MATHEMATICS

SECTION-A

1. Let [t] denote the greatest integer less than or equal to t. Let f(x) = x – [x], g(x) = 1 – x + [x], and h(x) = min{f(x), g(x)}, x ∈ [–2, 2]. Then h is :

(1) continuous in [–2, 2] but not differentiable at more than four points in (–2, 2)

(2) not continuous at exactly three points in [–2, 2]

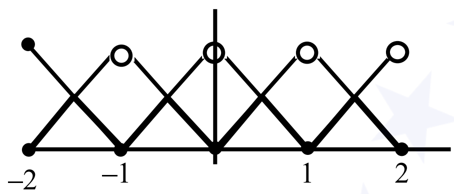
(3) continuous in [–2, 2] but not differentiable at exactly three points in (–2, 2)

(4) not continuous at exactly four points in [–2, 2]

Official Ans. by NTA (1)

Sol. $\min\{x-[x],\ 1-x+[x]\}$

$h(x)=\min\{x-[x],\ 1-[x-[x]]\}$



⇒ always continuous in [–2, 2]

but non differentiable at 7 Points

2. Let $A=\begin{pmatrix}1&0&0\\0&1&1\\1&0&0\end{pmatrix}$. Then $A^{2025}-A^{2020}$ is equal to :

(1) A^6-A (2) A^5

(3) A^5-A (4) A^6

Official Ans. by NTA (1)

Sol. $A=\begin{bmatrix}1&0&0\\0&1&1\\1&0&0\end{bmatrix} \Rightarrow A^2=\begin{bmatrix}1&0&0\\1&1&1\\1&0&0\end{bmatrix}$

$A^3=\begin{bmatrix}1&0&0\\2&1&1\\1&0&0\end{bmatrix} \Rightarrow A^4=\begin{bmatrix}1&0&0\\3&1&1\\1&0&0\end{bmatrix}$

$A^n=\begin{bmatrix}1&0&0\\n-1&1&1\\1&0&0\end{bmatrix}$

$A^{2025}-A^{2020}=\begin{bmatrix}0&0&0\\5&0&0\\0&0&0\end{bmatrix}$

$A^6-A=\begin{bmatrix}0&0&0\\5&0&0\\0&0&0\end{bmatrix}$

3. The local maximum value of the function $f(x)=\left(\frac{2}{x}\right)^{x^2}$, $x>0$, is

(1) $\left(2\sqrt{e}\right)^{\frac{1}{e}}$ (2) $\left(\frac{4}{\sqrt{e}}\right)^{\frac{e}{4}}$

(3) $(e)^{\frac{2}{e}}$ (4) 1

Official Ans. by NTA (3)

Sol. $f(x)=\left(\frac{2}{x}\right)^{x^2}$; $x>0$

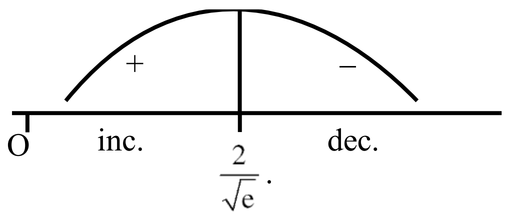
$\ell n\, f(x)=x^2\,(\ell n2-\ell nx)$

$f'(x)=f(x)\ \{-x+(\ell n\,2-\ell n\,x)2x\}$

$f'(x)=f(x).\ x\ \underbrace{(2\ell n2-2\ell nx-1)}_{g(x)}$
$\quad +\quad +$

$g(x)=2\ell\, n^2-2\ell\, n\,x-1$

$=\ell n\frac{4}{x^2}-1=0 \Rightarrow x=\frac{2}{\sqrt{e}}$



$LM=\frac{2}{\sqrt{e}}$

Local maximum value $=\left(\frac{2}{2/\sqrt{e}}\right)^{\frac{4}{e}} \Rightarrow e^{\frac{2}{e}}$

