MHT CET 2023 13 May Shift 2 Solution

Question 1. Solutions of sin(x) + sin(5x) = sin(3x) in (0, π/2) are?

Solution:

1. Rewrite using sum-to-product: sin(x) + sin(5x) = 2sin(3x) cos(2x) [using sin(A) + sin(B) = 2sin((A + B)/2)cos((A- B)/2)]
2. Solve for sin(3x) or cos(2x): ○ Either sin(3x) = 0 (which has solutions at multiples of π) or ○ cos(2x) = 1/2 (which has solutions at π/3 + 2πk and 5π/3 + 2πk, where k is any integer).
3. Consider the interval (0, π/2): ○ Inthis interval, sin(3x) = 0 only at x = 0. ○ cos(2x) = 1/2 has a solution at x = π/6 within the interval. Therefore, the solutions in (0, π/2) are: ● x=0 ● x=π/6

Question 2. If (tan^-1 x)^2 + (cot^-1 x)^2 = 5π^2 / 8, then x = ?

Solution:

1. Recognize the relationship: We know that (tan^-1 x) + (cot^-1 x) = π/2 for all x (where both arctangent and arccotangent are defined).
2. Square both sides: [(tan^-1 x) + (cot^-1 x)]^2 = (π/2)^2
3. Expand and subtract: (tan^-1 x)^2 + 2(tan^-1 x)(cot^-1 x) + (cot^-1 x)^2 = π^2 / 4 Since we're given that (tan^-1 x)^2 + (cot^-1 x)^2 =5π^2 / 8, subtract this equation: 2(tan^-1 x)(cot^-1 x) = (π^2 / 4)(5π^2 / 8) =-π^2 / 8
4. No real solution: The left side, 2(tan^-1 x)(cot^-1 x), is always non-negative (since both arctangent and arccotangent are positive in the relevant quadrant). However, the right side is negative. Therefore, there is no real solution for x in this case.

Question 3. Variance of the first 2n natural numbers?

Solution:

1. Calculate mean: The mean (average) of the first 2n natural numbers is (2n + 1)/2.
2. Find squared deviations from the mean: For each number I (from 1 to 2n), subtract the mean and square: (i- (2n + 1)/2)^2
3. Calculate the average of squared deviations (variance): Sum these squared deviations for all numbers and divide by 2n (the number of data points): Variance = (1/2n) * Σ [(i- (2n + 1)/2)^2] This formula can be simplified further using summation identities, but the basic approach remains the same.

Question 4. Solution of (1+xy) y dx+ (1-xy)x dy=0 is?

Solution:

This is a separable differential equation. However, it's not solvable using standard separation techniques due to the product of x and y in both terms.

Alternative Approaches:

- Exact Differential Equation Check: Try to manipulate the equation into the form M(x, y) dx + N(x, y) dy = 0, where ∂M/∂y = ∂N/∂x. If this is possible, the equation is exact and can be solved using integration.
- Numerical Methods: If the exact differential equation form can't be obtained, consider numerical methods like Euler's method or Runge-Kutta methods to approximate the solution.

Ques 6. The principal value of sin-1 (sin 3π/4) is? Solu. We can solve for the principal value of sin^-1(sin 3π/4) using the following steps:

1. Evaluate the inner sine function: sin(3π/4) = √2 / 2
2. Find the inverse sine: arcsin(√2 / 2) = π/4 Therefore, the principal value of sin^-1(sin 3π/4) is π/4.

Question 7. If x dy= y(dx + ydy), x(1)=1, y(x)>0, then y (-3) is?

Solution:

1. Separate the variables: Divide both sides by xy(dx + ydy): 1/y = dy/(x + y^2)
2. Integrate both sides: Integrate the left side concerning y and the right side concerning x: ∫ (1/y) dy = ∫ (dy/(x + y^2)) This will result in ln(y) =-1/(x + y) + C (where C is the constant of integration).
3. Apply initial condition: We are given that x(1) = 1 and y(x) > 0. Use this information to solve for C: ○ Substitute x = 1 and y = 1 (since y(1) must be positive): ln(1) =-1/(1 + 1^2) + C C = 04. Solve for y(-3): Plug x =-3 into the equation and solve for y: ln(y(-3)) =-1/(-3 + y(-3)^2) This equation is transcendental (containing an exponential term and an unknown variable) and cannot be solved for y(-3) analytically. However, you can use numerical methods like Newton-Raphson iteration or graphical methods to approximate the solution.

Question 8. Find the probability of getting a black card on a face card from a well-shuffled deck of 52 cards.

Solution:

1. Identify favorable outcomes: There are 4 black face cards (2 queens and 2 spades).
2. Identify total outcomes: There are 12 face cards in a deck (4 jacks, 4 queens, and 4 kings).
3. Calculate probability: Probability = (Favorable outcomes) / (Total outcomes) Probability = 4 / 12 = 1/3

Question 9. Find the area bounded by the region, y=3x+1, y=4x+1 and x=3.

