CollegeDekho

MATHEMATICS

SECTION-A

1. The sum of all those terms which are rational numbers in the expansion of $(2^{1/3}+3^{1/4})^{12}$ is:

(1) 89 (2) 27 (3) 35 (4) 43

Official Ans. by NTA (4)

Sol. $T_{r+1} = {}^{12}C_r\left(2^{1/3}\right)^r.\left(3^{1/4}\right)^{12-r}$

T_{r+1} will be rational number

when r = 0, 3, 6, 9, 12

& r = 0, 4, 8, 12

$\Rightarrow$ r = 0, 12

$T_1 + T_{13} = 1 \times 3^3 + 1 \times 2^4 \times 1$

= 24 + 16 = 43

2. The first of the two samples in a group has 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with mean 15.6 and standard deviation $\sqrt{13.44}$, then the standard deviation of the second sample is :

(1) 8 (2) 6 (3) 4 (4) 5

Official Ans. by NTA (3)

Sol. $n_1 = 100$ $m = 250$

$\bar{X}_1 = 15$ $\bar{X} = 15.6$

$V_1(x) = 9$ $Var(x) = 13.44$

$$\sigma^2 = \frac{n_1\sigma_1^2+n_2\sigma_2^2}{n_1+n_2}+\frac{n_1n_2}{(n_1+n_2)^2}(\bar{x}_1-\bar{x}_2)^2$$

$n_2 = 150$, $\bar{x}_2 = 16$, $V_2(x) = \sigma_2$

$$13.44 = \frac{100\times9+150\times\sigma_2^2}{250}+\frac{100\times150}{(250)^2}\times1$$

$\Rightarrow \sigma_2^2 = 16 \Rightarrow \sigma_2 = 4$

3. If $f(x)=\begin{cases}\int_0^x (5+|1-t|)dt, & x>2\\ 5x+1, & x\le 2\end{cases}$, then

(1) f(x) is not continuous at x = 2

(2) f(x) is everywhere differentiable

(3) f(x) is continuous but not differentiable at x = 2

(4) f(x) is not differentiable at x = 1

Official Ans. by NTA (3)

Sol. $f(x) = \int_0^1 (5+(1-t))\,dt + \int_1^x (5+(t-1))\,dt$

$$= 6-\frac{1}{2}+\left(4t+\frac{t^2}{2}\right)\Bigg|_1^x$$

$$=\frac{11}{2}+4x+\frac{x^2}{2}-4-\frac{1}{2}$$

$$=\frac{x^2}{2}+4x+1$$

$f(2^+) = 2 + 8 + 1 = 11$

$f(2) = f(2^-) = 5 \times 2 + 1 = 11$

$\Rightarrow$ continuous at x = 2

Clearly differentiable at x = 1

$Lf'(2) = 5$

$Rf'(2) = 6$

$\Rightarrow$ not differentiable at x = 2

4. If the greatest value of the term independent of 'x' in the expansion of $\left(x\sin\alpha + a\frac{\cos\alpha}{x}\right)^{10}$ is $\frac{10!}{(5!)^2}$, then the value of 'a' is equal to:

(1) –1 (2) 1 (3) –2 (4) 2

Official Ans. by NTA (4)

Sol. $T_{r+1} = {}^{10}C_r(x\sin\alpha)^{10-r}\left(\frac{a\cos\alpha}{x}\right)^r$

r = 0, 1, 2, …, 10

T_{r+1} will be independent of x

when 10 – 2r = 0 $\Rightarrow$ r = 5

$$T_6 = {}^{10}C_5(x\sin\alpha)^5\times\left(\frac{a\cos\alpha}{x}\right)^5$$

$$= {}^{10}C_5\times a^5\times\frac{1}{2^5}(\sin2\alpha)^5$$

will be greatest when sin2α = 1

$$\Rightarrow {}^{10}C_5\frac{a^5}{2^5} = {}^{10}C_5 \Rightarrow a = 2$$

5. Consider the statement "The match will be played only if the weather is good and ground is not wet". Select the correct negation from the following:

(1) The match will not be played and weather is not good and ground is wet.

(2) If the match will not be played, then either weather is not good or ground is wet.

(3) The match will be played and weather is not good or ground is wet.

(4) The match will not be played or weather is good and ground is not wet.

