

MATHEMATICS

SECTION-A

1. If $0 < x < 1$, then $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \dots$, is equal to :

(1) $x \left(\frac{1+x}{1-x} \right) + \log_e(1-x)$

(2) $x \left(\frac{1-x}{1+x} \right) + \log_e(1-x)$

(3) $\frac{1-x}{1+x} + \log_e(1-x)$

(4) $\frac{1+x}{1-x} + \log_e(1-x)$

Official Ans. by NTA (1)

Sol. Let $t = \frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \dots \infty$
 $= \left(2 - \frac{1}{2}\right)x^2 + \left(2 - \frac{1}{3}\right)x^3 + \left(2 - \frac{1}{4}\right)x^4 + \dots \infty$
 $= 2(x^2 + x^3 + x^4 + \dots \infty) - \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots \infty\right)$
 $= \frac{2x^2}{1-x} - (\ln(1-x) - x)$
 $\Rightarrow t = \frac{2x^2}{1-x} + x - \ln(1-x)$
 $\Rightarrow t = \frac{x(1+x)}{1-x} - \ln(1-x)$

2. If for $x, y \in \mathbf{R}, x > 0$,
 $y = \log_{10}x + \log_{10}x^{1/3} + \log_{10}x^{1/9} + \dots$ upto ∞ terms

and $\frac{2+4+6+\dots+2y}{3+6+9+\dots+3y} = \frac{4}{\log_{10}x}$, then the ordered

pair (x, y) is equal to :

- (1) $(10^6, 6)$ (2) $(10^4, 6)$
- (3) $(10^2, 3)$ (4) $(10^6, 9)$

Official Ans. by NTA (4)

Sol. $\frac{2(1+2+3+\dots+y)}{3(1+2+3+\dots+y)} = \frac{4}{\log_{10}x}$

$\Rightarrow \log_{10}x = 6 \Rightarrow x = 10^6$

Now,

$y = (\log_{10}x) + \left(\log_{10}x^{\frac{1}{3}}\right) + \left(\log_{10}x^{\frac{1}{9}}\right) + \dots \infty$

$= \left(1 + \frac{1}{3} + \frac{1}{9} + \dots \infty\right) \log_{10}x$

$= \left(\frac{1}{1 - \frac{1}{3}}\right) \log_{10}x = 9$

So, $(x, y) = (10^6, 9)$

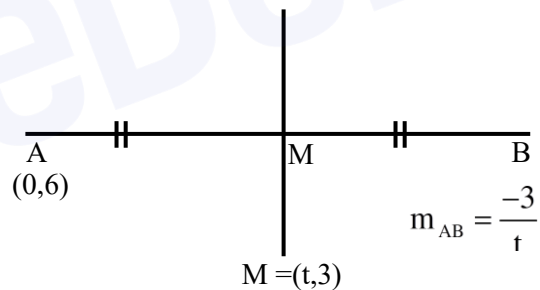
3. Let A be a fixed point $(0, 6)$ and B be a moving point $(2t, 0)$. Let M be the mid-point of AB and the perpendicular bisector of AB meets the y-axis at C.

The locus of the mid-point P of MC is :

- (1) $3x^2 - 2y - 6 = 0$ (2) $3x^2 + 2y - 6 = 0$
- (3) $2x^2 + 3y - 9 = 0$ (4) $2x^2 - 3y + 9 = 0$

Official Ans. by NTA (3)

Sol. A(0,6) and B(2t,0)



Perpendicular bisector of AB is

$(y - 3) = \frac{t}{3}(x - t)$

So, $C = \left(0, 3 - \frac{t^2}{3}\right)$

Let P be (h, k)

$h = \frac{t}{2}; k = \left(3 - \frac{t^2}{6}\right)$

$\Rightarrow k = 3 - \frac{4h^2}{6} \Rightarrow 2x^2 + 3y - 9 = 0$ option (3)

