FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 25th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

SECTION-A

1. For $z \in \mathbb{C}$ if the minimum value of $\left(|z-3\sqrt{2}|+|z-p\sqrt{2}i|\right)$ is $5\sqrt{2}$, then a value of p is ________

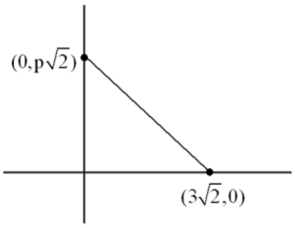
(A) 3 (B) $\frac{7}{2}$

(C) 4 (D) $\frac{9}{2}$

Official Ans. by NTA (C)

Ans. (C)

Sol.



$$\sqrt{(3\sqrt{2})^2+(p\sqrt{2})^2}=5\sqrt{2}$$

$18 + 2p^2 = 50$

$p^2 = 16$

$p = \pm 4$

As per option (C) is correct

2. The number of real values λ, such that the system of linear equations

$2x - 3y + 5z = 9$

$x + 3y - z = -18$

$3x - y + (\lambda^2 - |\lambda|)z = 16$

has no solution, is :-

(A) 0 (B) 1

(C) 2 (D) 4

Official Ans. by NTA (C)

Ans. (C)

Sol. $2x - 3y + 5z = 9$

$x + 3y - z = -18$

$3x - y + (\lambda^2 - |\lambda|)z = 16$

$$D=\begin{vmatrix}2 & -3 & 5\\ 1 & 3 & -1\\ 3 & -1 & \lambda^2-|\lambda|\end{vmatrix}=0$$

$\Rightarrow 3\lambda^2 - 3|\lambda| - 11 = 0$

Clearly one negative and one positive root since $|\lambda|$ is there so negative not possible and two values of λ corresponding to positive value

$$D_3=\begin{vmatrix}2 & -3 & 9\\ 1 & 3 & -18\\ 3 & -1 & 16\end{vmatrix}\neq 0 \text{ so no solution.}$$

3. The number of bijective functions f : {1, 3, 5, 7,........99} → {2, 4, 6, 8,......., 100}, such that $f(3) \geq f(9) \geq f(15) \geq f(21) \geq ..... \geq f(99)$, is _____

(A) $^{50}P_{17}$ (B) $^{50}P_{33}$

(C) 33! × 17! (D) $\frac{50!}{2}$

Official Ans. by NTA (B)

Ans. (B)

Sol. f : {1, 3, 5, 7, ...99} → {2, 4, 6, 8,.....,100}

$f(3) \geq f(9) \geq f(15) \geq .... f(99)$ ..... (1)

3, 9, 15,.....99 ⇒ 17 numbers

for condition one we have $^{50}C_{17}$ × 1 way rest 33 elements 33 !

$= {}^{50}C_{17} \times 33!$

$= {}^{50}C_{33} \times 33!$

$= {}^{50}P_{33}$.

4. The remainder when $(11)^{1011} + (1011)^{11}$ is divided by 9 is

(A) 1 (B) 4

(C) 6 (D) 8

Official Ans. by NTA (D)

Ans. (D)

Sol. $\frac{(9+2)^{1011}}{9}+\frac{(1008+3)^{11}}{9}$

$=\frac{2^{1011}}{9}+\frac{3^{11}}{9}$

$=\frac{(8)^{337}}{9}+\frac{3^{11}}{9}$

$=\frac{(9-1)^{337}}{9}+0$

$= (-1)^{337} + 9$

$= 8$

5. The sum $\sum_{n=1}^{21}\frac{3}{(4n-1)(4n+3)}$ is equal to

(A) $\frac{7}{87}$ (B) $\frac{7}{29}$

(C) $\frac{14}{87}$ (D) $\frac{21}{29}$

Official Ans. by NTA (B)

Ans. (B)

Sol. $\sum_{n=1}^{21}\frac{3}{(4n-1)(4n+3)}$

$=\frac{3}{4}\sum_{n=1}^{21}\frac{(4n+3)-(4n-1)}{(4n-1)(4n+3)}$

$=\frac{3}{4}\sum_{n=1}^{21}\frac{1}{4n-1}-\frac{1}{4n+3}$

$=\frac{3}{4}\left(\frac{1}{3}-\frac{1}{7}+\frac{1}{7}-\frac{1}{11}+\frac{1}{11}-....+\frac{1}{83}-\frac{1}{87}\right)$

$=\frac{3}{4}\left(\frac{1}{3}-\frac{1}{87}\right)=\frac{7}{29}$

6. $\lim_{x\to\frac{\pi}{4}}\frac{8\sqrt{2}-(\cos x+\sin x)^7}{\sqrt{2}-\sqrt{2}\sin 2x}$ is equal to

(A) 14 (B) 7

(C) $14\sqrt{2}$ (D) $7\sqrt{2}$

Official Ans. by NTA (A)