Official Ans. by NTA (3)

Sol. p : weather is food

q : ground is not wet

$\sim(p \wedge q) \equiv \sim p \vee \sim q$

$\equiv$ weather is not good or ground is wet

6. The value of $\cot\frac{\pi}{24}$ is:

(1) $\sqrt{2}+\sqrt{3}+2-\sqrt{6}$ (2) $\sqrt{2}+\sqrt{3}+2+\sqrt{6}$

(3) $\sqrt{2}-\sqrt{3}-2+\sqrt{6}$ (4) $3\sqrt{2}-\sqrt{3}-\sqrt{6}$

Official Ans. by NTA (2)

Sol. $\cot\theta = \frac{1+\cos 2\theta}{\sin 2\theta} = \frac{1+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$

$\theta = \frac{\pi}{24}$

$\Rightarrow \cot\left(\frac{\pi}{24}\right) = \frac{1+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$

$= \frac{(2\sqrt{2}+\sqrt{3}+1)}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$

$= \frac{2\sqrt{6}+2\sqrt{2}+3+\sqrt{3}+\sqrt{3}+1}{2}$

$= \sqrt{6}+\sqrt{2}+\sqrt{3}+2$

7. The lowest integer which is greater than $\left(1+\frac{1}{10^{100}}\right)^{10^{100}}$ is ________.

(1) 3 (2) 4 (3) 2 (4) 1

Official Ans. by NTA (1)

Sol. Let $P = \left(1+\frac{1}{10^{100}}\right)^{10^{100}}$,

Let $x = 10^{100}$

$\Rightarrow P = \left(1+\frac{1}{x}\right)^{x}$

$\Rightarrow P = 1+(x)\left(\frac{1}{x}\right)+\frac{(x)(x-1)}{\lfloor 2}\cdot\frac{1}{x^2}$

$+\frac{(x)(x-1)(x-2)}{\lfloor 3}\cdot\frac{1}{x^3}+\ldots$

(upto $10^{100}+1$ terms)

$\Rightarrow P = 1+1+\left(\frac{1}{\lfloor 2}-\frac{1}{\lfloor 2\, x^2}\right)+\left(\frac{1}{\lfloor 3}-\ldots\right)+\ldots$ so on

$\Rightarrow P = 2+\left(\text{Positive value less then } \frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}+\frac{1}{\lfloor 4}+\ldots\right)$

Also $e = 1+\frac{1}{\lfloor 1}+\frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}+\frac{1}{\lfloor 4}+\ldots$

$\Rightarrow \frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}+\frac{1}{\lfloor 4}+\ldots = e-2$

$\Rightarrow$ P = 2 + (positive value less then e – 2)

$\Rightarrow P \in (2, 3)$

$\Rightarrow$ least integer value of P is 3

8. The value of the integral $\int_{-1}^{1}\log\left(x+\sqrt{x^2+1}\right)dx$ is:

(1) 2 (2) 0 (3) –1 (4) 1

Official Ans. by NTA (2)

Sol. Let $I = \int_{-1}^{1}\log\left(x+\sqrt{x^2+1}\right)dx$

$\because \log\left(x+\sqrt{x^2+1}\right)$ is an odd function

$\therefore I = 0$

9. Let a, b and c be distinct positive numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are co-planar, then c is equal to:

(1) $\frac{2}{\frac{1}{a}+\frac{1}{b}}$ (2) $\frac{a+b}{2}$ (3) $\frac{1}{a}+\frac{1}{b}$ (4) $\sqrt{ab}$

Official Ans. by NTA (4)

Sol. Because vectors are coplanar

Hence $\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$

$\Rightarrow c^2 = ab \Rightarrow c = \sqrt{ab}$

10. If [x] be the greatest integer less than or equal to x, then $\sum_{n=8}^{100}\left[\frac{(-1)^n n}{2}\right]$ is equal to:

(1) 0 (2) 4 (3) –2 (4) 2

Official Ans. by NTA (2)

Sol. $\sum_{n=8}^{100}\left[\frac{(-1)^n . n}{2}\right]$

$= 4 - 5 + 5 - 6 + 6 + \ldots - 50 + 50 = 4$

11. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is:

(1) 4 (2) 1 (3) 2 (4) 3

Sol. $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$, $\frac{-\pi}{4} \le x \le \frac{\pi}{4}$

Apply : $R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$\begin{vmatrix} \sin x - \cos x & \cos x - \sin x & 0 \\ 0 & \sin x - \cos x & \cos x - \sin x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

$(\sin x - \cos x)^2 \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

$(\sin x - \cos x)^2(\sin x + 2\cos x) = 0$

$\therefore x = \frac{\pi}{4}$

Official Ans. by NTA (2)

12. If $|\vec{a}| = 2, |\vec{b}| = 5$ and $|\vec{a}\times\vec{b}| = 8$, then $|\vec{a}\cdot\vec{b}|$ is equal to :

(1) 6 (2) 4 (3) 3 (4) 5

Official Ans. by NTA (1)

Sol. $|\vec{a}| = 2$, $|\vec{b}| = 5$

$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta = \pm 8$

$\sin\theta = \pm\frac{4}{5}$

$\therefore\ \vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$= 10.\left(\pm\frac{3}{5}\right) = \pm 6$

$|\vec{a}.\vec{b}| = 6$

13. The number of real solutions of the equation, $x^2 - |x| - 12 = 0$ is:

(1) 2 (2) 3 (3) 1 (4) 4

Official Ans. by NTA (1)

Sol. $|x|^2 - |x| - 12 = 0$

$(|x| + 3)(|x| - 4) = 0$

$|x| = 4 \Rightarrow x = \pm 2$

14. Consider function $f : A \to B$ and $g : B \to C$ (A, B, C $\subseteq$ **R**) such that $(gof)^{-1}$ exists, then:

(1) f and g both are one-one

(2) f and g both are onto

(3) f is one-one and g is onto

(4) f is onto and g is one-one

Official Ans. by NTA (3)

Sol. $\therefore$ $(gof)^{-1}$ exist $\Rightarrow$ gof is bijective

$\Rightarrow$ 'f' must be one-one and 'g' must be ONTO

15. If $P = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix}$, then P^{50} is:

(1) $\begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & 25 \\ 0 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}$

Official Ans. by NTA (1)

Sol. $P = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$

$$P^2 = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$\vdots$

$$\therefore\ P^{50} = \begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$$

16. Let x be a random variable such that the probability function of a distribution is given by $P(X = 0) = \frac{1}{2}$, $P(X = j) = \frac{1}{3^j}$ $(j = 1, 2, 3, \ldots, \infty)$. Then the mean of the distribution and P(X is positive and even) respectively are:

(1) $\frac{3}{8}$ and $\frac{1}{8}$ (2) $\frac{3}{4}$ and $\frac{1}{8}$

(3) $\frac{3}{4}$ and $\frac{1}{9}$ (4) $\frac{3}{4}$ and $\frac{1}{16}$

Official Ans. by NTA (2)

Sol. mean $= \Sigma x_i p_i = \sum_{r=0}^{\infty} r.\frac{1}{3^r} = \frac{3}{4}$

$$p(x \text{ is even}) = \frac{1}{3^2} + \frac{1}{3^4} + \ldots\infty$$

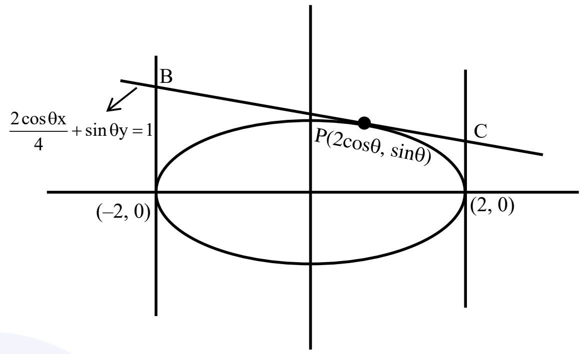
$$= \frac{\frac{1}{9}}{1-\frac{1}{9}} = \frac{1/9}{8/9} = \frac{1}{8}$$

17. If a tangent to the ellipse $x^2 + 4y^2 = 4$ meets the tangents at the extremities of its major axis at B and C, then the circle with BC as diameter passes through the point :

(1) $(\sqrt{3}, 0)$ (2) $(\sqrt{2}, 0)$ (3) (1, 1) (4) (–1, 1)

Official Ans. by NTA (1)

Sol.