4. If the value of the integral $\int_0^5 \frac{x+[x]}{e^{x-[x]}}dx = \alpha e^{-1} + \beta$, where $\alpha, \beta \in \mathbf{R}$, $5\alpha + 6\beta = 0$, and $[x]$ denotes the greatest integer less than or equal to x; then the value of $(\alpha + \beta)^2$ is equal to :

(1) 100 (2) 25

(3) 16 (4) 36

Official Ans. by NTA (2)

Sol. $I = \int_0^5 \frac{x+[x]}{e^{x-[x]}}dx$

$$\int_0^1 \frac{x}{e^x}dx + \int_1^2 \frac{x+1}{e^{x-1}}dx + \int_2^3 \frac{x+2}{e^{x-2}}dx + \ldots .. \int_4^5 \frac{x+4}{e^{x-4}}dx$$

$\Downarrow$ $\Downarrow$ $\Downarrow$

$x = t+1$ $x = z+2$ $x = y+4$

$$\int_0^1 \frac{t+2}{e^t}dt + \int_0^1 \frac{z+4}{e^z}dz + \ldots .. + \int_0^1 \frac{y+8}{e^y}dx$$

$$\Rightarrow \int_0^5 \frac{5x+20}{e^x}dt = 5\int_0^1 \frac{x+4}{e^x}dx$$

$$\Rightarrow 5\int_0^1 (x+4)e^{-x}dx$$

$$\Rightarrow 5e^{-x}(-x-5)|_0^1 \Rightarrow -\frac{30}{e} + 25$$

$\alpha = -30$

$\beta = 25 \Rightarrow 5\alpha + 6\beta = 0$

$(\alpha + \beta)^2 = 5^2 = 25$

5. The point $P\left(-2\sqrt{6}, \sqrt{3}\right)$ lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having eccentricity $\frac{\sqrt{5}}{2}$. If the tangent and normal at P to the hyperbola intersect its conjugate axis at the point Q and R respectively, then QR is equal to :

(1) $4\sqrt{3}$ (2) 6 (3) $6\sqrt{3}$ (4) $3\sqrt{6}$

Official Ans. by NTA (3)

Sol. $P(-2\sqrt{6}, \sqrt{3})$ lies on hyperbola

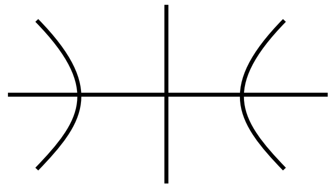
$$\Rightarrow \frac{24}{a^2} - \frac{3}{b^2} = 1 \quad \ldots\ldots.(i)$$

$$e = \frac{\sqrt{5}}{2} \Rightarrow b^2 = a^2\left(\frac{5}{4} - 1\right) \Rightarrow 4b^2 = a^2$$

Put in (i) $\Rightarrow \frac{6}{b^2} - \frac{3}{b^2} = 1 \Rightarrow b = \sqrt{3}$

$\Rightarrow a = \sqrt{12}$

$\frac{x^2}{12} - \frac{y^2}{3} = 1$



Tangent at P :

$$\frac{-x}{\sqrt{6}} - \frac{y}{\sqrt{3}} = 1 \Rightarrow Q\ (0, \sqrt{3})$$

Slope of $T = -\frac{1}{\sqrt{2}}$

Normal at P :

$y - \sqrt{3} = \sqrt{2}(x + 2\sqrt{6})$

$\Rightarrow R = (0, 5\sqrt{3})$

$QR = 6\sqrt{3}$

6. Let y(x) be the solution of the differential equation $2x^2dy + (e^y - 2x)dx = 0$, $x > 0$. If $y(e) = 1$, then $y(1)$ is equal to :

(1) 0 (2) 2

(3) $\log_e 2$ (4) $\log_e(2e)$

Official Ans. by NTA (3)