4. If $(\sin^{-1} x)^2 - (\cos^{-1} x)^2 = a$; $0 < x < 1$, $a \neq 0$, then the value of $2x^2 - 1$ is :

(1) $\cos\left(\frac{4a}{\pi}\right)$ (2) $\sin\left(\frac{2a}{\pi}\right)$

(3) $\cos\left(\frac{2a}{\pi}\right)$ (4) $\sin\left(\frac{4a}{\pi}\right)$

Official Ans. by NTA (2)

Sol. Given $a = (\sin^{-1} x)^2 - (\cos^{-1} x)^2$
 $= (\sin^{-1} x + \cos^{-1} x)(\sin^{-1} x - \cos^{-1} x)$
 $= \frac{\pi}{2} \left(\frac{\pi}{2} - 2 \cos^{-1} x \right)$
 $\Rightarrow 2 \cos^{-1} x = \frac{\pi}{2} - \frac{2a}{\pi}$

$\Rightarrow \cos^{-1}(2x^2 - 1) = \frac{\pi}{2} - \frac{2a}{\pi}$

$\Rightarrow 2x^2 - 1 = \cos\left(\frac{\pi}{2} - \frac{2a}{\pi}\right)$ option (2)

5. If the matrix $A = \begin{pmatrix} 0 & 2 \\ K & -1 \end{pmatrix}$ satisfies $A(A^3 + 3I) = 2I$, then the value of K is :

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) -1 (4) 1

Official Ans. by NTA (1)

Sol. Given matrix $A = \begin{bmatrix} 0 & 2 \\ k & -1 \end{bmatrix}$

$A^4 + 3IA = 2I$

$\Rightarrow A^4 = 2I - 3A$

Also characteristic equation of A is

$|A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 0 - \lambda & 2 \\ k & -1 - \lambda \end{vmatrix} = 0$

$\Rightarrow \lambda + \lambda^2 - 2k = 0$

$\Rightarrow A + A^2 = 2KI$

$\Rightarrow A^2 = 2KI - A$

$\Rightarrow A^4 = 4K^2I + A^2 - 4AK$

Put $A^2 = 2KI - A$

and $A^4 = 2I - 3A$

$2I - 3A = 4K^2I + 2KI - A - 4AK$

$\Rightarrow I(2 - 2K - 4K^2) = A(2 - 4K)$

$\Rightarrow -2I(2K^2 + K - 1) = 2A(1 - 2K)$

$\Rightarrow -2I(2K - 1)(K + 1) = 2A(1 - 2K)$

$\Rightarrow (2K - 1)(2A) - 2I(2K - 1)(K + 1) = 0$

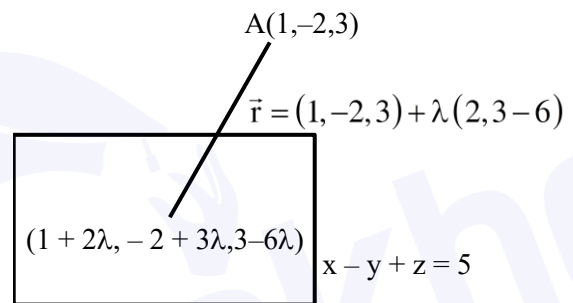
$\Rightarrow (2K - 1)[2A - 2I(K + 1)] = 0$

$\Rightarrow K = \frac{1}{2}$

6. The distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to a line, whose direction ratios are $2, 3, -6$ is :

(1) 3 (2) 5 (3) 2 (4) 1

Official Ans. by NTA (4)



Sol.

