FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Thursday 18th March, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The differential equation satisfied by the system of parabolas $y^2 = 4a(x + a)$ is :

(1) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) - y = 0$

(2) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) + y = 0$

(3) $y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$

(4) $y\left(\frac{dy}{dx}\right) + 2x\left(\frac{dy}{dx}\right) - y = 0$

Official Ans. by NTA (3)

Sol. $y^2 = 4ax + 4a^2$

differentiate with respect to x

$$\Rightarrow 2y\frac{dy}{dx} = 4a$$

$$\Rightarrow a = \left(\frac{y}{2}\frac{dy}{dx}\right)$$

so, required differential equation is

$$y^2 = \left(4 \times \frac{y}{2}\frac{dy}{dx}\right)x + 4\left(\frac{y}{2}\frac{dy}{dx}\right)^2$$

$$\Rightarrow y^2\left(\frac{dy}{dx}\right)^2 + 2xy\left(\frac{dy}{dx}\right) - y^2 = 0$$

$$\Rightarrow y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$$

2. The number of integral values of m so that the abscissa of point of intersection of lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is :

(1) 1 (2) 2 (3) 3 (4) 0

Official Ans. by NTA (2)

Sol. $3x + 4y = 9$

$y = mx + 1$

$\Rightarrow 3x + 4mx + 4 = 9$

$\Rightarrow (3 + 4m)x = 5$

$\Rightarrow$ x will be an integer when

$3 + 4m = 5, -5, 1, -1$

$$\Rightarrow m = \frac{1}{2}, -2, -\frac{1}{2}, -1$$

so, number of integral values of m is 2

3. Let $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + ... + a_{40}x^{40}$. then $a_1 + a_3 + a_5 + ... + a_{37}$ is equal to

(1) $2^{20}(2^{20} - 21)$

(2) $2^{19}(2^{20} - 21)$

(3) $2^{19}(2^{20} + 21)$

(4) $2^{20}(2^{20} + 21)$

Official Ans. by NTA (2)

Sol. $(1 + x + 2x^2)^{20} = a_0 + a_1x + ... + a_{40}x^{40}$ put $x = 1, -1$

$\Rightarrow a_0 + a_1 + a_2 + ... + a_{40} = 2^{20}$

$a_0 - a_1 + a_2 + ... + a_{40} = 2^{20}$

$$\Rightarrow a_1 + a_3 + ... + a_{39} = \frac{4^{20} - 2^{20}}{2}$$

$$\Rightarrow a_1 + a_3 + ... + a_{37} = 2^{39} - 2^{19} - a_{39}$$

$$\text{here } a_{39} = \frac{20!(2)^{19} \times 1}{19!} = 20 \times 2^{19}$$

$\Rightarrow a_1 + a_3 + ... + a_{37} = 2^{19}(2^{20} - 1 - 20)$

$= 2^{19}(2^{20} - 21)$

4. The solutions of the equation

$$\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0, (0 < x < \pi), \text{ are}$$

(1) $\frac{\pi}{12}, \frac{\pi}{6}$ (2) $\frac{\pi}{6}, \frac{5\pi}{6}$

(3) $\frac{5\pi}{12}, \frac{7\pi}{12}$ (4) $\frac{7\pi}{12}, \frac{11\pi}{12}$

Sol. $\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$

use $R_1 \to R_1 + R_2 + R_3$

$\Rightarrow (2+4\sin 2x)\begin{vmatrix} 1 & 1 & 1 \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$

$\Rightarrow \sin 2x = -\frac{1}{2}$

$\Rightarrow 2x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$x = \frac{\pi}{2} + \frac{\pi}{12}, \pi - \frac{\pi}{12}$

5. Choose the correct statement about two circles whose equations are given below :

$x^2 + y^2 - 10x - 10y + 41 = 0$

$x^2 + y^2 - 22x - 10y + 137 = 0$

(1) circles have same centre

(2) circles have no meeting point

(3) circles have only one meeting point

(4) circles have two meeting points

Official Ans. by NTA (3)

Sol. $x^2 + y^2 - 10x - 10y + 41 = 0$

$A(5,5),\ R_1 = 3$

$x^2 + y^2 - 22x - 10y + 137 = 0$

$B(11,5),\ R_2 = 3$

$AB = 6 = R_1 + R_2$

Touch each other externally

$\Rightarrow$ circles have only one meeting point.

