mv)^2} = -\frac{ah}{mv^2} = -\frac{h}{|e|Et^2}$$

21. Starling at temperature 300 K, one mole of an ideal diatomic gas ($\gamma = 1.4$) is first compressed

adiabatically from volume V_1 to $V_2 = \frac{V_1}{16}$. It is

then allowed to expand isobarically to volume $2V_2$. If all the processes are the quasi-static then the final temperature of the gas (in °K) is (to the nearest integer) _____.

NTA Ans. (1816.00 to 1820)

Sol. $PV^{\gamma} = constant$ $TV^{\gamma - 1} = C$

$$300 \times V^{\frac{7}{5}-1} = T_2 \left(\frac{V}{16}\right)^{\frac{7}{5}-1}$$

$$300 \times 2^{4 \times \frac{2}{5}} = T_2$$

$$V = \frac{nRT}{P}$$

$$V_2 = kT_2 \qquad \dots (1)$$

$$2V_2 = KT_f \qquad \dots (2)$$

$$\frac{1}{2} = \frac{T_2}{T_f} \Longrightarrow T_f = 2T_2$$

$$T_{\rm f} = 2 \times 300 \times 2^{\frac{8}{5}} = 1818.85$$

- : Correct answer 1819
- 22. In a meter bridge experiment S is a standard resistance. R is a resistance wire. It Is found that balancing length is l = 25 cm. If R is replaced by a wire of half length and half diameter that of R of same material, then the balancing distance l' (in cm) will now be_____.





Sol. In balancing $\frac{R}{S} = \frac{25}{75}$ New resistance $R' = \frac{\rho \ell}{A}$ $= \frac{\rho \times \frac{\ell}{2}}{\frac{A}{4}} = \frac{\rho \ell}{2} \times 4A$ R' = 2R $\frac{2R}{S} = \frac{\ell'}{100 - \ell'}$ $2 \times \frac{1}{3} = \frac{\ell'}{100 - \ell'} = 3\ell' = 200 - 2\ell'$ $5\ell' = 200$ $\ell' = 40$ $\therefore \text{ Correct answer 40}$

23. The circuit shown below is working as a 8 V dc regulated voltage source. When 12 V is used as input, the power dissipated (in mW) in each diode is; (considering both zener diodes are identical) _____.

4V

8V

NTA Ans. (12.00)
Ans. (40.00)

$$12V$$

Sol. $12V$
Current in circuit = $\frac{4}{100}$

So power dissipited in each diode = VI

$$= 4 \times \frac{1}{100} W = 40 \times 10^{-3} mW$$

: Correct answer 40

24. In a Young's double slit experiment 15 fringes are observed on a small portion of the screen when light of wavelength 500 nm is used. Ten fringes are observed on the same section of the screen when another light source of wavelength λ is used. Then the value of λ is (in nm) _____.

NTA Ans. (750.00)

Sol. The length of the screen used portion for 15 fringes, and also for ten fringes

$$15 \times 500 \times \frac{D}{\lambda} = 10 \times \frac{\lambda D}{\lambda}$$

15 × 50 = λ
 λ = 750 nm
 \therefore Correct answer 750

25. An electric field $\vec{E} = 4x\hat{i} - (y^2 + 1)\hat{j}N/C$ passes through the box shown in figure. The flux of the electric field through surfaces ABCD and BCGF are marked as ϕ_I and ϕ_{II} respectively. The difference between $(\phi_I - \phi_{II})$ is (in Nm²/C) _____.



NTA Ans. (-48.00)

Sol. The flux passes through ABCD (x – y) plane is zero, because electric field parallel to surface. Flux of the electric field through surface BCGF (y - z)

At BCGF (electric field) $\Rightarrow \vec{E} = 12\hat{i} - (y^2 + 1)\hat{j}$ (x = 3m) Flux $\phi_{II} = 12 \times 4 = 48 \text{ Nm}^2/\text{C}$

So $\phi_{I} - \phi_{II} = 0 - 48 = -48 \text{ Nm}^{2}/\text{C}$

∴ Correct answer –48