$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$

Equation of tangent is $(\cos\theta)x + 2\sin\theta y = 2$

$B\left(-2, \frac{1+\cos\theta}{\sin\theta}\right)$, $C\left(2, \frac{1-\cos\theta}{\sin\theta}\right)$

$B\left(-2, \cot\frac{\theta}{2}\right)$ $C\left(2, \tan\frac{\theta}{2}\right)$

Equation of circle is

$$(x+2)(x-2) + \left(y - \cot\frac{\theta}{2}\right)\left(y - \tan\frac{\theta}{2}\right) = 0$$

$$x^2 - 4 + y^2 - \left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)y + 1 = 0$$

so, $(\sqrt{3}, 0)$ satisfying option (1)

18. Let the equation of the pair of lines, y = px and y = qx, can be written as (y – px) (y – qx) = 0. Then the equation of the pair of the angle bisectors of the lines $x^2 - 4xy - 5y^2 = 0$ is:

(1) $x^2 - 3xy + y^2 = 0$ (2) $x^2 + 4xy - y^2 = 0$

(3) $x^2 + 3xy - y^2 = 0$ (4) $x^2 - 3xy - y^2 = 0$

Official Ans. by NTA (3)

Sol. $\frac{x^2 - y^2}{1-(-5)} = \frac{xy}{-2}$

$$\frac{x^2 - y^2}{6} = \frac{xy}{-2}$$

$\Rightarrow x^2 - y^2 = -3xy \Rightarrow x^2 + 3xy - y^2 = 0$

19. If ${}^{n}P_{r} = {}^{n}P_{r+1}$ and ${}^{n}C_{r} = {}^{n}C_{r-1}$, then the value of r is equal to:

(1) 1 (2) 4 (3) 2 (4) 3

Official Ans. by NTA (3)

Sol. ${}^{n}P_r = {}^{n}P_{r+1} \Rightarrow \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!}$

$\Rightarrow (n-r) = 1 \quad \ldots(1)$

${}^{n}C_r = {}^{n}C_{r-1}$

$\Rightarrow \frac{n!}{r!(n-r)!} = \frac{n!}{(r-1)!(n-r+1)!}$

$\Rightarrow \frac{1}{r(n-r)!} = \frac{1}{(n-r+1)(n-r)!}$

$\Rightarrow n - r + 1 = r$

$\Rightarrow n + 1 = 2r \quad \ldots(2)$

$(1) \Rightarrow 2r - 1 - r = 1 \Rightarrow r = 2$

20. Let y = y (x) be the solution of the differential equation $xdy = (y + x^3 \cos x)dx$ with $y(\pi) = 0$, then $y\left(\frac{\pi}{2}\right)$ is equal to:

(1) $\frac{\pi^2}{4} + \frac{\pi}{2}$ (2) $\frac{\pi^2}{2} + \frac{\pi}{4}$

(3) $\frac{\pi^2}{2} - \frac{\pi}{4}$ (4) $\frac{\pi^2}{4} - \frac{\pi}{2}$

Official Ans. by NTA (1)

Sol. $xdy = (y + x^3\cos x)dx$

$xdy = ydx + x^3\cos x dx$

$\frac{xdy - ydx}{x^2} = \frac{x^3 \cos x\, dx}{x^2}$

$\frac{d}{dx}\left(\frac{y}{x}\right) = \int x \cos x\, dx$

$\Rightarrow \frac{y}{x} = x\sin x - \int 1.\sin x\, dx$

$\frac{y}{x} = x\sin x + \cos x + C$

$\Rightarrow 0 = -1 + C \Rightarrow C = 1, x = \pi, y = 0$

so $\frac{y}{x} = x\sin x + \cos x + 1$

$y = x^2\sin x + x\cos x + x \qquad x = \frac{\pi}{2}$

$y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2}$

SECTION-B

1. Let $n \in \mathbf{N}$ and [x] denote the greatest integer less than or equal to x. If the sum of (n + 1) terms ${}^{n}C_0, 3\cdot{}^{n}C_1, 5\cdot{}^{n}C_2, 7\cdot{}^{n}C_3, \ldots..$ is equal to $2^{100} \cdot 101$, then $2\left[\frac{n-1}{2}\right]$ is equal to __________.