$(1 + 2\lambda) + 2 - 3\lambda + 3 - 6\lambda = 5$

$\Rightarrow 6 - 7\lambda = 5 \Rightarrow \lambda = \frac{1}{7}$

so, $P = \left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right)$

$AP = \sqrt{\left(1 - \frac{9}{7}\right)^2 + \left(-2 + \frac{11}{7}\right)^2 + \left(3 - \frac{15}{7}\right)^2}$

$AP = \sqrt{\left(\frac{4}{49}\right) + \frac{9}{49} + \frac{36}{49}} = 1$

7. If $S = \left\{z \in \mathbb{C} : \frac{z-i}{z+2i} \in \mathbb{R}\right\}$, then :

(1) S contains exactly two elements

(2) S contains only one element

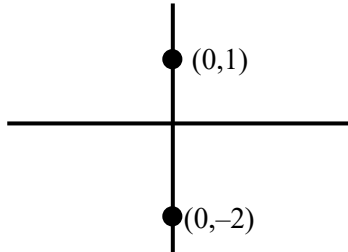
(3) S is a circle in the complex plane

(4) S is a straight line in the complex plane

Official Ans. by NTA (4)

Sol. Given $\frac{z-i}{z+2i} \in \mathbb{R}$

Then $\arg\left(\frac{z-i}{z+2i}\right)$ is 0 or π



$\Rightarrow S$ is straight line in complex

8. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = 2(y + 2 \sin x - 5) x - 2 \cos x$ such that $y(0) = 7$. Then $y(\pi)$ is equal to :

- (1) $2e^{\pi^2} + 5$ (2) $e^{\pi^2} + 5$
 (3) $3e^{\pi^2} + 5$ (4) $7e^{\pi^2} + 5$

Official Ans. by NTA (1)

Sol. $\frac{dy}{dx} - 2xy = 2(2 \sin x - 5)x - 2 \cos x$

IF = e^{-x^2}

so, $y \cdot e^{-x^2} = \int e^{-x^2} (2x(2 \sin x - 5) - 2 \cos x) dx$

$\Rightarrow y \cdot e^{-x^2} = e^{-x^2} (5 - 2 \sin x) + c$

$\Rightarrow y = 5 - 2 \sin x + c \cdot e^{x^2}$

Given at $x = 0, y = 7$

$\Rightarrow 7 = 5 + c \Rightarrow c = 2$

So, $y = 5 - 2 \sin x + 2e^{x^2}$

Now at $x = \pi,$

$y = 5 + 2e^{\pi^2}$

9. Equation of a plane at a distance $\sqrt{\frac{2}{21}}$ from the origin, which contains the line of intersection of the planes $x - y - z - 1 = 0$ and $2x + y - 3z + 4 = 0$, is :

- (1) $3x - y - 5z + 2 = 0$ (2) $3x - 4z + 3 = 0$
 (3) $-x + 2y + 2z - 3 = 0$ (4) $4x - y - 5z + 2 = 0$

Official Ans. by NTA (4)

Sol. Required equation of plane

$P_1 + \lambda P_2 = 0$

$(x - y - z - 1) + \lambda(2x + y - 3z + 4) = 0$

Given that its dist. From origin is $\frac{2}{\sqrt{21}}$

Thus $\frac{|4\lambda - 1|}{\sqrt{(2\lambda + 1)^2 + (\lambda - 1)^2 + (-3\lambda - 1)^2}} = \frac{\sqrt{2}}{\sqrt{21}}$

$\Rightarrow 21(4\lambda - 1)^2 = 2(14\lambda^2 + 8\lambda + 3)$

$\Rightarrow 336\lambda^2 - 168\lambda + 21 = 28\lambda^2 + 16\lambda + 6$

$\Rightarrow 308\lambda^2 - 184\lambda + 15 = 0$

$\Rightarrow 308\lambda^2 - 154\lambda - 30\lambda + 15 = 0$

$\Rightarrow (2\lambda - 1)(154\lambda - 15) = 0$

$\Rightarrow \lambda = \frac{1}{2}$ or $\frac{15}{154}$

for $\lambda = \frac{1}{2}$ reqd. plane is

$4x - y - 5z + 2 = 0$

10. If $U_n = \left(1 + \frac{1}{n^2}\right) \left(1 + \frac{2^2}{n^2}\right) \dots \left(1 + \frac{n^2}{n^2}\right)^n$, then