Ans. (A)

Sol. sinx + cosx = t

1 + sin2x = t^2

sin2x = $t^2 - 1$

$\lim_{t\to\sqrt{2}}\frac{8\sqrt{2}-t^7}{\sqrt{2}-\sqrt{2}(t^2-1)}$

$\lim_{t\to\sqrt{2}}\frac{8\sqrt{2}-t^7}{2\sqrt{2}-\sqrt{2}t^2}$ (L-Hospital Rule)

$\lim_{t\to\sqrt{2}}\frac{-7t^6}{-2\sqrt{2}t}=\lim_{t\to\sqrt{2}}\frac{7}{2\sqrt{2}}\times t^5$

$=\frac{7}{2\sqrt{2}}\times(\sqrt{2})^5=14$

7. $\lim_{n\to\infty}\frac{1}{2^n}\left(\frac{1}{\sqrt{1-\frac{1}{2^n}}}+\frac{1}{\sqrt{1-\frac{2}{2^n}}}+\frac{1}{\sqrt{1-\frac{3}{2^n}}}+......+\frac{1}{\sqrt{1-\frac{2^n-1}{2^n}}}\right)$

is equal to

(A) $\frac{1}{2}$ (B) 1

(C) 2 (D) –2

Official Ans. by NTA (C)

Ans. (C)

Sol. $\lim_{n\to\infty}\frac{1}{2^n}\sum_{r=1}^{2^n}\frac{1}{\sqrt{1-\frac{r}{2^n}}}$

$\therefore \frac{1}{2^n}\to dx \Leftarrow \frac{r}{2^n}=x \quad (\frac{r}{n'}=x,\frac{1}{x}=dx)$

$2^n = n'$

$\lim_{n'\to\infty}\frac{1}{n'}\sum_{r=1}^{n'-1}\frac{1}{\sqrt{1-\frac{r}{n'}}}=\int_0^1\frac{1}{\sqrt{1-x}}dx$

$=-\left.\frac{(1-x)^{1/2}}{1/2}\right]_0^1=-2[0-1]=2$

8. If A and B are two events such that $P(A)=\frac{1}{3}$, $P(B)=\frac{1}{5}$ and $P(A\cup B)=\frac{1}{2}$, then $P(A|B')+P(B|A')$ is equal to

(A) $\frac{3}{4}$ (B) $\frac{5}{8}$

(C) $\frac{5}{4}$ (D) $\frac{7}{8}$

Official Ans. by NTA (B)

Ans. (B)

Sol. $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\frac{1}{2}=\frac{1}{3}+\frac{1}{5}-P(A\cap B)$

$P\left(\frac{A}{\bar{B}}\right)+P\left(\frac{B}{\bar{A}}\right)=\frac{P(A\cap\bar{B})}{P(\bar{B})}+\frac{P(B\cap\bar{A})}{P(\bar{A})}$

$=\frac{P(A)-P(A\cap B')}{1-P(B)}+\frac{P(B)-P(A\cap B)}{1-P(A)}$

$=\frac{\frac{1}{3}-\frac{1}{30}}{\frac{4}{5}}+\frac{\frac{1}{5}-\frac{1}{30}}{\frac{2}{3}}=\frac{5}{8}$

9. Let [t] denote the greatest integer less than or equal to t. Then the value of the integral $\int_{-3}^{101}([\sin(\pi x)]+e^{[\cos(2\pi x)]})dx$ is equal to

(A) $\frac{52(1-e)}{e}$ (B) $\frac{52}{e}$

(C) $\frac{52(2+e)}{e}$ (D) $\frac{104}{e}$

Official Ans. by NTA (B)

Ans. (B)

Sol. $\int_{-3}^{101}([\sin \pi x]+e^{[\cos 2\pi x]})\,dx$

$52\int_{0}^{2}([\sin \pi x]+e^{[\cos 2\pi x]})\,dt$

$\frac{52}{\pi}\int_{0}^{2\pi}([\sin t]+e^{[\cos 2t]})\,dt$

$\frac{52}{\pi}\left(\int_{0}^{2\pi}([\sin t]dt+\int_{0}^{2\pi}e^{[\cos 2t]}\,dt\right)$

$I_1=\int_{0}^{2\pi}[\sin t]dt$

Using King

$I_1=\int_{0}^{2\pi}[-\sin t]dt$

$2I_1=\int_{0}^{2\pi}(-1)\,dt=-2\pi$

$I_1=-\pi$

$I_2=2\int_{0}^{\pi}e^{[\cos 2t]}dt$

$=2.2\int_{0}^{\pi/2}e^{[\cos 2t]}dt$

$=4\left(\int_{0}^{\pi/4}e^{0}.dt+\int_{\pi/4}^{\pi/2}e^{-1}dt\right)$

$4\left(\frac{\pi}{4}+e^{-1}\left(\frac{\pi}{4}\right)\right)=\pi(1+e^{-1})$

$I=\frac{52}{\pi}(-\pi+\pi+\pi e^{-1})=\frac{52}{e}$

10. Let the point P (α, β) be at a unit distance from each of the two lines L_1 : 3x – 4y + 12 = 0, and L_2 : 8x + 6y + 11 = 0. If P lies below L_1 and above L_2, then 100 $(\alpha + \beta)$ is equal to