6. Let α,β,γ be the real roots of the equation, $x^3 + ax^2 + bx + c = 0$, $(a,b,c \in R$ and $a,b \neq 0)$. If the system of equations (in, u,v,w) given by $\alpha u + \beta v + \gamma w = 0$, $\beta u + \gamma v + \alpha w = 0$; $\gamma u + \alpha v + \beta w = 0$ has non-trivial solution, then the value of $\frac{a^2}{b}$ is

Sol. $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$

$\Rightarrow -(\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \Sigma\alpha\beta) = 0$

$\Rightarrow -(-a)(a^2 - 2b - b) = 0$

$\Rightarrow a(a^2 - 3b) = 0$

$\Rightarrow a^2 = 3b \Rightarrow \frac{a^2}{b} = 3$

7. The integral $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{4x^2-4x+6}}dx$ is equal to

(where c is a constant of integration)

(1) $\frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$

(2) $\frac{1}{2}\cos\sqrt{(2x+1)^2+5}+c$

(3) $\frac{1}{2}\cos\sqrt{(2x-1)^2+5}+c$

(4) $\frac{1}{2}\sin\sqrt{(2x+1)^2+5}+c$

Official Ans. by NTA (1)

Sol. $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{(2x-1)^2+5}}dx$

$(2x-1)^2 + 5 = t^2$

$2(2x-1)\,2dx = 2t\,dt$

$2\sqrt{t^2-5}dx = t\,dt$

So $\int \frac{\sqrt{t^2-5}\cos t}{2\sqrt{t^2-5}}dt = \frac{1}{2}\sin t + c$

$= \frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$

8. The equation of one of the straight lines which passes through the point (1,3) and makes an angles $\tan^{-1}\left(\sqrt{2}\right)$ with the straight line, $y+1=3\sqrt{2}\,x$ is

(1) $4\sqrt{2}x+5y-\left(15+4\sqrt{2}\right)=0$

(2) $5\sqrt{2}x+4y-\left(15+4\sqrt{2}\right)=0$

(3) $4\sqrt{2}x+5y-4\sqrt{2}=0$

(4) $4\sqrt{2}x-5y-\left(5+4\sqrt{2}\right)=0$

Official Ans. by NTA (1)

Sol. $y=mx+c$

$3 = m + c$

$$\sqrt{2}=\left|\frac{m-3\sqrt{2}}{1+3\sqrt{2}m}\right|$$

$=6m+\sqrt{2}=m-3\sqrt{2}$

$=\sin=-4\sqrt{2} \;\rightarrow\; m=\dfrac{-4\sqrt{2}}{5}$

$= \;=6m-\sqrt{2}=m-3\sqrt{2}$

$=7m-2\sqrt{2} \;\rightarrow\; m=\dfrac{2\sqrt{2}}{7}$

According to options take $m=\dfrac{-4\sqrt{2}}{5}$

So $y=\dfrac{-4\sqrt{2}x}{5}+\dfrac{3+4\sqrt{2}}{5}$

$4\sqrt{2}x+5y-\left(15+4\sqrt{2}\right)=0$

9. If $\lim\limits_{x\to 0}\dfrac{\sin^{-1}x-\tan^{-1}x}{3x^3}$ is equal to L, then the value of (6L + 1) is

(1) $\dfrac{1}{6}$ (2) $\dfrac{1}{2}$

(3) 6 (4) 2

Official Ans. by NTA (4)

Sol. $$\lim_{x\to 0}\frac{\left(x+\frac{x^3}{3!}\ldots\right)-\left(x-\frac{x^3}{3}\ldots\right)}{3x^3}=\frac{1}{6}$$

So 6L + 1 = 2

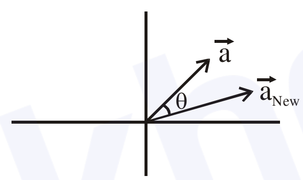
10. A vector $\vec{a}$ has components 3p and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to new system, $\vec{a}$ has components p + 1 and $\sqrt{10}$, then a value of p is equal to:

(1) 1 (2) $-\dfrac{5}{4}$ (3) $\dfrac{4}{5}$ (4) –1

Official Ans. by NTA (4)

Sol. $\vec{a}_{Old}=3p\hat{i}+\hat{j}$

$\vec{a}_{New}=(p+1)\hat{i}+\sqrt{10}\hat{j}$



$\Rightarrow\left|\vec{a}_{Old}\right|=\left|\vec{a}_{New}\right|$

$\Rightarrow ap^2 + 1 = p^2 + 2p + 1 + 10$

$8p^2 - 2p - 10 = 0$

$4p^2 - p - 5 = 0$

$(4p - 5)(p + 1) = 0 \rightarrow p = \dfrac{5}{4}, -1$

11. If the equation $a|z|^2+\overline{\overline{\alpha}z+\alpha\overline{z}}+d=0$ represents a circle where a,d are real constants then which of the following condition is correct ?