$\lim_{n \rightarrow \infty} (U_n)^{\frac{-4}{n^2}}$ is equal to :

- (1) $\frac{e^2}{16}$ (2) $\frac{4}{e}$ (3) $\frac{16}{e^2}$ (4) $\frac{4}{e^2}$

Official Ans. by NTA (1)

Sol. $U_n = \prod_{r=1}^n \left(1 + \frac{r^2}{n^2}\right)^r$

$L = \lim_{n \rightarrow \infty} (U_n)^{-4/n^2}$

$\log L = \lim_{n \rightarrow \infty} \frac{-4}{n^2} \sum_{r=1}^n \log \left(1 + \frac{r^2}{n^2}\right)^r$

$\Rightarrow \log L = \lim_{n \rightarrow \infty} \sum_{r=1}^n -\frac{4r}{n} \cdot \frac{1}{n} \log \left(1 + \frac{r^2}{n^2}\right)$

$\Rightarrow \log L \Rightarrow -4 \int_0^1 x \log(1 + x^2) dx$

put $1 + x^2 = t$

Now, $2x dx = dt$

$= -2 \int_1^2 \log(t) dt = -2 [t \log t - t]_1^2$

$\Rightarrow \log L = -2(2 \log 2 - 1)$

$$\begin{aligned} \therefore L &= e^{-2(2\log 2-1)} \\ &= e^{-2\left(\log\left(\frac{4}{e}\right)\right)} \\ &= e^{\log\left(\frac{4}{e}\right)^{-2}} \\ &= \left(\frac{e}{4}\right)^2 = \frac{e^2}{16} \end{aligned}$$

11. The statement $(p \wedge (p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow r$ is :

- (1) a tautology
- (2) equivalent to $p \rightarrow \sim r$
- (3) a fallacy
- (4) equivalent to $q \rightarrow \sim r$

Official Ans. by NTA (1)

Sol. $(p \wedge (p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow r$
 $\equiv (p \wedge (\sim p \vee q) \vee (\sim q \vee r)) \rightarrow r$
 $\equiv ((p \wedge q) \wedge (\sim p \vee r)) \rightarrow r$
 $\equiv (p \wedge q \wedge r) \rightarrow r$
 $\equiv \sim (p \wedge q \wedge r) \vee r$
 $\equiv (\sim p) \vee (\sim q) \vee (\sim r) \vee r$
 \Rightarrow tautology

12. Let us consider a curve, $y = f(x)$ passing through the point $(-2, 2)$ and the slope of the tangent to the curve at any point $(x, f(x))$ is given by $f(x) + xf'(x) = x^2$.

Then :

- (1) $x^2 + 2xf(x) - 12 = 0$
- (2) $x^3 + xf(x) + 12 = 0$
- (3) $x^3 - 3xf(x) - 4 = 0$
- (4) $x^2 + 2xf(x) + 4 = 0$

Official Ans. by NTA (3)

Sol. $y + \frac{xdy}{dx} = x^2$ (given)
 $\Rightarrow \frac{dy}{dx} + \frac{y}{x} = x$
 If $e^{\int \frac{1}{x} dx} = x$

Solution of DE

$$\begin{aligned} \Rightarrow y \cdot x &= \int x \cdot x \, dx \\ \Rightarrow xy &= \frac{x^3}{3} + \frac{c}{3} \end{aligned}$$

Passes through $(-2, 2)$, so

$$-12 = -8 + c \Rightarrow c = -4$$

$$\therefore 3xy = x^3 - 4$$

$$\text{ie. } 3x \cdot f(x) = x^3 - 4$$

13. $\sum_{k=0}^{20} \binom{20}{k}^2$ is equal to :

- (1) ${}^{40}C_{21}$
- (2) ${}^{40}C_{19}$
- (3) ${}^{40}C_{20}$
- (4) ${}^{41}C_{20}$

Official Ans. by NTA (3)

