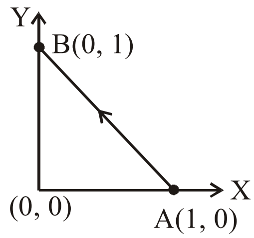
FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Thursday 09th JANUARY, 2020) TIME : 9 : 30 AM to 12 : 30 PM

PHYSICS — TEST PAPER WITH ANSWER & SOLUTION

1. Consider a force $\vec{F} = -x\hat{i} + y\hat{j}$. The work done by this force in moving a particle from point A(1, 0) to B(0, 1) along the line segment is : (all quantities are in SI units)



(1) $\frac{3}{2}$ (2) 1 (3) 2 (4) $\frac{1}{2}$

NTA Ans. (2)

Sol. $W = \int_{\vec{r}_i}^{\vec{r}_g} \vec{F}.d\vec{r}$

$$W = \int_1^0 -x\,dx + \int_0^1 y\,dy$$

$$W = \left.\frac{-x^2}{2}\right|_1^0 + \left.\frac{y^2}{2}\right|_0^1$$

$$= -\left(\frac{0^2}{2} - \frac{1^2}{2}\right) + \left(\frac{1^2}{2} - \frac{0^2}{2}\right)$$

W = 1J

2. A quantity f is given by $f = \sqrt{\frac{hc^5}{G}}$ where c is speed of light, G universal gravitational constant and h is the Planck's constant. Dimension of f is that of :

(1) Momentum (2) Area

(3) Energy (4) Volume

NTA Ans. (3)

Sol. $[h] = M^1L^2T^{-1}$

$[C] = L^1T^{-1}$

$[G] = M^{-1}L^3T^{-2}$

$[f] = \sqrt{\frac{M^1L^2T^{-1} \times L^5T^{-5}}{{}^{-1}\ {}^{3}\ {}^{-2}}} = M^1L^2T^{-2}$

3. A body A of mass m is moving in a circular orbit of radius R about a planet. Another body B of mass $\frac{m}{2}$ collides with A with a velocity which is half $\left(\frac{\vec{v}}{2}\right)$ the instantaneous velocity $\vec{v}$ of A. The collision is completely inelastic. Then, the combined body :

(1) starts moving in an elliptical orbit around the planet.

(2) continues to move in a circular orbit

(3) Falls vertically downwards towards the planet

(4) Escapes from the Planet's Gravitational field.

NTA Ans. (1)

Sol. Initially, the body of mass m is moving in a circular orbit of radius R. So it must be moving with orbital speed.

$$v_0 = \sqrt{\frac{GM}{R}}$$

After collision, let the combined mass moves with speed v_1

$$mv_0 + \frac{m}{2}\frac{v_0}{2} = \left(\frac{3m}{2}\right)v_1$$

$$v_1 = \frac{5v_0}{6}$$

Since after collision, the speed is not equal to orbital speed at that point. So motion cannot be circular. Since velocity will remain tangential, so it cannot fall vertically towards the planet. Their speed after collision is less than escape speed $\sqrt{2}v_0$, so they cannot escape gravitational field.

So their motion will be elliptical around the

4. The electric fields of two plane electromagnetic plane waves in vacuum are given by

$\vec{E}_1 = E_0\hat{j}\cos(\omega t - kx)$ and

$\vec{E}_2 = E_0\hat{k}\cos(\omega t - ky)$

At t = 0, a particle of charge q is at origin with a velocity $\vec{v} = 0.8c\hat{j}$ (c is the speed of light in vacuum). The instantaneous force experienced by the particle is :

(1) $E_0q(-0.8\hat{i} + \hat{j} + \hat{k})$

(2) $E_0q(0.8\hat{i} - \hat{j} + 0.4\hat{k})$

(3) $E_0q(0.8\hat{i} + \hat{j} + 0.2\hat{k})$

(4) $E_0q(0.4\hat{i} - 3\hat{j} + 0.8\hat{k})$

NTA Ans. (3)