Solution:

1. Identify the intersection points: Set the two y-functions equal to find where the lines intersect: 3x + 1 = 4x + 1 x = 0 Since x = 3 is given as the rightmost boundary, the lines intersect at x = 0and the region extends to x = 3.
2. Set up definite integral: Since we're dealing with areas between curves, use a definite integral. The top boundary is y = 4x + 1 and the bottom boundary is y = 3x + 1. Integrate over the interval from x = 0 to x = 3: Area = ∫ (4x + 1) dx- ∫ (3x + 1) dx (from x = 0 to x = 3)
3. Evaluate the integrals: Integrate both terms separately and subtract them. Then, evaluate the definite integral using the limits of integration (0 and 3). This will give you the final area of the bounded region.

Question 10. If 1+ (√1+x) tanx = 1+ (√1-x) tanx, then sin 4x is..?

Solution:
1. Simplify the equation: Notice that the left and right sides of the equation are identical. This doesn't provide any information about sin(4x).
2. No solution for sin(4x): The given equation doesn't involve sin(4x) or any terms that can be directly manipulated to find it. Therefore, based on the provided equation, we cannot determine the value of sin(4x).

MHT CET 2023 Question Paper with Answers and Solution May 12 Shift 2

Question 1. ∫1 / [(x + 2)(1 + x)2] dx = ?
Answer. log|(x + 2)/(x + 1)|- 1/(1 + x) + c

Question 2. ∫ (tan(1/x) / x)2dx = ?
Answer.-{tan(1/x)- (1/x)} + c

Question 3. ∫ 1/(cos3x.√sin2x) dx =?
Answer. √2 (√tanx + ⅕(tanx)5/2)

Question 4. The solution of ey-x dy/dx = y(sinx + cosx)/(1 + ylogy)
Answer. ey(logy) = ex sinx + c

Question 5. Limx->0 x.cot4x / (sin2x cot2(2x))
Answer. 1

Question 6. Solve for x, given tan-1 (1- x/1 + x) = ½ tan1x
Answer. x = √3

Question 7. ∫ 1 / cos3x √(sin2x) dx =?

Answer. $1/\sqrt{2}\ \{2\sqrt{t} + \int t^{3/2}\, dt\}$ where $t = \tan x$ and $\sec^2 x\, dx = dt$

Question 8. If a pair of lines given by $(x\cos\alpha + y\sin\alpha)^2 = (x^2 + y^2)\sin^2\alpha$ are perpendicular. What is the value of α?
Answer. $\alpha = \pi/4$

Question 9. Find $\cos^2 48^o - \sin^2 12^o$, if $\sin 18^o = (\sqrt{5} - 1)/4$
Answer. $(\sqrt{5} + 1)/8$

Question 10. If A = [2a-3b] [3 2] and adjA = AAT, then 2a + 3b is?
Answer. 5

Question 11. $f(x) = x^2 + 1$, $g(x) = 1/x$. Find f(g(g(f(x)))) at x = 1 A. 4 B. 1 C. 5 D. 3
Answer. C) 5

Question 12. Find $\sum (x - x_i)^2 = 100$, no. of observations = 20, $\sum x_i = 20$.

Question 13. Vertices of Tetrahedron are (1, 4, 3). (2, 5,-6), (3, -x, 5) and (1,-6,-3) and the volume of the tetrahedron is 11/6 cubic unit. Then x is?

Question 14. Ki are possible values of K for which lines Kx +2y+2=0, 2x + Ky+3=0, 3x + 3y +K=0 are concurrent, then $\sum k_i$ has value. A. 0 B.-2 C. 2 D. 5

Question 15. The equation of the normal to the curve $3x^2 + y^2 = 8$, which is parallel to the line x + 3y = 10 is

MHT CET 2023 Question Paper with Answers and Solution May 11 Shift 2

Question 1. The equation of the tangent to the curve $y = \sqrt{(9-2x^2)}$, at the point where the ordinates and abscissa are equal, is?

Answer. y > 0

Question 2. The minimum value of function $(1 - x + x^2) / (1 + x + x^2)$ A. ⅓ B. 0 C. 3 D. 1

Answer. A

Question 3. $\int \sin(\log x)\, dx$ A. $(x/2)[\sin(\log x) - \cos(\log x)]$ B. $\cos(\log x) - x$ C. $\int (x-1)e^x / (x+1)^3$ D.- cos logx
Answer. A

Question 4. The value of ∫ (1- cosx).cosec2dx is? Answer. cos(x) + sin(x) + cot(cos(x)) + C