Sol. $\sum_{k=0}^{20} {}^{20}C_k \cdot {}^{20}C_{20-k}$

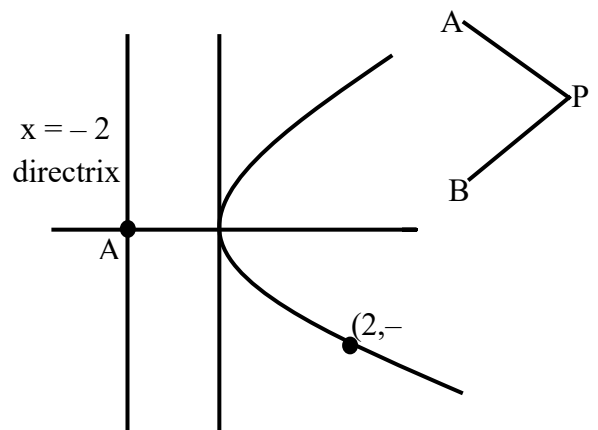
sum of suffix is const. so summation will be ${}^{40}C_{20}$

14. A tangent and a normal are drawn at the point $P(2, -4)$ on the parabola $y^2 = 8x$, which meet the directrix of the parabola at the points A and B respectively. If $Q(a, b)$ is a point such that $AQBP$ is a square, then $2a + b$ is equal to :

- (1) -16
- (2) -18
- (3) -12
- (4) -20

Official Ans. by NTA (1)

Sol.



Equation of tangent at $(2, -4)$ ($T = 0$)

$$-4y = 4(x + 2)$$

Sol. Probability of obtaining total sum 7 = probability of getting opposite faces.

Probability of getting opposite faces

$$= 2 \left[\left(\frac{1}{6} - x \right) \left(\frac{1}{6} + x \right) + \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} \right]$$

$$\Rightarrow 2 \left[\left(\frac{1}{6} - x \right) \left(\frac{1}{6} + x \right) + \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} \right] = \frac{13}{96}$$

(given)

$$x = \frac{1}{8}$$

18. If $x^2 + 9y^2 - 4x + 3 = 0$, $x, y \in \mathbb{R}$, then x and y respectively lie in the intervals:

(1) $\left[-\frac{1}{3}, \frac{1}{3} \right]$ and $\left[-\frac{1}{3}, \frac{1}{3} \right]$

(2) $\left[-\frac{1}{3}, \frac{1}{3} \right]$ and $[1, 3]$

(3) $[1, 3]$ and $[1, 3]$

(4) $[1, 3]$ and $\left[-\frac{1}{3}, \frac{1}{3} \right]$

Official Ans. by NTA (4)

Sol. $x^2 + 9y^2 - 4x + 3 = 0$

$$(x^2 - 4x) + (9y^2) + 3 = 0$$

$$(x^2 - 4x + 4) + (9y^2) + 3 - 4 = 0$$

$$(x - 2)^2 + (3y)^2 = 1$$

$$\frac{(x-2)^2}{(1)^2} + \frac{y^2}{\left(\frac{1}{3}\right)^2} = 1 \text{ (equation of an ellipse).}$$

As it is equation of an ellipse, x & y can vary inside the ellipse.

So, $x - 2 \in [-1, 1]$ and $y \in \left[-\frac{1}{3}, \frac{1}{3} \right]$

$x \in [1, 3]$ $y \in \left[-\frac{1}{3}, \frac{1}{3} \right]$

19. $\int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e (x^2 - 44x + 484)} dx$ is equal to:

(1) 6 (2) 8

(3) 5 (4) 10

Official Ans. by NTA (3)

Sol. Let $I = \int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e (x^2 - 44x + 484)} dx$

$$I = \int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e (x - 22)^2} dx \dots (1)$$

We know

$$\int_a^b f(x) dx = \int_a^b f(a + b - x) dx \text{ (king)}$$

So $I = \int_6^{16} \frac{\log_e (22 - x)^2}{\log_e (22 - x)^2 + \log_e (22 - (22 - x))^2} dx$

$$I = \int_0^{16} \frac{\log_e (22 - x)^2}{\log_e x^2 + \log_e (22 - x)^2} dx \dots (2)$$