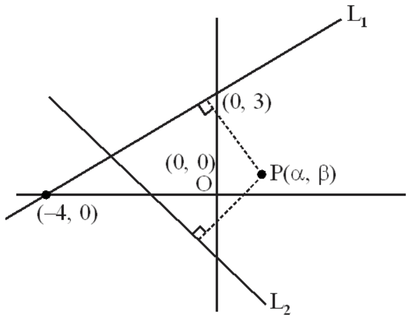
(A) –14 (B) 42

(C) –22 (D) 14

Official Ans. by NTA (D)

Ans. (D)

Sol.



By observing origin and P lies in same region.

$L_1(0, 0) > 0;\ L_1(\alpha, \beta) > 0 \Rightarrow 3\alpha-4\beta+12>0$

$1=\left|\frac{3\alpha-4\beta+12}{5}\right|$

$3\alpha-4\beta+12=5$ ....(1)

Similarly for L_2

$L_2(0, 0) > 0;\ L_2(\alpha, \beta) > 0$

$1=\left|\frac{8\alpha+6\beta+11}{10}\right| \Rightarrow 8\alpha+6\beta+11=10$ .....(2)

Solving (1) and (2)

$\alpha=-\frac{23}{25};\beta=\frac{106}{100}$

$100(\alpha+\beta)=100\left(\frac{-92}{100}+\frac{106}{100}\right)=14$

11. Let a smooth curve y = f(x) be such that the slope of the tangent at any point (x, y) on it is directly proportional to $\left(\frac{-y}{x}\right)$. If the curve passes through the point (1, 2) and (8, 1), then $\left|y\left(\frac{1}{8}\right)\right|$ is equal to

(A) $2\log_e 2$ (B) 4

(C) 1 (D) $4\log_e 2$

Official Ans. by NTA (B)

Ans. (B)

Sol. $\frac{dy}{dx}=-\frac{\alpha y}{x}$

$\frac{dy}{y}=-\frac{\alpha}{x}dx$

$\Rightarrow \frac{dy}{y}+\frac{\alpha}{x}dx=0$

$\Rightarrow \ell ny+\alpha\ell nx=\ell nc$

$\Rightarrow yx^{\alpha}=c$

For (1, 2) $\Rightarrow 2.1^{\alpha} = c \Rightarrow c = 2$

For (8, 1) $\Rightarrow 1.8^{\alpha} = 2 \Rightarrow \alpha = \frac{1}{3}$

$\therefore$ curve is $y = 2x^{-1/3}$

At x = 1/8, y(1/8) = $2\left(\frac{1}{8}\right)^{-\frac{1}{3}} \Rightarrow y = 4$

12. If the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the line $\frac{x}{7}+\frac{y}{2\sqrt{6}}=1$ on the x-axis and the line $\frac{x}{7}-\frac{y}{2\sqrt{6}}=1$ on the y-axis, then the eccentricity of the ellipse is

(A) $\frac{5}{7}$ (B) $\frac{2\sqrt{6}}{7}$

(C) $\frac{3}{7}$ (D) $\frac{2\sqrt{5}}{7}$

Official Ans. by NTA (A)

Ans. (A)

Sol. Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through the point (7, 0) and $(0,-2\sqrt{6})$

Now $\frac{49}{a^2}+0=1 \Rightarrow a^2=49$

and $0+\frac{24}{b^2}=1 \Rightarrow b^2=24$

Now a > b $\Rightarrow b^2 = a^2(1-e^2)$

$\Rightarrow 24 = 49(1-e^2) \Rightarrow e^2 = \frac{25}{49}$

$\Rightarrow e = \frac{5}{7}$

13. The tangents at the point A(1, 3) and B(1, –1) on the parabola $y^2 - 2x - 2y = 1$ meet at the point P. Then the area (in unit²) of the triangle PAB is :-

(A) 4 (B) 6

(C) 7 (D) 8

Official Ans. by NTA (D)

Ans. (D)

Sol. Both point A(1, 3), B(1, –1) lies on the parabola $y^2 - 2y - 2x - 1 = 0$

Equation of tangent aty A(1, 3) is T = 0

$x - 2y + 5 = 0$

and equation of tangent at B(1, –1) is T = 0

$x + 2y + 1 = 0$

So point P is (–3, 1)

$$\Rightarrow A = \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 1 & -1 & 1 \\ -3 & 1 & 1 \end{vmatrix} = 8$$