Sol. $\vec{E}_1 = E_0\hat{j}\cos(\omega t - kx)$

Its corresponding magnetic field will be

$$\vec{B}_1 = \frac{E_0}{c}\hat{k}\cos(\omega t - kx)$$

$$\vec{E}_2 = E_0\hat{k}\cos(\omega t - ky)$$

$$\vec{B}_2 = \frac{E_0}{c}\hat{i}\cos(\omega t - ky)$$

Net force on charge particle

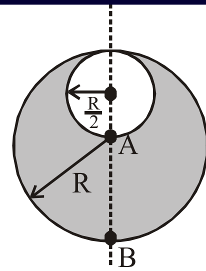
$$= q\vec{E}_1 + q\vec{E}_2 + q\vec{v}\times B_1 + q\vec{v}\times\vec{B}_2$$

$$= qE_0\hat{j} + qE_0\hat{k} + q(0.8c\hat{j})\times\left(\frac{E_0}{c}\hat{k}\right) + q(0.8c\hat{j})\times\left(\frac{E_0}{c}\hat{i}\right)$$

$$= qE_0\hat{j} + qE_0\hat{k} + 0.8qE_0\hat{i} - 0.8qE_0\hat{k}$$

$$\vec{F} = qE_0[0.8\hat{i} + 1\hat{j} + 0.2\hat{k}]$$

5. Consider a sphere of radius R which carries a uniform charge density ρ. If a sphere of radius $\frac{R}{2}$ is carved out of it, as shown, the ratio $\frac{|\vec{E}_A|}{|\vec{E}_B|}$ of magnitude of electric field $\vec{E}_A$ and $\vec{E}_B$, respectively, at points A and B due to the remaining portion is :



(1) $\frac{18}{54}$ (2) $\frac{21}{34}$ (3) $\frac{17}{54}$ (4) $\frac{18}{34}$

NTA Ans. (4)

Sol. Fill the empty space with +ρ and –ρ charge density.

$$|E_A| = 0 + \frac{k\rho.\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\left(\frac{R}{2}\right)^2} = k\rho\frac{4}{3}\pi\left(\frac{R}{2}\right)$$

$$|E_B| = \frac{k\rho.\frac{4}{3}\pi R^3}{R^2} - \frac{k\rho.\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\left(\frac{3R}{2}\right)^2}$$

$$= k\rho\frac{4}{3}\pi R - k\rho\frac{4}{3}\pi\frac{R}{18} = k\rho.\frac{4}{3}\pi\left(\frac{17R}{18}\right)$$

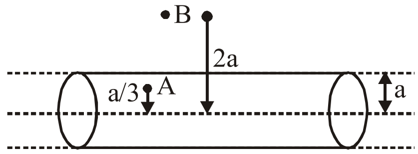
$$\frac{E_A}{E_B} = \frac{9}{17} = \frac{18}{34}$$

6. A long, straight wire of radius a carries a current distributed uniformly over its cross-section. The ratio of the magnetic fields due to the wire at distance $\frac{a}{3}$ and 2a, respectively from the axis of the wire is :

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) $\frac{1}{2}$ (4) 2

NTA Ans. (1)

Sol.



Let current density be J.

$\therefore$ Applying Ampere's law.

$$\oint \vec{B}.d\vec{\ell} = \mu_0 i \Rightarrow B_A 2\pi \frac{a}{3} = \mu_0 J\pi\left(\frac{a}{3}\right)^2$$

$$\therefore \quad B_A = \frac{\mu_0 Ja}{6}$$

Similarly, $B_B = \dfrac{\mu_0 Ja}{4}$

$$\therefore \quad \frac{B_A}{B_B} = \frac{\mu_0 Ja \times 4}{\mu_0 J6a} = \frac{2}{3}$$

7. Consider two ideal diatomic gases A and B at some temperature T. Molecules of the gas A are rigid, and have a mass m. Molecules of the gas B have an additional vibrational mode, and have a mass $\frac{m}{4}$. The ratio of the specific heats $(C_V^A \text{ and } C_V^B)$ of gas A and B, respectively is :