Sol. $2x^2dy + (e^y - 2x)dx = 0$

$$\frac{dy}{dx} + \frac{e^y - 2x}{2x^2} = 0 \Rightarrow \frac{dy}{dx} + \frac{e^y}{2x^2} - \frac{1}{x} = 0$$

$$e^{-y}\frac{dy}{dx} - \frac{e^{-y}}{x} = -\frac{1}{2x^2} \Rightarrow \text{Put } e^{-y} = z$$

$$\frac{-dz}{dx} - \frac{z}{x} = -\frac{1}{2x^2} \Rightarrow xdz + zdx = \frac{dx}{2x}$$

$$d(xz) = \frac{dx}{2x} \Rightarrow xz = \frac{1}{2}\log_e x + c$$

$xe^{-y} = \frac{1}{2}\log_e x + c$, passes through $(e,1)$

$$\Rightarrow C = \frac{1}{2}$$

$$xe^{-y} = \frac{\log_e ex}{2}$$

$$e^{-y} = \frac{1}{2} \Rightarrow y = \log_e 2$$

7. Consider the two statements :

$(S1): (p \to q) \vee (\sim q \to p)$ is a tautology.

$(S2): (p \wedge \sim q) \wedge (\sim p \vee q)$ is a fallacy.

Then :

(1) only (S1) is true.

(2) both (S1) and (S2) are false.

(3) both (S1) and (S2) are true.

(4) only (S2) is true.

Official Ans. by NTA (3)

Sol. $S_1 : (\sim p \vee q) \vee (q \vee p) = (q \vee \sim p) \vee (q \vee p)$

$S_1 = q \vee (\sim p \vee p) = qvt = t =$ tautology

$S_2 : (p \wedge \sim q) \wedge (\sim p \vee q) = (p \wedge \sim q) \wedge \sim (p \wedge \sim q) = C$

$=$ fallacy

8. The domain of the function $\operatorname{cosec}^{-1}\left(\frac{1+x}{x}\right)$ is :

(1) $\left(-1, -\frac{1}{2}\right] \cup (0, \infty)$ (2) $\left[-\frac{1}{2}, 0\right) \cup [1, \infty)$

(3) $\left(-\frac{1}{2}, \infty\right) - \{0\}$ (4) $\left[-\frac{1}{2}, \infty\right) - \{0\}$

Official Ans. by NTA (4)

Sol. $\frac{1+x}{x} \in (-\infty, -1] \cup [1, \infty)$

$$\frac{1}{x} \in (-\infty, -2] \cup [0, \infty)$$

$$x \in \left[-\frac{1}{2}, 0\right) \cup (0, \infty)$$

$$x \in \left[-\frac{1}{2}, \infty\right) - \{0\}$$

9. A fair die is tossed until six is obtained on it. Let X be the number of required tosses, then the conditional probability $P(X \geq 5 \mid X > 2)$ is :

(1) $\frac{125}{216}$ (2) $\frac{11}{36}$ (3) $\frac{5}{6}$ (4) $\frac{25}{36}$

Official Ans. by NTA (4)

Sol. $P(x \geq 5 \mid x > 2) = \frac{P(x \geq 5)}{P(x > 2)}$

$$\frac{\left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^5 \cdot \frac{1}{6} + \ldots\ldots + \infty}{\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \ldots\ldots + \infty}$$

$$\frac{\dfrac{\left(\frac{5}{6}\right)^4 \cdot \frac{1}{6}}{1 - \frac{5}{6}}}{\dfrac{\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}}{1 - \frac{5}{6}}} = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

10. If $\sum_{r=1}^{50} \tan^{-1}\frac{1}{2r^2} = p$, then the value of tan p is :

(1) $\frac{101}{102}$ (2) $\frac{50}{51}$ (3) 100 (4) $\frac{51}{50}$

Official Ans. by NTA (2)