Official Ans. by NTA (98)

Sol. $1.{}^{n}C_0 + 3.{}^{n}C_1 + 5.{}^{n}C_2 + \ldots + (2n+1).{}^{n}C_n$

$T_r = (2r + 1){}^{n}C_r$

$S = \Sigma T_r$

$S = \Sigma(2r+1){}^{n}C_r = \Sigma 2r{}^{n}C_r + \Sigma{}^{n}C_r$

$S = 2(n.2^{n-1}) + 2^n = 2^n(n + 1)$

$2^n(n + 1) = 2^{100}.101 \Rightarrow n = 100$

$2\left[\frac{n-1}{2}\right] = 2\left[\frac{99}{2}\right] = 98$

2. Consider the function $f(x) = \frac{P(x)}{\sin(x-2)}, \quad x \neq 2$

$= 7 \qquad , \quad x = 2$

Where P(x) is a polynomial such that P″ (x) is always a constant and P(3) = 9. If f(x) is continuous at x = 2, then P(5) is equal to _______.

Official Ans. by NTA (39)

Sol. $f(x) = \begin{cases} \frac{P(x)}{\sin(x-2)}, & x \neq 2 \\ 7 \quad , & x = 2 \end{cases}$

P”(x) = const. $\Rightarrow$ P(x) is a 2 degree polynomial

$f(x)$ is cont. at x = 2

$f(2^+) = f(2^-)$

$\lim_{x\to 2^+} \frac{P(x)}{\sin(x-2)} = 7$

$\lim_{x\to 2^+} \frac{(x-2)(ax+b)}{\sin(x-2)} = 7 \Rightarrow \boxed{2a+b=7}$

$P(x) = (x - 2)(ax + b)$

$P(3) = (3 - 2)(3a + b) = 9 \Rightarrow \boxed{3a+b=9}$

$\boxed{a = 2,\ b = 3}$

$P(5) = (5 - 2)(2.5 + 3) = 3.13 = 39$

3. The equation of a circle is $Re(z^2) + 2(Im(z))^2 + 2Re(z) = 0$, where $z = x + iy$. A line which passes through the center of the given circle and the vertex of the parabola, $x^2 - 6x - y + 13 = 0$, has y-intercept equal to ____.

Official Ans. by NTA (1)

Sol. Equation of circle is $(x^2 - y^2) + 2y^2 + 2x = 0$

$x^2 + y^2 + 2x = 0$

Centre : $(-1, 0)$

Parabola : $x^2 - 6x - y + 13 = 0$

$(x - 3)^2 = y - 4$

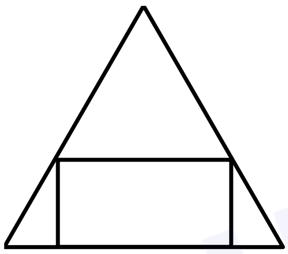
Vertex : $(3, 4)$

Equation of line $\equiv y - 0 = \frac{4-0}{3+1}(x + 1)$

$y = x + 1$

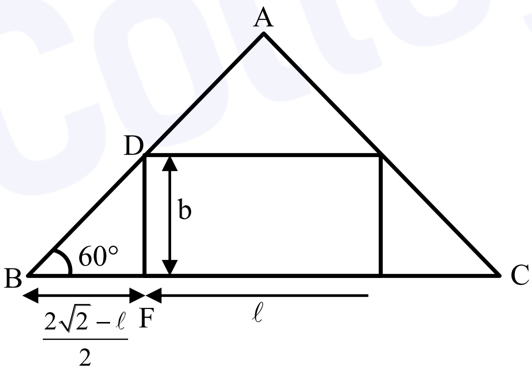
y-intercept = 1

4. If a rectangle is inscribed in an equilateral triangle of side length $2\sqrt{2}$ as shown in the figure, then the square of the largest area of such a rectangle is_______.



Official Ans. by NTA (3)

Sol.