(1) $|\alpha|^2-ad\neq 0$

(2) $|\alpha|^2 - ad > 0$ and $a \in R - \{0\}$

(3) $|\alpha|^2 - ad \geq 0$ and $a \in R$

(4) $\alpha = 0$, $a,d \in R^+$

Official Ans. by NTA (2)

Sol. $az\overline{z}+\alpha\overline{z}+\overline{\alpha}z+d=0 \rightarrow$ Circle

centre $=\dfrac{-\alpha}{a}$ $\quad 2=\sqrt{\dfrac{\alpha\overline{\alpha}}{a^2}-\dfrac{d}{a}}=\sqrt{\dfrac{\alpha\overline{\alpha}-ad}{a^2}}$

So $|\alpha|^2 - ad > 0$ & $a \in R - \{0\}$

12. For the four circles M, N, O and P, following four equations are given :

Circle M : $x^2 + y^2 = 1$

Circle N : $x^2 + y^2 - 2x = 0$

Circle O : $x^2 + y^2 - 2x - 2y + 1 = 0$

Circle P : $x^2 + y^2 - 2y = 0$

If the centre of circle M is joined with centre of the circle N, further centre of circle N is joined with centre of the circle O, centre of circle O is joined with the centre of circle P and lastly, centre of circle P is joined with centre of circle M, then these lines form the sides of a :

(1) Rhombus
(2) Square
(3) Rectangle
(4) Parallelogram

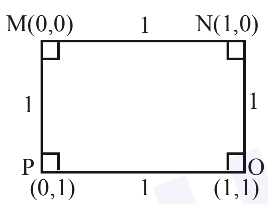
Official Ans. by NTA (2)

Sol. M : $x^2 + y^2 = 1$ (0,0)

N : $x^2 + y^2 - 2x = 0$ (1,0)

O : $x^2 + y^2 - 2x - 2y + 1 = 0$ (1,1)

P : $x^2 + y^2 - 2y = 0$ (0,1)



13. If α, β are natural numbers such that $100^\alpha - 199\beta = (100)(100) + (99)(101) + (98)(102) + ... + (1)(199)$, then the slope of the line passing through (α,β) and origin is :

(1) 540
(2) 550
(3) 530
(4) 510

Official Ans. by NTA (2)

Sol. $S = (100)(100) + (99)(101) + (98)(102) .....$

$$...(2)(198)+(1)(199)$$

99

2 2

$$= 100^3 - \frac{99 \times 100 \times 199}{6}$$

α = 3 β = 1650

$$\text{slope} = \frac{1650}{3} = 550$$

14. The real valued function $f(x) = \frac{\operatorname{cosec}^{-1} x}{\sqrt{x - [x]}}$, where [x] denotes the greatest integer less than or equal to x, is defined for all x belonging to :

(1) all reals except integers
(2) all non-integers except the interval [–1,1]
(3) all integers except 0,–1,1
(4) all reals except the Interval [–1,1]

Official Ans. by NTA (2)

Sol. $f(x) = \frac{\operatorname{cosec}^{-1} x}{\sqrt{\{x\}}}$

Domain $\in (-\infty, -1] \cup [1, \infty)$

$\{x\} \neq 0$ so $x \neq$ integers

15. $\frac{1}{3^2 - 1} + \frac{1}{5^2 - 1} + \frac{1}{7^2 - 1} + ... + \frac{1}{(201)^2 - 1}$ is equal to

(1) $\frac{101}{404}$ (2) $\frac{25}{101}$ (3) $\frac{101}{408}$ (4) $\frac{99}{400}$

Official Ans. by NTA (2)

Sol. $$T_n = \frac{1}{(2n+1)^2 - 1} \frac{1}{(2n+2)2n} = \frac{1}{4(n)(n+1)}$$

$$= \frac{(n+1) - n}{4n(n+1)} = \frac{1}{4}\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$S = \frac{1}{4}\left(1 - \frac{1}{101}\right) = \frac{1}{4}\left(\frac{100}{101}\right) = \frac{25}{101}$$