Sol. $\sum_{r=1}^{50} \tan^{-1}\left(\frac{2}{4r^2}\right) = \sum_{r=1}^{50} \tan^{-1}\left(\frac{(2r+1)-(2r-1)}{1+(2r+1)(2r-1)}\right)$

$$\sum_{r=1}^{50} \tan^{-1}(2r+1) - \tan^{-1}(2r-1)$$

$$\tan^{-1}(101) - \tan^{-1}1 \Rightarrow \tan^{-1}\frac{50}{51}$$

11. Two fair dice are thrown. The numbers on them are taken as λ and μ, and a system of linear equations

$x + y + z = 5$

$x + 2y + 3z = \mu$

$x + 3y + \lambda z = 1$

is constructed. If p is the probability that the system has a unique solution and q is the probability that the system has no solution, then:

(1) $p=\frac{1}{6}$ and $q=\frac{1}{36}$ (2) $p=\frac{5}{6}$ and $q=\frac{5}{36}$

(3) $p=\frac{5}{6}$ and $q=\frac{1}{36}$ (4) $p=\frac{1}{6}$ and $q=\frac{5}{36}$

Official Ans. by NTA (2)

Sol. $D\neq 0 \Rightarrow \begin{vmatrix}1&1&1\\1&2&3\\1&3&\lambda\end{vmatrix}\neq 0 \Rightarrow \lambda\neq 5$

For no solution $D=0\Rightarrow \lambda=5$

$D_1=\begin{vmatrix}1&1&5\\1&2&\mu\\1&3&1\end{vmatrix}\neq 0\Rightarrow \mu\neq 3$

$p=\frac{5}{6}$

$q=\frac{1}{6}\times\frac{5}{6}=\frac{5}{36}$

Option (2)

12. The locus of the mid points of the chords of the hyperbola $x^2-y^2=4$, which touch the parabola $y^2=8x$, is :

(1) $y^3(x-2)=x^2$ (2) $x^3(x-2)=y^2$

(3) $y^2(x-2)=x^3$ (4) $x^2(x-2)=y^3$

Official Ans. by NTA (3)

Sol. $T=S_1$

$xh-yk=h^2-k^2$

$y=\frac{xh}{k}-\frac{(h^2-k^2)}{k}$

this touches $y^2=8x$ then $c=\frac{a}{m}$

$\left(\frac{k^2-h^2}{k}\right)=\frac{2k}{h}$

$2y^2=x(y^2-x^2)$

$y^2(x-2)=x^3$

13. The value of

$$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$$

is :

(1) $\frac{1}{4\sqrt{2}}$ (2) $\frac{1}{4}$

(3) $\frac{1}{8}$ (4) $\frac{1}{8\sqrt{2}}$

Official Ans. by NTA (3)

Sol. $2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$

$2\sin^2\frac{\pi}{8}\sin^2\frac{2\pi}{8}\sin^2\frac{3\pi}{8}$

$\sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8}$

$\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}$

$\frac{1}{4}\sin^2\left(\frac{\pi}{4}\right)=\frac{1}{8}$

14. If $\left(\sqrt{3}+i\right)^{100}=2^{99}(p+iq)$, then p and q are roots of the equation :

(1) $x^2-\left(\sqrt{3}-1\right)x-\sqrt{3}=0$

(2) $x^2+\left(\sqrt{3}+1\right)x+\sqrt{3}=0$

(3) $x^2+\left(\sqrt{3}-1\right)x-\sqrt{3}=0$

(4) $x^2-\left(\sqrt{3}+1\right)x+\sqrt{3}=0$

Official Ans. by NTA (1)