(1) 7 : 9 (2) 5 : 7 (3) 3 : 5 (4) 5 : 9

NTA Ans. (2)

Sol. Degree of freedom of a diatomic molecule if vibration is absent = 5

Degree of freedom of a diatomic molecule if vibration is present = 7

$$\therefore \quad C_v^A = \frac{f_A}{2}R = \frac{5}{2}R \;\&\; C_v^B = \frac{f_B}{2}R = \frac{7}{2}R$$

$$\therefore \quad \frac{C_v^A}{C_v^B} = \frac{5}{7}$$

8. A particle moving with kinetic energy E has de Broglie wavelength λ. If energy ΔE is added to its energy, the wavelength become $\lambda/2$. Value of ΔE, is :

(1) 2E (2) E (3) 3E (4) 4E

Sol. Given, de-Broglie wavelength = $\dfrac{h}{\sqrt{2mE}} = \lambda$

Also, $\dfrac{h}{\sqrt{2m(E+\Delta E)}} = \dfrac{\lambda}{2}$

$$\therefore \quad \frac{E+\Delta E}{E} = 4 \Rightarrow \Delta E = 3E.$$

9. If the screw on a screw-gauge is given six rotations, it moves by 3 mm on the main scale. If there are 50 divisions on the circular scale the least count of the screw gauge is :

(1) 0.001 mm (2) 0.001 cm

(3) 0.02 mm (4) 0.01 cm

NTA Ans. (2)

Sol. Given on six rotation, reading of main scale changes by 3mm.

$\therefore$ 1 rotation corresponds to $\frac{1}{2}$mm

Also no. of division on circular scale = 50.

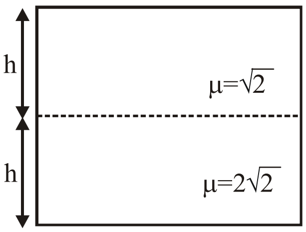
$\therefore$ Least count of the screw gauge will be $\frac{0.5}{50}$mm = 0.001 cm.

10. A vessel of depth 2h is half filled with a liquid of refractive index $2\sqrt{2}$ and the upper half with another liquid of refractive index $\sqrt{2}$. The liquids are immiscible. The apparent depth of the inner surface of the bottom of vessel will be :

(1) $\dfrac{h}{\sqrt{2}}$ (2) $\dfrac{3}{4}h\sqrt{2}$

(3) $\dfrac{h}{2\left(\sqrt{2}+1\right)}$ (4) $\dfrac{h}{3\sqrt{2}}$

Sol.



For near normal incidence,

$$h_{app} = \frac{h_{actual}}{\left(\frac{\mu_{in}}{\mu_{ref.}}\right)}$$

$$\therefore \quad h_{apparent} = \frac{\frac{h}{\left(\frac{2\sqrt{2}}{\sqrt{2}}\right)} + h}{\frac{\sqrt{2}}{1}} = \frac{3h}{2\sqrt{2}} = \frac{3}{4}h\sqrt{2}$$

11. Radiation, with wavelength 6561 Å falls on a metal surface to produce photoelectrons. The electrons are made to enter a uniform magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by the electrons is 10 mm, the work function of the metal is close to :

(1) 1.8eV (2) 1.1eV (3) 0.8eV (4) 1.6eV

NTA Ans. (2)

Sol. Let the work function be ϕ.

$$\therefore \quad KE_{max} = \frac{hc}{\lambda} - \phi$$

Again, $R_{max} = \frac{\sqrt{2mKE_{max}}}{qB} = \frac{\sqrt{2m\left(\frac{hc}{\lambda} - \phi\right)}}{qB}$

$$\therefore \quad \frac{R_{max}^2 q^2 B^2}{2m} = \frac{hc}{\lambda} - \phi$$

$$\therefore \quad \phi = \frac{hc}{\lambda} - \frac{R_{max}^2 q^2 B^2}{2m} = 1.0899 \text{ eV} \approx 1.1\text{eV}$$