16. If the functions are defined as $f(x)=\sqrt{x}$ and $g(x)=\sqrt{1-x}$, then what is the common domain of the following functions :

$f + g$, $f - g$, f/g, g/f, $g - f$ where $(f \pm g)(x) = f(x) \pm g(x)$, $(f/g)(x) = \frac{f(x)}{g(x)}$

(1) $0 \le x \le 1$
(2) $0 \le x < 1$
(3) $0 < x < 1$
(4) $0 < x \le 1$

Official Ans. by NTA (3)

Sol. $f(x) + g(x) = \sqrt{x}+\sqrt{1-x}$, domain [0, 1]

$f(x) - g(x) = \sqrt{x}-\sqrt{1-x}$, domain [0, 1]

$g(x) - f(x) = \sqrt{1-x}-\sqrt{x}$, domain [0, 1]

$\frac{f(x)}{g(x)}=\frac{\sqrt{x}}{\sqrt{1-x}}$, domain [0, 1)

$\frac{g(x)}{f(x)}=\frac{\sqrt{1-x}}{\sqrt{x}}$, domain (0, 1]

So, common domain is (0, 1)

17. If $f(x)=\begin{cases}\frac{1}{|x|} & ; \ |x|\ge 1\\ ax^2+b & ; \ |x|<1\end{cases}$ is differentiable at every point of the domain, then the values of a and b are respectively :

(1) $\frac{1}{2},\frac{1}{2}$

(2) $\frac{1}{2},-\frac{3}{2}$

(3) $\frac{5}{2},-\frac{3}{2}$

(4) $-\frac{1}{2},\frac{3}{2}$

Official Ans. by NTA (4)

Sol. $f(x)=\begin{cases}\frac{1}{|x|}, & |x|\ge 1\\ ax^2+b, & |x|<1\end{cases}$

at x = 1 function must be continuous

So, 1 = a + b ...(1)

differentiability at x = 1

$\left(-\frac{1}{x^2}\right)_{x=1}=(2ax)_{x=1}$

$\Rightarrow -1 = 2a \Rightarrow a = -\frac{1}{2}$

(1) $\Rightarrow b = 1 + \frac{1}{2} = \frac{3}{2}$

18. Let $A+2B=\begin{bmatrix}1 & 2 & 0\\ 6 & -3 & 3\\ -5 & 3 & 1\end{bmatrix}$ and $2A-B=\begin{bmatrix}2 & -1 & 5\\ 2 & -1 & 6\\ 0 & 1 & 2\end{bmatrix}$. If Tr(A) denotes the sum of all diagonal elements of the matrix A, then Tr(A) – Tr(B) has value equal to

(1) 1 (2) 2 (3) 0 (4) 3

Official Ans. by NTA (2)

Sol. $A+2B=\begin{pmatrix}1 & 2 & 0\\ 6 & -3 & 3\\ -5 & 3 & 1\end{pmatrix}$ ...(1)

$2A-B=\begin{pmatrix}2 & -1 & 5\\ 2 & -1 & 6\\ 0 & 1 & 2\end{pmatrix}$

$\Rightarrow 4A-2B=\begin{pmatrix}4 & -2 & 10\\ 4 & -2 & 12\\ 0 & 2 & 4\end{pmatrix}$ ...(2)

$(1) + (2) \Rightarrow 5A = \begin{pmatrix} 5 & 0 & 10 \\ 10 & -5 & 15 \\ -5 & 5 & 5 \end{pmatrix}$

$A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{pmatrix}$ and $2A = \begin{pmatrix} 2 & 0 & 4 \\ 4 & -2 & 6 \\ -2 & 2 & 2 \end{pmatrix}$

$\therefore B = \begin{pmatrix} 2 & 0 & 4 \\ 4 & -2 & 6 \\ -2 & 2 & 2 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{pmatrix}$

$B = \begin{pmatrix} 0 & 1 & -1 \\ 2 & -1 & 0 \\ -2 & 1 & 0 \end{pmatrix}$

$tr(A) = 1 - 1 + 1 = 1$

$tr(B) = -1$

tr(A) = 1 and tr(B) = –1

$\therefore$ tr(A) – tr(B) = 2

19. The sum of all the 4-digit distinct numbers that can be formed with the digits 1, 2, 2 and 3 is:

(1) 26664 (2) 122664

(3) 122234 (4) 22264

Official Ans. by NTA (1)

Sol. Digits are 1, 2, 2, 3

total distinct numbers $\frac{4!}{2!} = 12$.

total numbers when 1 at unit place is 3.