In ΔDBF

$\tan 60° = \frac{2b}{2\sqrt{2}-\ell}$ $\quad b = \frac{\sqrt{3}\left(2\sqrt{2}-\ell\right)}{2}$

A = Area of rectangle = $\ell \times b$

$A = \ell \times \frac{\sqrt{3}}{2}\left(2\sqrt{2}-\ell\right)$

$\frac{dA}{d\ell} = \frac{\sqrt{3}}{2}\left(2\sqrt{2}-\ell\right) - \frac{\ell.\sqrt{3}}{2} = 0$

$\boxed{\ell = \sqrt{2}}$

$A = \ell \times b = \sqrt{2} \times \frac{\sqrt{3}}{2}\left(\sqrt{2}\right) = \sqrt{3}$

$\Rightarrow \boxed{A^2 = 3}$

5. If $(\vec{a}+3\vec{b})$ is perpendicular to $(7\vec{a}-5\vec{b})$ and $(\vec{a}-4\vec{b})$ is perpendicular to $(7\vec{a}-2\vec{b})$, then the angle between $\vec{a}$ and $\vec{b}$ (in degrees) is ________.

Official Ans. by NTA (60)

Sol. $(\vec{a}+3\vec{b}) \perp (7\vec{a}-5\vec{b})$

$(\vec{a}+3\vec{b}).(7\vec{a}-5\vec{b}) = 0$

$7|\vec{a}|^2 - 15|\vec{b}|^2 + 16\vec{a}.\vec{b} = 0$ ...(1)

$(\vec{a}-4\vec{b}).(7\vec{a}-2\vec{b}) = 0$

$7|\vec{a}|^2 + 8|\vec{b}|^2 - 30\vec{a}.\vec{b} = 0$ ...(2)

from (1) & (2)

$|\vec{a}| = |\vec{b}|$

$\cos\theta = \frac{|\vec{b}|}{2|\vec{a}|}$ $\quad \therefore \theta = 60°$

6. Let a curve $y = f(x)$ pass through the point $(2, (\log_e 2)^2)$ and have slope $\frac{2y}{x\log_e x}$ for all positive real value of x. Then the value of f(e) is equal to_______.

Official Ans. by NTA (1)

Sol. $y' = \frac{2y}{x\ell nx}$

$\Rightarrow \frac{dy}{y} = \frac{2dx}{x\ell nx}$

$\Rightarrow \ell n|y| = 2\ell n|\ell nx| + C$

put $x = 2, y = (\ell n2)^2$

$\Rightarrow c = 0 \Rightarrow y = (\ell nx)^2$

$\Rightarrow f(e) = 1$

7. If a + b + c = 1, ab + bc + ca = 2 and abc = 3, then the value of $a^4 + b^4 + c^4$ is equal to ________.

Official Ans. by NTA (13)

Sol. $a^2 + b^2 + c^2 = (a + b + c)^2 - 2\Sigma ab = -3$

$(ab + bc + ca)^2 = \Sigma(ab)^2 + 2abc\Sigma a$

$\Rightarrow \Sigma(ab)^2 = -2$

$a^4 + b^4 + c^4 = (a^2 + b^2 + c^2)^2 - 2\Sigma(ab)^2$

$= 9 - 2(-2) = 13$

8. A fair coin is tossed n-times such that the probability of getting at least one head is at least 0.9. Then the minimum value of n is _______.

Official Ans. by NTA (4)

Sol. $P(\text{Head}) = \frac{1}{2}$

$1 - P(\text{All tail}) \geq 0.9$

$1 - \left(\frac{1}{2}\right)^n \geq 0.9$

$\Rightarrow \left(\frac{1}{2}\right)^n \leq \frac{1}{10}$

$\Rightarrow n_{min} = 4$

9. If the co-efficient of x^7 and x^8 in the expansion of $\left(2 + \frac{x}{3}\right)^n$ are equal, then the value of n is equal to _______.

Official Ans. by NTA (55)

Sol. ${}^nC_7 2^{n-7} \frac{1}{3^7} = {}^nC_8 2^{n-8} \frac{1}{3^8}$

$\Rightarrow n - 7 = 48 \Rightarrow n = 55$

10. If the lines $\frac{x-k}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x+1}{3} = \frac{y+2}{2} = \frac{z+3}{1}$ are co-planar, then the value of k is _______.

Official Ans. by NTA (1)

Sol. $\begin{vmatrix} k+1 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0$

$(k + 1)[2 - 6] - 4[1 - 9] + 6[2 - 6] = 0$

$k = 1$