Sol. $\left(2e^{i\pi/6}\right)^{100}=2^{99}(p+iq)$

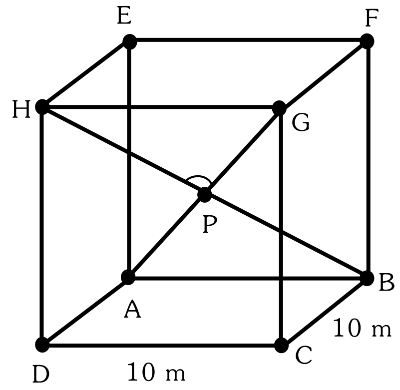
$2^{100}\left(\cos\frac{50\pi}{3}+i\sin\frac{50\pi}{3}\right)=2^{99}(p+iq)$

$p+iq=2\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$

$p=-1,\ q=\sqrt{3}$

$x^2-(\sqrt{3}-1)x-\sqrt{3}=0.$

15. A hall has a square floor of dimension 10m × 10m (see the figure) and vertical walls. If the angle GPH between the diagonals AG and BH is $\cos^{-1}\frac{1}{5}$, then the height of the hall (in meters) is :



(1) 5 (2) $2\sqrt{10}$ (3) $5\sqrt{3}$ (4) $5\sqrt{2}$

Official Ans. by NTA (4)

Sol. $A(\hat{j})$. $B(10\hat{i})$

$\mathbf{H}\ (h\hat{j}+10\hat{k})$

$\mathbf{G}\ (10\hat{i}+h\hat{j}+10\hat{k})$

$\overrightarrow{AG} = 10\hat{i}+h\hat{j}+10\hat{k}$

$\overrightarrow{BH} = -10\hat{i}+h\hat{j}+10\hat{k}$

$$\cos\theta = \frac{\overrightarrow{AG}\,\overrightarrow{BH}}{|\overrightarrow{AG}||\overrightarrow{BH}|}$$

$$\frac{1}{5} = \frac{h^2}{h^2+200}$$

$4h^2 = 200 \Rightarrow h = 5\sqrt{2}$

16. Let P be the plane passing through the point (1,2,3) and the line of intersection of the planes $\vec{r}\cdot(\hat{i}+\hat{j}+4\hat{k}) = 16$ and $\vec{r}\cdot(-\hat{i}+\hat{j}+\hat{k}) = 6$. Then which of the following points does **NOT** lie on P ?

(1) (3, 3, 2) (2) (6, –6, 2)

(3) (4, 2, 2) (4) (–8, 8, 6)

Official Ans. by NTA (3)

Sol. $(x+y+4z-16)+\lambda(-x+y+z-6)=0$

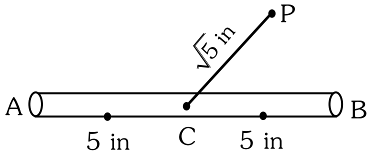
Passes through (1,2,3)

$-1+\lambda(-2) \Rightarrow \lambda = -\frac{1}{2}$

$2(x+y+4z-16)-(-x+y+z-6)=0$

$3x + y + 7z - 26 = 0$

17. A 10 inches long pencil AB with mid point C and a small eraser P are placed on the horizontal top of a table such that $PC = \sqrt{5}$ inches and $\angle PCB = \tan^{-1}(2)$. The acute angle through which the pencil must be rotated about C so that the perpendicular distance between eraser and pencil becomes exactly 1 inch is :

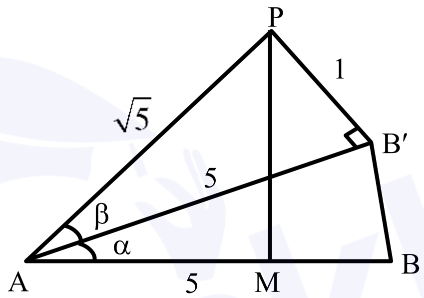


(1) $\tan^{-1}\left(\frac{3}{4}\right)$ (2) $\tan^{-1}(1)$

(3) $\tan^{-1}\left(\frac{4}{3}\right)$ (4) $\tan^{-1}\left(\frac{1}{2}\right)$

Official Ans. by NTA (1)

Sol.