14. Let the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{7}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{\alpha}=\frac{1}{25}$ coincide. Then the length of the latus rectum of the hyperbola is:-

(A) $\frac{32}{9}$ (B) $\frac{18}{5}$

(C) $\frac{27}{4}$ (D) $\frac{27}{10}$

Official Ans. by NTA (D)

Ans. (D)

Sol. $\frac{x^2}{16}+\frac{y^2}{7}=1$

$\Rightarrow 7 = 16(1-e^2) \Rightarrow e = \frac{3}{4}$

Foci of ellipse is $(\pm ae, 0) \Rightarrow (\pm 3, 0)$

Now hyperbola be $\frac{x^2}{144}-\frac{y^2}{\alpha}=\frac{1}{25}$

$$\frac{x^2}{\frac{144}{25}}-\frac{y^2}{\frac{\alpha}{25}}=1$$

Now $a = \frac{12}{5}$, $b^2 = \frac{\alpha}{25}$

Let eccentricity of hyperbola be e

ae = 3 (Given)

$\Rightarrow \frac{12}{5}e = 3 \Rightarrow e = \frac{5}{4}$

$b^2 = a^2(e^2 - 1)$

$\frac{\alpha}{25} = \frac{144}{25}\left(\frac{25}{16}-1\right) \Rightarrow \alpha = 81$

Hyperbola is $\frac{x^2}{\frac{144}{25}}-\frac{y^2}{\frac{81}{25}}=1$

Now length of LR = $\frac{2b^2}{a} = \frac{27}{10}$

15. A plane E is perpendicular to the two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$, and passes through the point P(1, –1, 1). If the distance of the plane E from the point Q(a, a, 2) is $3\sqrt{2}$, then $(PQ)^2$ is equal to

(A) 9 (B) 12

(C) 21 (D) 33

Official Ans. by NTA (C)

Ans. (C)

Sol. Let equation of plane be

a(x – 1) + b(y + 1) + c(z – 1) = 0 .....(1)

It is perpendicular to the given two planes

2a – 2b + c = 0

a – b + 2c = 0

$\Rightarrow \frac{a}{3} = \frac{b}{3} = \frac{c}{0}$

Equation of plane be x + y = 0

Now $\frac{|a+a|}{\sqrt{2}} = 3\sqrt{2} \Rightarrow |2a| = 6 \Rightarrow a = \pm 3$

P(3, 3, 2) or P(–3, –3, 2), Q(1, –1, 1)

$PQ^2 = (3-1)^2 + (3+1)^2 + (2-1)^2 = 21$

16. The shortest distance between the lines $\frac{x+7}{-6} = \frac{y-6}{7} = z$ and $\frac{7-x}{2} = y-2 = z-6$ is

(A) $2\sqrt{29}$ (B) 1

(C) $\sqrt{\frac{37}{29}}$ (D) $\frac{\sqrt{29}}{2}$

Official Ans. by NTA (A)

Ans. (A)

Sol. $\frac{x+7}{-6} = \frac{y-6}{7} = \frac{z-0}{1}$:

point (–7, 6, 0) dr's –6, 7, 1

$\frac{x-7}{2} = \frac{y-2}{-1} = \frac{z-6}{-1}$:

point (7, 2, 6) dr's 2, –1, –1

Shortest distance

$$= \left| \frac{\begin{vmatrix} 14 & -4 & 6 \\ -6 & 7 & 1 \\ 2 & -1 & -1 \end{vmatrix}}{\sqrt{(-7+1)^2 + (6-2)^2 + (6-14)^2}} \right| = 2\sqrt{29}$$

17. Let $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b}$ be a vector such that $\vec{a} \times \vec{b} = 2\hat{i} - \hat{k}$ and $\vec{a}.\vec{b} = 3$. Then the projection of $\vec{b}$ on the vector $\vec{a} - \vec{b}$ is :-

(A) $\frac{2}{\sqrt{21}}$ (B) $2\sqrt{\frac{3}{7}}$

(C) $\frac{2}{3}\sqrt{\frac{7}{3}}$ (D) $\frac{2}{3}$

Official Ans. by NTA (A)

Ans. (A)

Sol. Projection of $\vec{b}$ r $\vec{a} - \vec{b}$

$= \frac{\vec{b}.(\vec{a} - \vec{b})}{|\vec{a} - \vec{b}|}$

$= \frac{\vec{a}.\vec{b} - |b|^2}{\sqrt{a^2 + b^2 - 2a \cdot b}} = \frac{3 - b^2}{\sqrt{6 + b^2 - 6}} = \frac{3 - b^2}{b}$

$\left|\vec{a} \times \vec{b}\right|^2 = 5$

$a^2b^2 - (a \cdot b)^2 = 5$

$6b^2 = 14 \Rightarrow b^2 = \frac{7}{3}$

$\therefore \frac{3 - b^2}{b} = \frac{3 - \frac{7}{3}}{\sqrt{\frac{7}{3}}} = 2 \times \sqrt{21}$