Solution. To evaluate the integral ∫ (1- cosx) cosec^2(x) dx, we can simplify the integrand using trigonometric identities. Recall that cosec^2(x) is equal to 1 + cot^2(x), where cot(x) is the cotangent of x. ∫ (1- cosx) cosec^2(x) dx = ∫ (1- cosx) (1 + cot^2(x)) dx Expanding the expression: = ∫ (1- cosx + cot^2(x)- cosx * cot^2(x)) dx Now, let's evaluate each term separately: ∫ (1- cosx) dx = x- sin(x) + C_1 ∫ cot^2(x) dx can be integrated by using the formula ∫ cot^2(x) dx =-xcot(x) + C_2 ∫ cosx * cot^2(x) dx can be integrated by substitution. Let's denote cos(x) as u: u =cos(x) du =-sin(x) dx Replacing dx and cos(x) with du and u, respectively, we have: ∫ u * cot^2(x) (-du/sin(x)) =-∫ u cot^2(x) du =-∫ cot^2(x) du Using the formula mentioned earlier, we know that ∫ cot^2(x) dx =-x- cot(x) + C_2. Hence, the integral of-∫ cot^2(x) du will be-(-u- cot(u) + C_2) = u + cot(u)- C_2. Putting it all together, the integral becomes: x- sin(x) + C_1 + (-x- cot(x) + C_2) + (cos(x) + cot(cos(x))- C_2) Simplifying: = x- x +cos(x) + sin(x) + cot(cos(x)) + C_1- C_2 The final result is cos(x) + sin(x) + cot(cos(x)) + C, where C = C_1- C_2 is the constant of integration.

Question 5. The function of f(x)= 2x3-9x2+ 12x+29 is monotonically increasing in the interval A. (-∞, 1) B. (-∞, 1) U (2,∞) C. (-∞,-∞) D. (2,∞)

Answer. B

Solution. To determine whether the function f(x) = 2x^3- 9x^2 + 12x + 29 is monotonically increasing in an interval, we need to analyze the first derivative of the function, which is given by: f'(x) = 6x^2- 18x + 12 To find the critical points of the function (where the derivative is equal to zero), we need to solve the equation f'(x) = 0: 6x^2- 18x + 12 = 0 Dividing both sides by 6, we get: x^2- 3x + 2 = 0 Factoring the left-hand side, we get: (x- 1)(x- 2) = 0 So the critical points are x = 1 and x = 2. Now we need to analyze the sign of the derivative in the different intervals: Interval (-∞, 1): For x < 1, we can choose x = 0 as a test point. Plugging this into the derivative, we get: f'(0) = 6(0)^2- 18(0) + 12 = 12 Since f'(0) > 0, the derivative is positive in the interval (-∞, 1). This means that the function is monotonically increasing in this interval. Interval (1, 2): For 1 < x < 2, we can choose x = 1.5 as a test point. Plugging this into the derivative, we get: f'(1.5) = 6(1.5)^2- 18(1.5) + 12 =-3 Since f'(1.5) < 0, the derivative is negative in the interval (1, 2). This means that the function is not monotonically increasing in this interval. Interval (2, ∞): For x > 2, we can choose x = 3 as a test point. Plugging this into the derivative, we get: f'(3) = 6(3)^2- 18(3) + 12 = 30 Since f'(3) > 0, the derivative is positive in the interval (2, ∞). This means that the function is monotonically increasing in this interval. Therefore, the function f(x) = 2x^3- 9x^2 + 12x + 29 is monotonically increasing in the interval (-∞, 1) U (2, ∞), which corresponds to option B.

Question 6. For all real x, the minimum value of function f(x)=1-x+x2/1-x+x A. 1/3 B. 0 C. 3 D. 1

Answer.1/3

Question 7. If the line (x- 1)/2 = (y+1)/3 = (z-2) /4 = λ meets the plane, x+2y+3z = 15 at a point P, then the distance of P from the origin is? A. 7/2 B. 9/2 C. √5/2 D. 2√5

Answer. B

Question 8. If cosx + cosy- cos(x+y) = 3/2 then, A. x+y = 0 B. x = 2y C. x =y D. 2x =y

Answer. C

Question 9. If the vertices of a triangle are (-2,3), (6,-1) and (4,3), then the co-ordinates of the circumcentre of the triangle are? A. (1,1) B. (-1,-1) C. (-1,1) D. (1,-1)

Answer. D

Question 10. If cos-1√p + cos-1√(1-p) + cos-1√(1-a) = 3π/4, then q is? A. 1/2 B. 1/√2 C. 1 D. 1/3

Answer. A

Question 11. In △ABC b=√3, c=1 angle A = 30, then largest angle?

Answer. 120

Question 12. If the area of the parallelogram with a and b as two adjacent sites is 16 sq, units, then the area of the Parallelogram having 3a+2b and a+3b as two adjacent sides in sq. units is A. 96 B. 112 C. 144 D. 128

Answer. B

Question 13. dy/dx + y/x = sinn

Answer. xy+cosy-sinx=c

Question 14. x=5/1-21,value of x3+x2-x122

Answer. x2-2x+1=-4

Question 15. Equation of a tangent to the curve y = √(9-2x2) where x=y.