12. The aperture diameter of a telescope is 5m. The separation between the moon and the earth is 4×10^5 km. With light of wavelength of 5500 Å, the minimum separation between objects on the surface of moon, so that they are just resolved, is close to :

(1) 20 m (2) 600 m

(3) 60 m (4) 200 m

Sol. Let distance is x then

$$d\theta = \frac{1.22\lambda}{D} \quad (D = \text{diameter})$$

$$\frac{x}{d} = \frac{1.22\lambda}{D} \quad (d = \text{distance between earth \& moon})$$

$$x = \frac{1.22 \times (5500 \times 10^{-10}) \times (4 \times 10^8)}{5} = 53.68\,m$$

most appropriate is 60m.

13. Two particles of equal mass m have respective initial velocities $u\hat{i}$ and $u\left(\frac{\hat{i}+\hat{j}}{2}\right)$. They collide completely inelastically. The energy lost in the process is :

(1) $\frac{3}{4}mu^2$ (2) $\frac{1}{8}mu^2$ (3) $\sqrt{\frac{2}{3}}mu^2$ (4) $\frac{1}{3}mu^2$

NTA Ans. (2)

Sol. From momentum conservation

$$mu\hat{i} + mu\left(\frac{\hat{i}+\hat{j}}{2}\right) = (m + m)\vec{v}$$

$$\Rightarrow \quad \vec{v} = \frac{3}{4}u\hat{i} + \frac{u}{4}\hat{j}$$

$$\Rightarrow \quad |v| = \frac{u}{4}\sqrt{10}$$

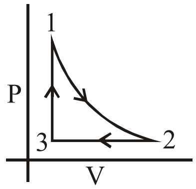
Final kinetic energy $= \frac{1}{2}2m\left(\frac{u}{4}\sqrt{10}\right)^2 = \frac{5}{8}mu^2$

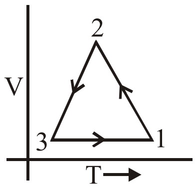
Initial kinetic energy

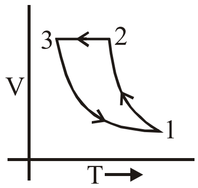
$$= \frac{1}{2}mu^2 + \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{6}{8}mu^2$$

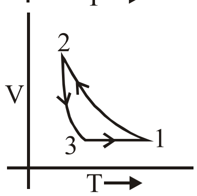
Loss in K.E. $= k_i - k_f = \frac{1}{}mu^2$

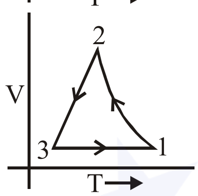
14. Which of the following is an equivalent cyclic process corresponding to the thermodynamic cyclic given in the figure ? where, $1 \rightarrow 2$ is adiabatic.
(Graphs are schematic and are not to scale)



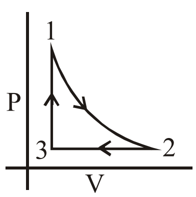
(1) 

(2) 

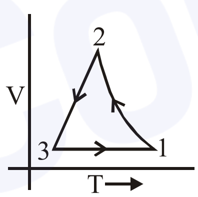
(3) 

(4) 

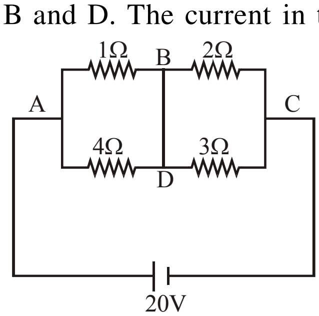
NTA Ans. (4)

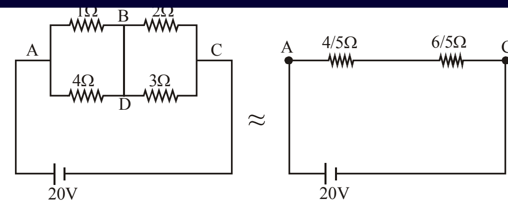
Sol. 