(1) + (2)

$$2I = \int_6^{16} 1 dx = 10$$

$I = 5$

20. A wire of length 20 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a regular hexagon. Then the length of the side (in meters) of the hexagon, so that the combined area of the square and the hexagon is minimum, is:

(1) $\frac{5}{2 + \sqrt{3}}$

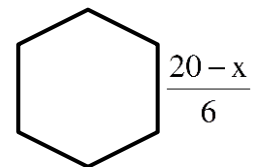
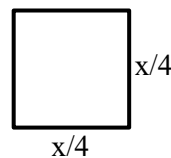
(2) $\frac{10}{2 + 3\sqrt{3}}$

(3) $\frac{5}{3 + \sqrt{3}}$

(4) $\frac{10}{3 + 2\sqrt{3}}$

Official Ans. by NTA (4)

Sol. Let the wire is cut into two pieces of length x and $20 - x$.



Area of square = $\left(\frac{x}{4} \right)^2$

Area of regular hexagon

$$= 6 \times \frac{\sqrt{3}}{4} \left(\frac{20-x}{6} \right)^2$$

$$\text{Total area} = A(x) = \frac{x^2}{16} + \frac{3\sqrt{3}(20-x)^2}{2 \cdot 36}$$

$$A'(x) = \frac{2x}{16} + \frac{3\sqrt{3} \times 2}{2 \times 36} (20-x)(-1)$$

$$A'(x) = 0 \text{ at } x = \frac{40\sqrt{3}}{3+2\sqrt{3}}$$

$$\text{Length of side of regular Hexagon} = \frac{1}{6}(20-x)$$

$$= \frac{1}{6} \left(20 - \frac{4\sqrt{3}}{3+2\sqrt{3}} \right)$$

$$= \frac{10}{2+2\sqrt{3}}$$

SECTION-B

1. Let $\vec{a} = \hat{i} + 5\hat{j} + \alpha\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + \beta\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} - 3\hat{k}$ be three vectors such that, $|\vec{b} \times \vec{c}| = 5\sqrt{3}$ and \vec{a} is perpendicular to \vec{b} . Then the greatest amongst the values of $|\vec{a}|^2$ is _____.

Official Ans. by NTA (90)

Sol. since, $\vec{a} \cdot \vec{b} = 0$

$$1 + 15 + \alpha\beta = 0 \Rightarrow \alpha\beta = -16 \dots(1)$$

Also,

$$|\vec{b} \times \vec{c}|^2 = 75 \Rightarrow (10 + \beta^2)14 - (5 - 3\beta)^2 = 75$$

$$\Rightarrow 5\beta^2 + 30\beta + 40 = 0$$

$$\Rightarrow \beta = -4, -2$$

$$\Rightarrow \alpha = 4, 8$$

$$\Rightarrow |\vec{a}|_{\max}^2 = (26 + \alpha^2)_{\max} = 90$$

2. The number of distinct real roots of the equation $3x^4 + 4x^3 - 12x^2 + 4 = 0$ is _____.

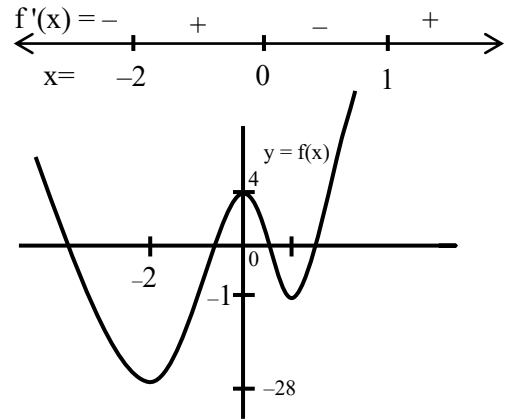
Official Ans. by NTA (4)

Sol. $3x^4 + 4x^3 - 12x^2 + 4 = 0$

$$\text{So, Let } f(x) = 3x^4 + 4x^3 - 12x^2 + 4$$

$$\therefore f(x) = 12x(x^2 + x - 2)$$

$$= 12x(x+2)(x-1)$$



3. Let the equation $x^2 + y^2 + px + (1-p)y + 5 = 0$ represent circles of varying radius $r \in (0, 5]$. Then the number of elements in the set $S = \{q : q = p^2 \text{ and } q \text{ is an integer}\}$ is _____.