18. If the mean deviation about median for the number 3, 5, 7, 2k, 12, 16, 21, 24 arranged in the ascending order, is 6 then the median is

(A) 11.5 (B) 10.5

(C) 12 (D) 11

Official Ans. by NTA (D)

Ans. (D)

Sol. 3, 5, 7, 2k, 12, 16, 21, 24

Median = $\frac{2k + 12}{2} = k + 6$

M.D. = $\frac{\Sigma}{}$ 6

= (k + 3) + (k + 1) + (k – 1) + (6 – k) + (6 – k) + (10 – k) + (15 – k) + (18 – k) = 48

= 58 – 2k = 48

k = 5

Median = k + 6 = 11

19. $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{}\right)$

is equal to

(A) $\frac{3}{16}$ (B) $\frac{1}{16}$

(C) $\frac{1}{32}$ (D) $\frac{9}{32}$

Official Ans. by NTA (B)

Ans. (B)

Sol. $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$

$$2\cos\left(\frac{\pi}{2}-\frac{\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{5\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{7\pi}{15}\right)$$

$$\cos\left(\frac{\pi}{2}-\frac{9\pi}{22}\right)$$

$$2\cos\left(\frac{10\pi}{22}\right)\cos\left(\frac{8\pi}{22}\right)\cos\left(\frac{6\pi}{22}\right)\cos\left(\frac{4\pi}{22}\right)\cos\left(\frac{2\pi}{22}\right)$$

$$2\cos\left(\frac{\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)\cos\left(\frac{3\pi}{11}\right)\cos\left(\frac{4\pi}{11}\right)\cos\left(\frac{5\pi}{11}\right)$$

$$2\cos\left(\frac{\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)\cos\left(\frac{4\pi}{11}\right)\cos\left(\pi-\frac{3\pi}{11}\right)\cos\left(\pi+\frac{5\pi}{11}\right)$$

$$2\cos\left(\frac{\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)\cos\left(\frac{4\pi}{11}\right)\cos\left(\frac{8\pi}{11}\right)\cos\left(\frac{16\pi}{11}\right)$$

$$\frac{2.\sin\left(2^5\times\frac{\pi}{11}\right)}{2^5\sin\frac{\pi}{11}}$$

$$\frac{2.\sin\left(\frac{32\pi}{11}\right)}{32\sin\frac{\pi}{11}}=\frac{1}{16}$$

20. Consider the following statements :

P : Ramu is intelligent

Q : Ramu is rich

R : Ramu is not honest

The negation of the statement "Ramu is intelligent and honest if and only if Ramu is not rich" can be expressed as :

(A) $((P\wedge(\sim R))\wedge Q)\wedge((\sim Q)\wedge((\sim P)\vee R))$

(B) $((P\wedge R)\wedge Q)\vee((\sim Q)\wedge((\sim P)\vee(\sim R)))$

(C) $((P\wedge R)\wedge Q)\wedge((\sim Q)\wedge((\sim P)\vee(\sim R)))$

(D) $((P\wedge(\sim R))\wedge Q)\vee((\sim Q)\wedge((\sim P)\vee R))$

Official Ans. by NTA (D)

Ans. (D)

Sol. Negation of $(P\wedge\sim R)\leftrightarrow(\sim Q)$

$\Rightarrow((P\wedge\sim R)\wedge Q)\vee(\sim Q\wedge\sim(P\wedge\sim R))$

$\Rightarrow((P\wedge\sim R)\wedge Q)\vee(\sim Q\wedge(\sim P\vee R))$

Answer D is correct

SECTION-B

1. Let A : {1, 2, 3, 4, 5, 6, 7}. Define B = $\{T\subseteq A$: either $1\notin T$ or $2\in T\}$ and C = $T\subseteq A$: T the sum of all the elements of T is a prime number}. Then the number of elements in the set $B\cup C$ is __________

Official Ans. by NTA (107)

Ans. (107)

Sol. A : {1, 2, 3, 4, 5, 6, 7}

Number of elements in set B

$= n(1\notin T)+n(2\in T)-n[(1\notin T)\cap(2\in T)]$

$=2^6+2^6-2^5=96$

Number of elements in set C

= {{2}, {3}, {5}, {7}, {1, 2}, {1, 4}, {1, 6}, {2, 3}, {2, 5}, {3, 4}, {4, 7}, {5, 6}, {6, 7}, {1, 2, 4}, {1, 3, 7}, {1, 4, 6}, {1, 5, 7}, {2, 3, 6}, {2, 4, 5}, {2, 4, 7}, {2, 5, 6}, {3, 4, 6}, {4, 6, 7}, {1, 2, 4, 6}, {2, 4, 6, 7}, {2, 4, 6, 5}, {3, 5, 7, 4}, {1, 3, 5, 4}, {1, 5, 7, 4}, {1, 2, 3, 5}, {1, 2, 3, 7}, {1, 3, 6, 7}, {1, 5, 6, 7}, {2, 3, 5, 7}, {1, 5, 7, 2, 4}, {3, 5, 7, 2, 6}, {1, 3, 7, 2, 4}, {1,4, 5, 6, 7}, {1, 3, 4, 5, 6}, {1, 2, 3, 6, 7}, {1, 2, 3, 5, 6}, {1, 2, 3, 4, 6, 7}