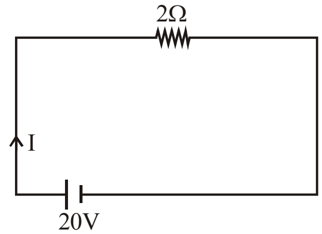
In process 2 to 3 pressure is constant & in process 3 to 1 volume is constant which is correct only in option 4.
Correct graph is



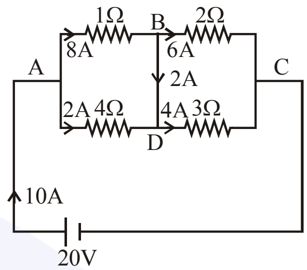
15. In the given circuit diagram, a wire is joining points B and D. The current in this wire is :



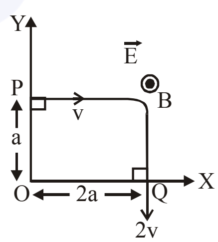
Sol. 

≈ 

$\Rightarrow I = \frac{20}{2} = 10A$

Now, 

16. A charged particle of mass 'm' and charge 'q' moving under the influence of uniform electric field $E\vec{i}$ and a uniform magnetic field $B\vec{k}$ follows a trajectory from point P to Q as shown in figure. The velocities at P and Q are respectively, $v\vec{i}$ and $-2v\vec{j}$. Then which of the following statements (A, B, C, D) are the correct? (Trajectory shown is schematic and not to scale):



(A) $E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$

(B) Rate of work done by the electric field at P is $\frac{3}{4}\left(\frac{mv^3}{a}\right)$

(C) Rate of work done by both the fields at Q is zero

(D) The difference between the magnitude of angular momentum of the particle at P and Q is 2 mav.

(1) (A), (B), (C), (D) (2) (A), (B), (C)
(3) (B), (C), (D) (4) (A), (C), (D)

Sol. Option (A)

$W = k_f - k_i$

$$qE(2a - 0) = \frac{1}{2}m(2V)^2 - \frac{1}{2}mV^2$$

$$qE2a = \frac{3}{2}mV^2$$

$$E = \frac{3}{4}\frac{mv^2}{qa}$$

Option (B)

Rate of work done $P = \vec{F}.\vec{V} = FV\cos\theta = FV$

Power = qEV

$$\text{Power} = q\left(\frac{3}{4}\frac{mV^2}{qa}\right)V$$

$$\text{Power} = q\frac{3}{4}\frac{mV^3}{qa}$$

$$\text{Power} = \frac{3}{4}\frac{mV^3}{a}$$

Option (C)

Angle between electric force and velocity is 90º, hence rate of work done will be zero at Q.

Option (D)

Initial angular momentum L_i = mVa

Final angular momentum L_f = m(2V) (2a)

Change in angular momentum $L_f - L_i$ = 3mVa

(Note : angular momentum is calculated about O)

17. Three harmonic waves having equal frequency ν and same intensity I_0, have phase angles 0, $\frac{\pi}{4}$ and $-\frac{\pi}{4}$ respectively. When they are superimposed the intensity of the resultant wave is close to :

(1) 5.8 I_0 (2) 0.2 I_0

(3) I_0 (4) 3 I_0

NTA Ans. (1)

Sol. Let amplitude of each wave is A.

Resultant wave equation

$$= A\sin\omega t + A\sin\left(\omega t - \frac{\pi}{4}\right) + A\sin\left(\omega t + \frac{\pi}{4}\right)$$

$$= \left(\sqrt{2}+1\right)A\sin\omega t$$

Resultant wave amplitude = $\left(\sqrt{2}+1\right)A$

as $I \propto A^2$

so $\frac{I}{I_0} = \left(\sqrt{2}+1\right)^2$

$I = 5.8\ I_0$

18. An electric dipole of moment $\vec{p} = (-\hat{i} - 3\hat{j} + 2\hat{k}) \times 10^{-29}$ C .m is at the origin (0, 0, 0). The electric field due to this dipole at $\vec{r} = +\hat{i} + 3\hat{j} + 5\hat{k}$ (note that $\vec{r}.\vec{p} = 0$) is parallel to:

(1) $(-\hat{i} + 3\hat{j} - 2\hat{k})$ (2) $(+\hat{i} - 3\hat{j} - 2\hat{k})$

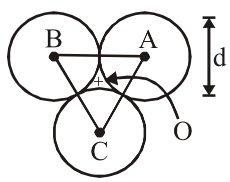
(3) $(+\hat{i} + 3\hat{j} - 2\hat{k})$ (4) $(-\hat{i} - 3\hat{j} + 2\hat{k})$

NTA Ans. (3)

Sol. Since $\vec{r}$ and $\vec{p}$ are perpendicular to each other therefore point lies on the equitorial plane. Therefore electric field at the point will be antiparallel to the dipole moment.

i.e. $\vec{E} \parallel -\vec{p}$

$\vec{E} \parallel (\hat{i} + 3\hat{j} - 2\hat{k})$

19. 

Three solid spheres each of mass m and diameter d are stuck together such that the lines connecting the centres form an equilateral triangle of side of length d. The ratio I_0/I_A of moment of inertia I_0 of the system about an axis passing the centroid and about center of any of the spheres I_A and perpendicular to the plane of the triangle is :

(1) $\frac{13}{23}$ (2) $\frac{15}{13}$ (3) $\frac{23}{13}$ (4) $\frac{13}{15}$

Sol. From parallel axis theorem

$$I_0 = 3\times\left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + M\left(\frac{d}{\sqrt{3}}\right)^2\right] = \frac{13}{10}Md^2$$

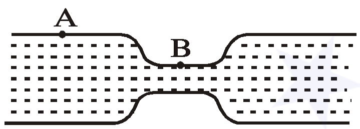
$$I_A = I_0 + 3M\left(\frac{d}{\sqrt{3}}\right)^2$$

$$= \frac{13}{10}Md^2 + Md^2$$

$$= \frac{23}{10}Md^2$$

$$\frac{I_0}{I_A} = \frac{13}{23}$$

20. Water flows in a horizontal tube (see figure). The pressure of water changes by 700 Nm^{-2} between A and B where the area of cross section are 40 cm^2 and 20 cm^2, respectively. Find the rate of flow of water through the tube. (density of water = 1000 kgm^{-3})



(Fig.)

(1) 1810 cm^3/s (2) 3020 cm^3/s

(3) 2720 cm^3/s (4) 2420 cm^3/s

NTA Ans. (3)

Sol. Rate of flow of water = $A_AV_A = A_BV_B$

$(40)V_A = (20)V_B$

$V_B = 2V_A$ .... (1)

Using Bernoulli's theorem

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$

$$P_A - P_B = \frac{1}{2}\rho(V_B^2 - V_A^2)$$

$$700 = \frac{1}{2}\times 1000(4V_A^2 - V_A^2)$$

V_A = 0.68 m/s = 68 cm/s

Rate of flow = A_AV_A

= (40) (68) = 2720 cm^3/s

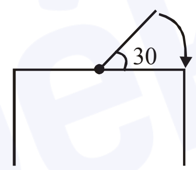
21. In a fluorescent lamp choke (a small transformer) 100 V of reverse voltage is produced when the choke current changes uniformly from 0.25 A to 0 in a duration of 0.025 ms. The self-inductance of the choke (in mH) is estimated to be __________ .

NTA Ans. (10.00)

Sol. $V = \left|L\frac{di}{dt}\right|$

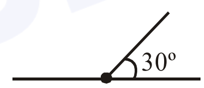
$$\Rightarrow L = \frac{V}{\left|\frac{di}{dt}\right|} = \frac{100}{\frac{0.25}{0.025\times 10^{-3}}} = 10mH$$

22. One end of a straight uniform 1m long bar is pivoted on horizontal table. It is released from rest when it makes an angle 30° from the horizontal (see figure). Its angular speed when it hits the table is given as $\sqrt{n}\,s^{-1}$, where n is an integer. The value of n is __________ .