Official Ans. by NTA (61)

$$\text{Sol. } r = \sqrt{\frac{p^2}{4} + \frac{(1-p)^2}{4}} - 5 = \frac{\sqrt{2p^2 - 2p - 19}}{2}$$

$$\text{Since, } r \in (0, 5]$$

$$\text{So, } 0 < 2p^2 - 2p - 19 \leq 100$$

$$\Rightarrow p \in \left[\frac{1-\sqrt{239}}{2}, \frac{1-\sqrt{39}}{2} \right) \cup \left(\frac{1+\sqrt{39}}{2}, \frac{1+\sqrt{239}}{2} \right]$$
 so, number of integral values of p^2 is 61

4. If $A = \{x \in \mathbf{R} : |x-2| > 1\}$, $B = \{x \in \mathbf{R} : \sqrt{x^2-3} > 1\}$, $C = \{x \in \mathbf{R} : |x-4| \geq 2\}$ and Z is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^c \cap Z$ is _____.

Official Ans. by NTA (256)

$$\text{Sol. } A = (-\infty, 1) \cup (3, \infty)$$

$$B = (-\infty, -2) \cup (2, \infty)$$

$$C = (-\infty, 2] \cup [6, \infty)$$

$$\text{So, } A \cap B \cap C = (-\infty, -2) \cup [6, \infty)$$

$$Z \cap (A \cap B \cap C)^c = \{-2, -1, 0, -1, 2, 3, 4, 5\}$$

$$\text{Hence no. of its subsets} = 2^8 = 256.$$

5. If $\int \frac{dx}{(x^2+x+1)^2} = a \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + b\left(\frac{2x+1}{x^2+x+1}\right) + C$, $x > 0$ where C is the constant of integration, then the value of $9(\sqrt{3}a + b)$ is equal to _____.

Official Ans. by NTA (15)

Sol.
$$I = \int \frac{dx}{\left[\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}\right]^2}$$

$$\int \frac{dt}{\left(t^2 + \frac{3}{4}\right)^2} \quad \left(\text{Put } x + \frac{1}{2} = t\right)$$

$$= \frac{\sqrt{3}}{2} \int \frac{\sec^2 \theta d\theta}{\frac{9}{16} \sec^4 \theta} \quad \left(\text{Put } t = \frac{\sqrt{3}}{2} \tan \theta\right)$$

$$= \frac{4\sqrt{3}}{9} \int (1 + \cos 2\theta) d\theta$$

$$= \frac{4\sqrt{3}}{9} \left[\theta + \frac{\sin 2\theta}{2}\right] + c$$

$$= \frac{4\sqrt{3}}{9} \left[\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{\sqrt{3}(2x+1)}{3+(2x+1)^2}\right] + c$$

$$= \frac{4\sqrt{3}}{9} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{1}{3} \left(\frac{2x+1}{x^2+x+1}\right) + c$$

Hence, $9(\sqrt{3}a + b) = 15$

6. If the system of linear equations

$$2x + y - z = 3$$

$$x - y - z = \alpha$$

$$3x + 3y + \beta z = 3$$

has infinitely many solution, then $\alpha + \beta - \alpha\beta$ is equal to _____.