From figure.

$$\sin\beta = \frac{1}{\sqrt{5}}$$

$$\therefore \tan\beta = \frac{}{2}$$

$\tan(\alpha+\beta) = 2$

$$\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta} = 2$$

$$\frac{\tan\alpha+\frac{1}{2}}{1-\tan\alpha\left(\frac{1}{2}\right)} = 2$$

$$\tan\alpha = \frac{3}{4}$$

$$\boxed{\alpha = \tan^{1}\left(\frac{3}{4}\right)}$$

18. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{1+\sin^2 x}{1+\pi^{\sin x}}\right)dx$ is

(1) $\frac{\pi}{2}$ (2) $\frac{5\pi}{4}$ (3) $\frac{3\pi}{4}$ (4) $\frac{3\pi}{2}$

Official Ans. by NTA (3)

Sol. $I=\int_0^{\pi/2}\frac{(1+\sin^2 x)}{(1+\pi^{\sin x})}+\frac{\pi^{\sin x}(1+\sin^2 x)}{(1+\pi^{\sin x})}dx$

$I=\int_0^{\pi/2}(1+\sin^2 x)dx$

$I=\frac{\pi}{2}+\frac{\pi}{2}\cdot\frac{1}{2}=\frac{3\pi}{4}$

19. A circle C touches the line x = 2y at the point (2,1) and intersects the circle $C_1 : x^2 + y^2 + 2y - 5 = 0$ at two points P and Q such that PQ is a diameter of C_1. Then the diameter of C is :

(1) $7\sqrt{5}$ (2) 15

(3) $\sqrt{285}$ (4) $4\sqrt{15}$

Official Ans. by NTA (1)

Sol. $(x-2)^2 + (y-1)^2 + \lambda(x-2y) = 0$

$C : x^2 + y^2 + x(\lambda-4) + y(-2-2\lambda) + 5 = 0$

$C_1 : x^2 + y^2 + 2y - 5 = 0$

$S_1 - S_2 = 0$ (Equation of PQ)

$(\lambda-4)x - (2\lambda+4)y + 10 = 0$ Passes through (0,–1)

$\Rightarrow \lambda = -7$

$C : x^2 + y^2 - 11x + 12y + 5 = 0$

$=\frac{\sqrt{245}}{4}$

Diometer = $7\sqrt{5}$

20. $\lim_{x\to 2}\left(\sum_{n=1}^{9}\frac{x}{n(n+1)x^2+2(2n+1)x+4}\right)$ is equal to :

(1) $\frac{9}{44}$ (2) $\frac{5}{24}$

(3) $\frac{1}{5}$ (4) $\frac{7}{36}$

Official Ans. by NTA (1)

Sol. $S=\lim_{x\to 2}\sum_{n=1}^{9}\frac{x}{n(n+1)x^2+2(2n+1)x+4}$

$S=\sum_{n=1}^{9}\frac{2}{4(n^2+3n+2)}=\frac{1}{2}\sum_{n=1}^{9}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$

$S=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{11}\right)=\frac{9}{44}$

SECTION-B

1. The sum of all 3-digit numbers less than or equal to 500, that are formed without using the digit "1" and they all are multiple of 11, is _______.

Official Ans. by NTA (7744)

Sol. 209, 220, 231, ..., 495

Sum = $\frac{27}{2}(209+495)=9504$

Number containing 1 at unit place
2 3 1
3 4 1
4 5 1

Number containing 1 at 10th place
3 1 9
4 1 8

Required = 9501 – (231 + 341 + 451 + 319 + 418)

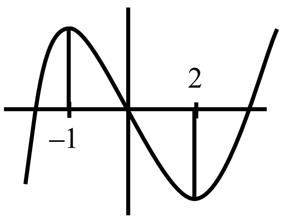
7744

2. Let a and b respectively be the points of local maximum and local minimum of the function $f(x) = 2x^3 - 3x^2 - 12x$. If A is the total area of the region bounded by y = f(x), the x-axis and the lines x = a and x = b, then 4A is equal to ________.