Number of elementrrs in C = 42

$\Rightarrow n(B\cup C)=n(B)+n(C)-n(B\cap C)$

= 96 + 42 – 31 = 107

2. Let f(x) be a quadratic polynomial with leading coefficient 1 such that f(0) = p, $p\neq 0$ and $f(1)=\frac{1}{3}$.

If the equation f(x) = 0 and fofofof(x) = 0 have a common real root, then f(–3) is equal to......................

Official Ans. by NTA (25)

Ans. (25)

Sol. Let $f(x)=x^2+bx+p$

$f(1)=\frac{1}{3}\Rightarrow 1+b+p=\frac{1}{3}$ ...(1)

Assume common root be α

$f(\alpha)=0$ & $f(f(f(f(\alpha))))=0$

$\Rightarrow f(f(p))=0$

$\Rightarrow f(p^2+bp+p)=0$

$\Rightarrow f(p(p+b+1))=0$

$\Rightarrow f\left(\frac{p}{3}\right)=0$

$\Rightarrow \frac{p^2}{9}+b\cdot\frac{p}{3}+p=0$

$\Rightarrow \frac{p}{9}+\frac{b}{3}+1=0$

$p+3b+9=0$ ...(2)

From (1) & (2) $\Rightarrow p = \frac{7}{2}$

Now, $f(-3) = 9 - 3b + p$

$= 9 - (-p - 9) + p$

$= 18 + 2p = 18 + 2 \times \frac{7}{2} = 25$

3. Let $A = \begin{bmatrix} 1 & a & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}, a, b \in \mathbb{R}$. If for some $n \in N$, $A^n = \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 96 \\ & & 1 \end{bmatrix}$ then n + a + b is equal to ________

Official Ans. by NTA (24)

Ans. (24)

Sol. $A^2 = \begin{bmatrix} 1 & a & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & a & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2a & 2a+ab \\ 0 & 1 & 2b \\ 0 & 0 & 1 \end{bmatrix}$

$A^2A = \begin{bmatrix} 1 & 2a & 2a+ab \\ 0 & 1 & 2b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & a & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 3a & 3a+3ab \\ 0 & 1 & 3b \\ 0 & 0 & 1 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 4a & 4a+6ab \\ 0 & 1 & 4b \\ 0 & 1 & 1 \end{bmatrix}$

$A^n = \begin{bmatrix} 1 & na & \frac{(n^2-n)}{2}ab + na \\ 0 & 1 & nb \\ 0 & 0 & 1 \end{bmatrix}$

na = 48, nb = 96

$na + \frac{nab}{2}(n-1) = 2160$

48 + 24b(n – 1) = 2160

48 + 24 × 96 – 24b = 2160

b = 8 and a = 4, n = 12

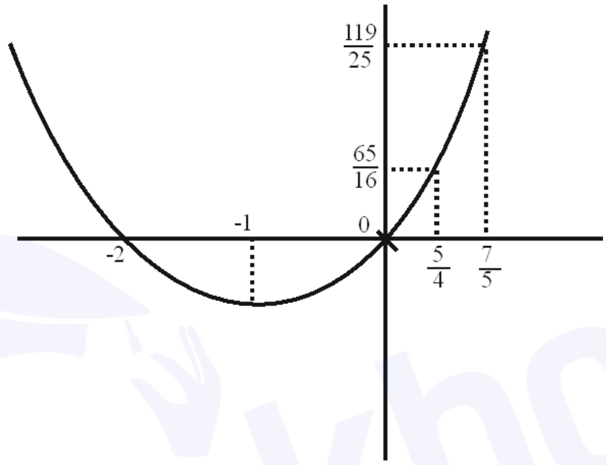
n + a + b = 24

4. The sum of the maximum and minimum values of the function f(x) = |5x – 7| + [x^2 + 2x] is the interval $\left[\frac{5}{4}, 2\right]$, where [t] is the greatest integer $\leq t$ is ________