NTA Ans. (15.00)

Sol.



From mechanical energy conservation,

$U_i + K_i = U_f + K_f$

$$\Rightarrow mg\frac{\ell}{2}\sin 30° + 0 = 0 + \frac{1}{2}I\omega^2$$

$$\Rightarrow mg \times \frac{1}{2} \times \frac{1}{2} + 0 = 0 + \frac{1}{2} \times \frac{m(1)^2}{3}\omega^2$$

$$\Rightarrow \omega^2 = \frac{3g}{2} \Rightarrow \omega = \sqrt{15}$$

$\therefore$ n = 15

23. The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + 2bt + c$. If the acceleration of the particle depends on x as x^{-n}, where n is an integer, the value of n is ________ .

NTA Ans. (3.00)

Sol. $x = \sqrt{at^2 + 2bt + c}$

Differentiating w.r.t. time

$$\frac{dx}{dt} = v = \frac{1}{2\sqrt{at^2 + 2bt + c}} \times (2at + 2b)$$

$$\Rightarrow v = \frac{at + b}{x}$$

$\Rightarrow vx = at + b$

Differentiating w.r.t. x

$$\Rightarrow \frac{dv}{dx} \times x + v = a \times \frac{dt}{dx}$$

Multiply both side by v

$$\Rightarrow \left(v\frac{dv}{dx}\right)x + v^2 = a$$

$$\Rightarrow a'x = a - v^2 \quad \text{[Here a' is acceleration]}$$

$$\Rightarrow a'x = a - \left(\frac{at + b}{x}\right)^2$$

$$\Rightarrow a'x = \frac{ax^2 - (at + b)^2}{x^2}$$

$$\Rightarrow a'x = \frac{a(at^2 + 2bt + c) - (at + b)^2}{x^2}$$

$$\Rightarrow a'x = \frac{ac - b^2}{x^2}$$

$$\Rightarrow a' = \frac{ac - b^2}{x^3}$$

$$\therefore a' \propto \frac{1}{x^3} \qquad \therefore n = 3$$

24. A body of mass m = 10 kg is attached to one end of a wire of length 0.3 m. The maximum angular speed (in rad s^{-1}) with which it can be rotated about its other end in space station is (Breaking stress of wire = 4.8×10^7 Nm^{-2} and area of cross-section of the wire = 10^{-2} cm^2) is:

NTA Ans. (4.00)

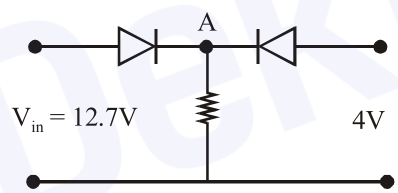
Sol. $T = m\omega^2\ell$

Breaking stress = $\frac{T}{A} = \frac{m\omega^2\ell}{A}$

$$\Rightarrow \omega^2 = \frac{4.8 \times 10^7 \times (10^{-2} \times 10^{-4})}{10 \times 0.3} = 16$$

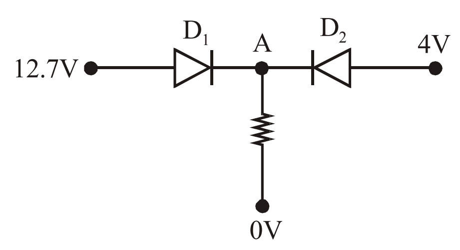
$\Rightarrow \omega = 4$

25. Both the diodes used in the circuit shown are assumed to be ideal and have negligible resistance when these are forward biased. Built in potential in each diode is 0.7 V. For the input voltages shown in the figure, the voltage (in Volts) at point A is ________ .



NTA Ans. (12.00)

Sol.



Diode D_1 is forward biased and D_2 is reverse biased.

$\therefore$ $V_A = 12.7 - 0.7 = 12V$.