Official Ans. by NTA (5)

Sol. $2 \times (i) - (ii) - (iii)$ gives :

$$-(1 + \beta)z = 3 - \alpha$$

For infinitely many solution

$$\beta + 1 = 0 = 3 - \alpha \Rightarrow (\alpha, \beta) = (3, -1)$$

Hence, $\alpha + \beta - \alpha\beta = 5$

7. Let n be an odd natural number such that the variance of $1, 2, 3, 4, \dots, n$ is 14. Then n is equal to _____.

Official Ans. by NTA (13)

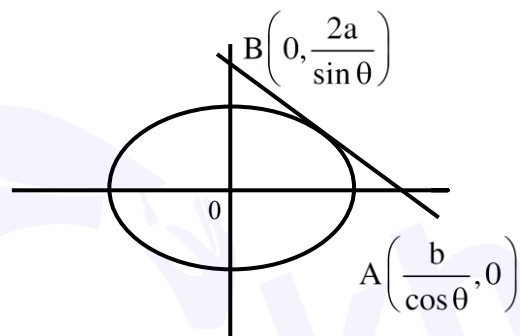
Sol. $\frac{n^2 - 1}{12} = 14 \Rightarrow n = 13$

8. If the minimum area of the triangle formed by a tangent to the ellipse $\frac{x^2}{b^2} + \frac{y^2}{4a^2} = 1$ and the co-ordinate axis is kab , then k is equal to _____.

Official Ans. by NTA (2)

Sol. Tangent

$$\frac{x \cos \theta}{b} + \frac{y \sin \theta}{2a} = 1$$



So, $\text{area}(\Delta OAB) = \frac{1}{2} \times \frac{b}{\cos \theta} \times \frac{2a}{\sin \theta}$

$$= \frac{2ab}{\sin 2\theta} \geq 2ab$$

$$\Rightarrow k = 2$$

9. A number is called a palindrome if it reads the same backward as well as forward. For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55, is _____.

Official Ans. by NTA (100)

Sol.

5	a	b	b	a	5
---	---	---	---	---	---

It is always divisible by 5 and 11.

So, required number = $10 \times 10 = 100$

10. If $y^{1/4} + y^{-1/4} = 2x$, and $(x^2 - 1)\frac{d^2y}{dx^2} + \alpha x \frac{dy}{dx} + \beta y = 0$, then $|\alpha - \beta|$ is equal to _____.

Official Ans. by NTA (17)

Sol. $y^{\frac{1}{4}} + \frac{1}{y^{\frac{1}{4}}} = 2x$

$$\Rightarrow \left(y^{\frac{1}{4}}\right)^2 - 2xy^{\left(\frac{1}{4}\right)} + 1 = 0$$

$$\Rightarrow y^{\frac{1}{4}} = x + \sqrt{x^2 - 1} \text{ or } x - \sqrt{x^2 - 1}$$

So, $\frac{1}{4} \frac{1}{y^{\frac{3}{4}}} \frac{dy}{dx} = 1 + \frac{x}{\sqrt{x^2 - 1}}$

$$\Rightarrow \frac{1}{4} \frac{1}{y^{\frac{3}{4}}} \frac{dy}{dx} = \frac{y^{\frac{1}{4}}}{\sqrt{x^2 - 1}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{4y}{\sqrt{x^2 - 1}} \dots(1)$$

Hence, $\frac{d^2y}{dx^2} = 4 \frac{(\sqrt{x^2 - 1})y' - \frac{yx}{\sqrt{x^2 - 1}}}{x^2 - 1}$

$$\Rightarrow (x^2 - 1)y'' = 4 \frac{(x^2 - 1)y' - xy}{\sqrt{x^2 - 1}}$$

$$\Rightarrow (x^2 - 1)y'' = 4 \left(\sqrt{x^2 - 1}y' - \frac{xy}{\sqrt{x^2 - 1}} \right)$$

$$\Rightarrow (x^2 - 1)y'' = 4 \left(4y - \frac{xy'}{4} \right) \text{ (from I)}$$

$$\Rightarrow (x^2 - 1)y'' + xy' - 16y = 0$$

So, $|\alpha - \beta| = 17$