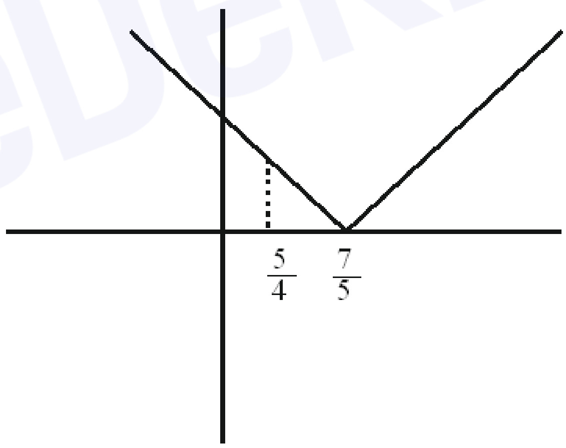
Official Ans. by NTA (15)

Ans. (15)

Sol. Graph of $x^2 + 2x$



Graph of |5x – 7|



$f(x)|_{min} = 4 + 0 = 4$, at $x = \frac{7}{5}$

$f(x)|_{max} = 8 + 3 = 11$, at x = 2

Required sum = 15

5. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = \frac{4y^3 + 2yx^2}{3xy^2 + x^3}, y(1) = 1$. If for some $n \in N, y(2) \in [n-1, n)$, then n is equal to ________

Official Ans. by NTA (3)

Ans. (3)

Sol. $\frac{dy}{dx}=\frac{4y^3+2yx^2}{3xy^2+x^3}, y(1)=1$

$\frac{dy}{dx}=\frac{4(y/x)^3+2(y/x)}{3(y/x)^2+1}$

$y = xp$

$x\frac{dp}{dx}+p=\frac{4p^3+2p}{3p^2+1}$

$x\frac{dp}{dx}=\frac{p^3+p}{3p^2+1}$

$\int\frac{3p^2+1}{p^3+p}dp=\int\frac{dx}{x}$

$ln(p^3 + p) = lnx + lnC$

$p^3 + p = xC$

$\left(\frac{y}{x}\right)^3+\left(\frac{y}{x}\right)=xC$

$y^3 + x^2y = x^4C$

$x = 1, y = 1$

$1 + 1 = C \Rightarrow C = 2$

$y^3 + x^2y = 2x^4$

Put $x = 2$

$y^3 + 4y - 32 = 0$

Having root between 2 and 3

$y(2) \in [2,3)$

6. Let f be a twice differentiable function on R. If $f'(0)=4$ and $f(x)+\int_0^x (x-t)f'(t)dt=(e^{2x}+e^{-2x})\cos 2x+\frac{2}{a}x$, then $(2a + 1)^5 a^2$ is equal to ________

Official Ans. by NTA (8)

Ans. (8)

Sol. $f'(0) = 4$

$f(x)+\int_0^x (x-t)f'(t)dt=(e^{2x}+e^{-2x})\cos 2x+\frac{2}{a}x$

Put $x = 0 : f(0) = 2$

$f'(x)+x(f'(x))+\int_0^x f'(t)dt-xf'(x)$

$=(e^{2x}+e^{-2x})(-2\sin 2x)+\cos 2x(2e^{2x}-2e^{-2x})+\frac{2}{a}$

$\Rightarrow f'(x)+f(x)-2=(e^{2x}+e^{-2x})(-2\sin 2x)$

$+\cos 2x(2e^{2x}-2e^{-2x})+\frac{2}{a}$

Put $x = 0$

$4 + 2 - 2 = 0 + (2 - 2) + 2/a$

$\Rightarrow a=\frac{1}{2}$

$(2a+1)^5a^2=2^5.\frac{1}{2^2}=8$

7. Let $a_n=\int_{-1}^{n}\left(1+\frac{x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+.......+\frac{x^{n-1}}{n}\right)dx$ for $n \in N$. Then the sum of all the elements of the set $\{n \in N : a_n \in (2,30)\}$ is ______

Official Ans. by NTA (5)

Ans. (5)

Sol. $\int_{-1}^{n}\left(1+\frac{x}{2}+\frac{x^2}{3}+...+\frac{x^{n-1}}{n}\right)dx$

$\left[x+\frac{x^2}{2}+\frac{x^3}{3^2}+...+\frac{x^n}{n^2}\right]_{-1}^{n}$

$\left(n+\frac{n^2}{2^2}+\frac{n^3}{3^2}+...+-\frac{n^n}{n^2}\right)$

$-\left(-1+\frac{1}{2^2}-\frac{1}{3^2}\quad\frac{}{4^2}\qquad\frac{}{n^2}\right)$