2 at unit place is 6

3 at unit place is 3.

So, sum $= (3 + 12 + 9)(10^3 + 10^2 + 10 + 1)$

$= (1111) \times 24$

$= 26664$

20. The value of $3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + \ldots\infty}}}}$ is equal to

(1) $1.5 + \sqrt{3}$ (2) $2 + \sqrt{3}$

(3) $3 + 2\sqrt{3}$ (4) $4 + \sqrt{3}$

Sol. Let $x = 3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + \ldots\infty}}}}$

So, $x = 3 + \cfrac{1}{4 + \cfrac{1}{x}} = 3 + \cfrac{1}{\cfrac{4x + 1}{x}}$

$\Rightarrow (x - 3) = \frac{x}{(4x + 1)}$

$\Rightarrow (4x + 1)(x - 3) = x$

$\Rightarrow 4x^2 - 12x + x - 3 = x$

$\Rightarrow 4x^2 - 12x - 3 = 0$

$x = \frac{12 \pm \sqrt{(12)^2 + 12 \times 4}}{2 \times 4} = \frac{12 \pm \sqrt{12(16)}}{8}$

$= \frac{12 \pm 4 \times 2\sqrt{3}}{8} = \frac{3 \pm 2\sqrt{3}}{2}$

$x = \frac{3}{2} \pm \sqrt{3} = 1.5 \pm \sqrt{3}$.

But only positive value is accepted

So, $x = 1.5 + \sqrt{3}$

SECTION-B

1. The number of times the digit 3 will be written when listing the integers from 1 to 1000 is

Official Ans. by NTA (300)

Sol. 3_ _ = 10 × 10 = 100

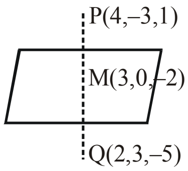
_3 _ = 10 × 10 = 100

_ _ 3 = 10 × 10 = $\frac{100}{300}$

2. Let the plane $ax + by + cz + d = 0$ bisect the line joining the points $(4,-3,1)$ and $(2, 3, -5)$ at the right angles. If a, b, c, d are integers, then the minimum value of $(a^2 + b^2 + c^2 + d^2)$ is

Official Ans. by NTA (28)

Sol.



Plane is $1(x - 3) - 3(y - 0) + 3(z + 2) = 0$

$x - 3y + 3z + 3 = 0$

$(a^2 + b^2 + c^2 + d^2)_{min} = 28$

3. Let $f(x)$ and $g(x)$ be two functions satisfying $f(x^2) + g(4 - x) = 4x^3$ and $g(4 - x) + g(x) = 0$, then the value of $\int_{-4}^{4} f(x)^2 dx$ is

Official Ans. by NTA (512)

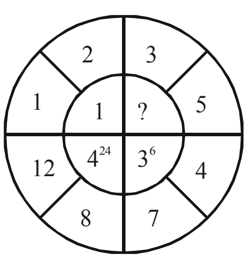
Sol. $I = 2\int_0^4 f(x^2)dx$ {Even funtion}

$= 2\int_0^4 (4x^3 - g(4-x))dx$

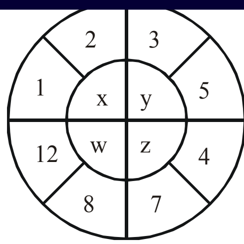
$= 2\left(\frac{4x^4}{4}\Bigg|_0^4 - \int_0^4 g(4-x)dx\right)$

$= 2(256 - 0) = 512$

4. The missing value in the following figure is



Sol. $x = (2 - 1)^{1!} = 1$



$w = (12 - 8)^{4!} = 4^{24}$

$z = (7 - 4)^{3!} = 3^6$

hence $y = (5 - 3)^{2!} = 2^2$

5. Let z_1, z_2 be the roots of the equation $z^2 + az + 12 = 0$ and z_1, z_2 form an equilateral triangle with origin. Then, the value of $|a|$ is

Official Ans. by NTA (6)