Official Ans. by NTA (114)

Sol. $f'(x) = 6x^2 - 6x - 12 = 6(x-2)(x+1)$

Point = (2,–20) & (–1,7)



$A=\int_{-1}^{0}(2x^3-3x^2-12x)dx+\int_0^2(12x+3x^2-2x^3)dx$

$$A=\left(\frac{x^4}{2}-x^3-6x^2\right)_{-1}^{0}+\left(6x^2+x^3-\frac{x^4}{2}\right)_{0}^{2}$$

$4A = 114$

3. If the projection of the vector $\hat{i}+2\hat{j}+\hat{k}$ on the sum of the two vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $-\lambda\hat{i}+2\hat{j}+3\hat{k}$ is 1, then λ is equal to ______.

Official Ans. by NTA (5)

Sol. $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$

$\vec{b}=(2-\lambda)\hat{i}+6\hat{j}-2\hat{k}$

$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=1,\ \vec{a}\cdot\vec{b}=12-\lambda$

$(\vec{a}\cdot\vec{b})=|\vec{b}|^2$

$\lambda^2-24\lambda+144=\lambda^2-4\lambda+4+40$

$20\lambda=100 \Rightarrow \lambda=5.$

4. Let $a_1, a_2,....,a_{10}$ be an AP with common difference -3 and $b_1, b_2,...., b_{10}$ be a GP with common ratio 2. Let $c_k = a_k + b_k$, $k = 1,2,..., 10$. If $c_2 = 12$ and $c_3 = 13$, then $\sum_{k=1}^{10} c_k$ is equal to _____.

Official Ans. by NTA (2021)

Sol. $c_2 = a_2 + b_2 = a_1 - 3 + 2b_1 = 12$

$a_1 + 2b_1 = 15$ ____(1)

$c_3 = a_3 + b_3 = a_1 - 6 + 4b_1 = 13$

$a_1 + 4b_1 = 19$ ____(2)

from (1) & (2) $b_1 = 2$, $a_1 = 11$

$$\sum_{k=1}^{10}c_k=\sum_{k=1}^{10}(a_k+b_k)=\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}b_k$$

$$=\frac{10}{2}(2\times 11+9\times(-3))+\frac{2(2^{10}-1)}{2-1}$$

$= 5(22 - 27) + 2(1023)$
$= 2046 - 25 = 2021$

5. Let Q be the foot of the perpendicular from the point P(7,–2,13) on the plane containing the lines $\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8}$ and $\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$. Then $(PQ)^2$, is equal to _______.

Official Ans. by NTA (96)

Sol. Containing the line $\begin{vmatrix} x+1 & y-1 & z-3 \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{vmatrix}=0$

$9(x + 1) - 18(y - 1) + 9(z - 3) = 0$

$x - 2y + z = 0$

$PQ=\left|\frac{7+4+13}{\sqrt{6}}\right|=4\sqrt{6}$

$PQ^2 = 96$

6. Let $\binom{n}{k}$ denotes ${}^{n}C_k$ and $\begin{bmatrix} n \\ k \end{bmatrix}=\begin{cases}\binom{n}{k}, & \text{if } 0\le k\le n\\ 0, & \text{otherwise}\end{cases}$

If $A_k=\sum_{i=0}^{9}\binom{9}{i}\begin{bmatrix}12\\12-k+i\end{bmatrix}+\sum_{i=0}^{8}\binom{8}{i}\begin{bmatrix}13\\13-k+i\end{bmatrix}$

and $A_4 - A_3 = 190$ p, then p is equal to :

Official Ans. by NTA (49)

Sol. $A_k=\sum_{i=0}^{9}{}^{9}C_i\ {}^{12}C_{k-i}+\sum_{i=0}^{8}{}^{8}C_i\ {}^{13}C_{k-i}$

$A_k={}^{21}C_k+{}^{21}C_k=2\cdot{}^{21}C_k$

$A_4-A_3=2\left({}^{21}C_4-{}^{21}C_3\right)=2(5985-1330)$

$190\,p = 2(5985 - 1330) \Rightarrow p = 49.$

7. Let $\lambda \neq 0$ be in **R**. If α and β are the roots of the equation $x^2 - x + 2\lambda = 0$, and α and γ are the roots of equation $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to ________.