$a_n=(n+1)+\frac{1}{2^2}(n^2-1)+\frac{1}{3^2}(n^3+1)$

$+...+\frac{1}{n^2}\left(n^n-(-1)^n\right)$

if $n = 1 \Rightarrow a_n = 2 \notin (2,30)$

if $n = 2 \Rightarrow a_n=(2+1)+\frac{1}{2^2}(2^2-1)=3+\frac{3}{4}<30$

if $n = 3$

$\Rightarrow a_n=(3+1)+\frac{1}{4}(8)+\frac{1}{9}(28)=11+\frac{28}{9}<$

If $n=4 \Rightarrow a_n=(4+1)+\frac{1}{4}(16-1)+\frac{1}{9}(64+1)+\frac{1}{16}$

$=5+\frac{15}{4}+\frac{65}{9}+\frac{255}{16}>30$

Test {2, 3} sum of elements 5

8. If the circles $x^2 + y^2 + 6x + 8y + 16 = 0$ and $x^2+y^2+2(3-\sqrt{3})x+x+2(4-\sqrt{6})y$ $=k+6\sqrt{3}+8\sqrt{6}, k>0$ touch internally at the point $P(\alpha, \beta)$, then $(\alpha+\sqrt{3})^2+(\beta+\sqrt{6})^2$ is equal to __________

Official Ans. by NTA (25)

Ans. (25)

Sol. $C_1\ (-3, -4)$

$r_1 = \sqrt{25-16} = 3$

$C_2 = (-3+\sqrt{3}, -4+\sqrt{6})$

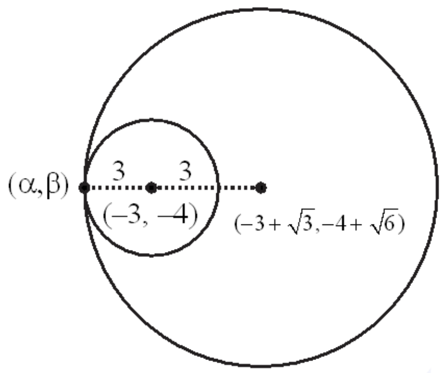
$r_2 = \sqrt{34+k}$

$C_1C_2 = |r_1 - r_2|$

$C_1C_2 = \sqrt{3+6} = 3$

$3 = |3 - \sqrt{34+k}| \Rightarrow k = 2$

$r_2 = 6$



$(\alpha, \beta) = (-\sqrt{3}-3, -4-\sqrt{6})$

$(\alpha+\sqrt{3})^2 + (\beta+\sqrt{6})^2 = 9+16 = 25$

9. Let the area enclosed by the x-axis, and the tangent and normal drawn to the curve $4x^3 - 3xy^2 + 6x^2 - 5xy - 8y^2 + 9x + 14 = 0$ at the point $(-2, 3)$ be A. Then 8A is equal to _____

Official Ans. by NTA (170)

Ans. (170)

Sol. $4x^3 - 3xy^2 + 6x^2 - 5xy - 8y^2 + 9x + 14 = 0$ at $P(-2, 3)$

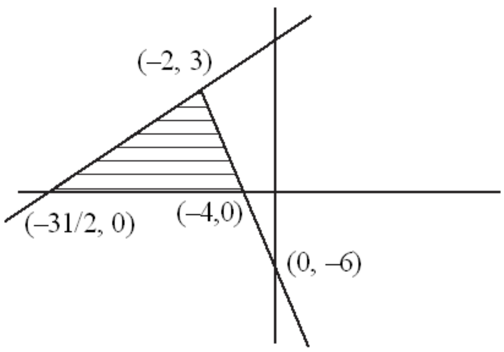
$12x^2 - 3(y^2 + 2yxy') + 12x - 5(xy' + y) - 16yy' + 9 = 0$

$48 - 3(9 - 12y') - 24 - 5(-2y' + 3) - 48y' + 9 = 0$

$y' = -9/2$

Tangent $y - 3 = -\frac{9}{2}(x+2) \Rightarrow 9x + 2y = -12$

Normal : $y - 3 = \frac{2}{9}(x+2) \Rightarrow 9y - 2x = 31$



$\text{Area} = \frac{1}{2}\left(\frac{31}{2} - 4\right) \times 3 = \frac{85}{4}$

$8A = 170$

10. Let $x = \sin(2\tan^{-1}\alpha)$ and $y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$. If $S = \{\alpha \in R : y^2 = 1 - x\}$, then $\sum_{\alpha \in S} 16\alpha^3$ is equal to _____

Official Ans. by NTA (130)

Ans. (130)

Sol. $x = \sin(2\tan^{-1}\alpha) = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\alpha}{1+\alpha^2}$

$\tan^{-1}\alpha = \theta \Rightarrow \tan\theta = \alpha$

$y^2 = \sin^2\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right) = \frac{1}{5}$

$y^2 + x = 1 \Rightarrow \frac{1}{5} + \frac{2\alpha}{1+\alpha^2} = 1$

$\frac{2\alpha}{1+\alpha^2} = \frac{4}{5}$

$(2\alpha - 1)(\alpha - 2) = 0$

$\Rightarrow 2\alpha^2 - 5\alpha + 2 = 0$

$\therefore \alpha = 2 \text{ or } \frac{1}{2}$

$S = \left\{2, \frac{1}{2}\right\}$

$\sum_{\alpha \in S} 16\alpha^3 = 16\left(8 + \frac{1}{8}\right) = 130$