Sol. If $0, z, z_2$ are vertices of equilateral triangles

$\Rightarrow a^2 + z_1^2 + z_2^2 = 0\ (z_1 + z_2) + z_1z_2$

$\Rightarrow (z_1 + z_2)^2 = 3z_1z_2$

$\Rightarrow a^2 = 3 \times 12$

$\Rightarrow |a| = 6$

6. The equation of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ which are at unit distance from the point (1, 2, 3) is $ax + by + cz + d = 0$. If $(b - d) = K(c - a)$, then the positive value of K is

Official Ans. by NTA (4)

Sol. Let plane is $x - 2y + 2z + \lambda = 0$

distance from (1,2,3) = 1

$\Rightarrow \frac{|\lambda + 3|}{5} = 1 \Rightarrow \lambda = 0, -6$

$\Rightarrow a = 1, b = -2, c = 2, d = -6$ or 0

$b - d = 4$ or -2, $c - a = 1$

$\Rightarrow k = 4$ or -2

7. The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If the mean age of the teachers in this school now is 39 years, then the age (in years) of the newly appointed teacher is_.

Official Ans. by NTA (35)

Sol. $\dfrac{\sum x_i}{25} = 40 \ \& \ \dfrac{\sum x_i - 60 + N}{25} = 39$

Let age of newly appointed teacher is N

$\Rightarrow 1000 - 60 + N = 975$

$\Rightarrow N = 35$ years

8. If $f(x) = \int \dfrac{5x^8 + 7x^6}{\left(x^2 + 1 + 2x^7\right)^2} dx, (x \geq 0), f(0) = 0$

and $f(1) = \dfrac{1}{K}$, then the value of K is

Official Ans. by NTA (4)

Sol. $f(x) = \int \dfrac{\left(5x^8 + 7x^6\right)dx}{x^{14}\left(x^{-5} + x^{-7} + 2\right)^2}$

Let $x^{-5} + x^{-7} + 2 = t$

$(-5x^{-6} - 7x^{-8})dx = dt$

$\Rightarrow f(x) = \int -\dfrac{dt}{t^2} = \dfrac{1}{t} + c$

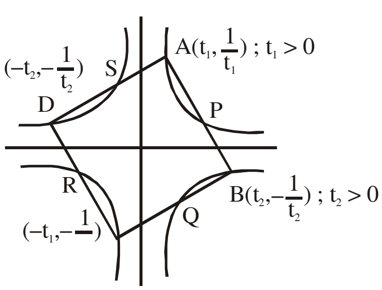
$f(x) = \dfrac{x^7}{x^2 + 1 + 2x^7}$

$f(1) = \dfrac{1}{4}$

9. A square ABCD has all its vertices on the curve $x^2y^2 = 1$. The midpoints of its sides also lie on the same curve. Then, the square of area of ABCD is

Official Ans. by NTA (80)

Sol. $xy = 1, -1$



$\dfrac{t_1 + t_2}{2} \cdot \dfrac{\frac{1}{t_1} - \frac{1}{t_2}}{2} = 1$

$\Rightarrow t_1^2 - t_2^2 = 4t_1t_2$

$\dfrac{1}{t_1^2} \times \left(-\dfrac{1}{t_2^2}\right) = -1 \Rightarrow t_1t_2 = 1$

$\Rightarrow (t_1t_2)^2 = 1 \Rightarrow t_1t_2 = 1$

$t_1^2 - t_2^2 = 4$

$\Rightarrow t_1^2 + t_2^2 = \sqrt{4^2 + 4} = 2\sqrt{5}$

$\Rightarrow t_1^2 = 2 + \sqrt{5} \Rightarrow \dfrac{1}{t_1^2} = \sqrt{5} - 2$

$AB^2 = (t_1 - t_2)^2 + \left(\dfrac{1}{t_1} + \dfrac{1}{t_2}\right)^2$

$= 2\left(t_1^2 + \dfrac{1}{t_1^2}\right) = 4\sqrt{5} \Rightarrow \text{Area}^2 = 80$

10. The number of solutions of the equation $|\cot x| = \cot x + \dfrac{1}{\sin x}$ in the interval $[0, 2\pi]$ is

Official Ans. by NTA (1)

Sol. If $\cot x > 0 \Rightarrow \dfrac{1}{\sin x} = 0$ (Not possible)

If $\cot x < 0 \Rightarrow 2\cot x + \dfrac{1}{\sin x} = 0$

$\Rightarrow 2\cos x = -1$

$\Rightarrow x = \dfrac{2\pi}{3} \text{ or } \dfrac{4\pi}{3}$ (reject)