Official Ans. by NTA (18)

Sol. $3\alpha^2 - 10\alpha + 27\lambda = 0$ _____(1)

$\alpha^2 - \alpha + 2\lambda = 0$ _____(2)

(1) – 3(2) gives

$-7\alpha + 21\lambda = 0 \Rightarrow \alpha = 3\lambda$

Put $\alpha = 3\lambda$ in equation (1) we get

$9\lambda^2 - 3\lambda + 2\lambda - 0$

$9\lambda^2=\lambda \Rightarrow \lambda=\frac{1}{9}$ as $\lambda\neq 0$

Now $\alpha=3\lambda \Rightarrow \lambda=\frac{1}{3}$

$\alpha + \beta = 1 \Rightarrow \beta = 2/3$

$\alpha + \gamma = \frac{10}{3} \Rightarrow \gamma = 3$

$$\frac{\beta\gamma}{\lambda} = \frac{\frac{2}{3} \times 3}{\frac{1}{9}} = 18$$

8. Let the mean and variance of four numbers 3, 7, x and y(x > y) be 5 and 10 respectively. Then the mean of four numbers 3 + 2x, 7 + 2y, x + y and x – y is _______.

Official Ans. by NTA (12)

Sol. $5 = \frac{3+7+x+y}{4} \Rightarrow x + y = 10$

$\text{Var}(x) = 10 = \frac{3^2 + 7^2 + x^2 + y^2}{4} - 25$

$140 = 49 + 9 + x^2 + y^2$

$x^2 + y^2 = 82$

$x + y = 10$

$\Rightarrow (x,y) = (9,1)$

Four numbers are 21,9,10,8

$\text{Mean} = \frac{48}{4} = 12$

9. Let A be a 3 × 3 real matrix.

If $\det(2\text{Adj}(2\ \text{Adj}(\text{Adj}(2A)))) = 2^{41}$, then the value of $\det(A^2)$ equal _____.

Official Ans. by NTA (4)

Sol. $\text{adj}\,(2A) = 2^2\,\text{adj}A$

$\Rightarrow \text{adj}(\text{adj}\,(2A)) = \text{adj}(4\,\text{adj}A) = 16\,\text{adj}\,(\text{adj}\,A)$

$= 16\,|A|\,A$

$\Rightarrow \text{adj}\,(32\,|A|\,A) = (32\,|A|)^2\,\text{adj}\,A$

$12(32|A|)^2\,|\text{adj}\,A| = 2^3\,(32|A|)^6\,|\text{adj}\ A|$

$2^3.2^{30}\,|A|^6\,.\,|A|^2 = 2^{41}$

$|A|^8 = 2^8 \Rightarrow |A| = \pm 2$

$|A|^2 = |A|^2 = 4$

10. The least positive integer n such that

$\frac{(2i)^n}{(1-i)^{n-2}}, i = \sqrt{-1}$ is a positive integer, is ______.

Official Ans. by NTA (6)

Sol. $\frac{(2i)^n}{(1-i)^{n-2}} = \frac{(2i)^n}{(-2i)^{\frac{n-2}{2}}}$

$= \frac{(2i)^{\frac{n+2}{2}}}{(-1)^{\frac{n-2}{2}}} = \frac{2^{\frac{n+2}{2}} i^{\frac{n+2}{2}}}{(-1)^{\frac{n-2}{2}}}$

This is positive integer for n = 6