MATHEMATICS
QUESTIONS : 40 FULL MARKS : 100
DURATION : 45 MINUTES

Directions: Ques No. 1 to 20 carry 2 marks each.
Select one correct answer for these questions.

1. If the straight line y = mx lies outside of the circle $x^2 + y^2 - 20y + 90 = 0$, then the value of m will satisfy
   (A) $m < 3$ **(B)** $|m| < 3$
   (C) $m > 3$ (D) $|m| > 3$

2. If the angles of a triangle are in the ratio 4 : 1 : 1, then the ratio of the longest side and perimeter is
   (A) $1 : 6$ (B) $1 : (2+\sqrt{3})$
   (C) $1 : 3$ **(D)** $\sqrt{3} : (2+\sqrt{3})$

3. If e_1, e_2 be the eccentricity of the curves $16x^2 + 9y^2 = 144$ and $9x^2 - 16y^2 = 144$, then
   (A) ${e_1}^2 + {e_2}^2 = 3$ (B) ${e_1}^2 + {e_2}^2 > 3$
   (C) ${e_1}^2 + {e_2}^2 < 3$ (D) None of these

4. The angle of intersection of the curves $y = x^2$ and $6y = 7 - x^3$ at $(1,1)$ is
   (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$
   (C) $\frac{\pi}{3}$ (D) π

5. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is (in sq. units)
   (A) $\sqrt{6}$ **(B) 12**
   (C) 6 (D) $\frac{3}{2}$

6. If the sum of first n positive integers is 15 times of the sum of their squares then $n = ?$
   (A) 5 (B) 6
   (C) 7 (D) 10

7. The difference between greatest and least value of the function $f(x) = \int_0^x (t^2 + t + 1)dt$ on [2, 3] is
   (A) 20/3 (B) 33/2
   (C) 59/6 (D) 101/6

8. The curve represented by the parametric equation $x = t^2 + t - 1,\ y = t^2 - t + 1$ is
   (A) Pair of straight line (B) An ellipse

(C) A parabola (D) A hyperbola

9. If $I_n = \int_0^{\pi/4} \tan^n x dx$, then $I_{100} + 2I_{98} + I_{96} = ?$

(A) 194/98.96 (B) 195/98.97
(C) 196/99.97 (D) 197/98.96

10. Three coins are thrown together. The probability of getting two or more heads is

(A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$

(C) $\frac{2}{3}$ (D) $\frac{3}{8}$

11. If the matrix $A = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & x \end{pmatrix}$ is orthogonal, then the value of x is

(A) 1 (B) 0
(C) $\cos\theta$ (D) $\sin\theta$

12. Domain of the function f (x) = x / log (1 + x) is

(A) x ε (– 1, 0) **(B) x ε (–1, 0) u (0, ∝)**
(C) x ε (ε, ∝) (D) x ε [–1, 0]

13. Let $X = \{1, 2, 3, \ldots, n\}$. The number of bijective mappings on X is

(A) 2^n (B) n^n
(C) $n!$ (D) n^2

14. If $f(x)$ satisfies the relation $2f(x) + f(1-x) = x^2$ for all real x, then $f(x)$ is

(A) $\frac{x^2+1}{5}$ **(B)** $\frac{x^2+2x-1}{3}$

(C) $x^2 + 2x$ (D) $3x^2 + 5$

15. If the vectors $\vec{a} + k\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ and $\vec{a} - 3\vec{b} + 5\vec{c}$ are coplanar, then the value of k is

(A) 2 (B) -1
(C) 1 **(D)** -2

16. The minimum value of $f(x) = \sin^4 x + \cos^4 x$, $0 \le x \le \frac{\pi}{2}$ is

(A) $2\sqrt{3}$ (B) 0
(C) $\frac{1}{2}$ (D) 1

17. The Mean Value Theorem is applicable in the interval $-1 \le x \le 1$ for the function

(A) $x^3 - 2x + 5$ (B) $|x| + 5$
(C) $\sin\frac{1}{x}$ (D) $x\cos\frac{1}{x}$

18. The directrix of the parabola $x^2 - 4x - 8y + 12 = 0$ is

(A) $y = 0$ (B) $x = 1$

(C) $x + y = 1$ **(D)** $y = -1$

19. The function $f(x) = 2x^3 + 3x^2 - 12x + 1$ decreases if

(A) $2 \leq x \leq 3$ (B) $x \leq 2$ and $x \geq 3$

(C) $-2 < x < 1$ (D) $x \leq -2$ and $x \geq 1$

20. The distance of the point (1, 2) from the line $x - 2y - 2 = 0$, measured along a line parallel to $4x - 3y + 2 = 0$ is

(A) 4 **(B) 5**

(C) 2 (D) 1

Directions: Ques No. 21 to 40 carry 3 marks each.
Select one correct answer for these questions.

21. If $\log_2 x + \log_2 y \geq 6$, then the least value of $(x + y)$ is

(A) 4 (B) 5

(C) 16 (D) 8

22. The value of $\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)dx$ *is*

(A) 0 (B) 1

(C) 2 (D) 3

23. The rank of the matrix $\begin{pmatrix} 4 & 1 & 0 \\ 3 & 0 & 1 \\ 6 & 0 & 2 \end{pmatrix}$ is

(A) 1 (B) 3

(C) 2 (D) None of these

24. If $z^2 + z + 1 = 0$, where z is a complex number, then the value of $(z + \frac{1}{z})^2 + (z^2 + \frac{1}{z^2})^2 + (z^3 + \frac{1}{z^3})^2 + (z^4 + \frac{1}{z^4})^2$ is

(A) 7 (B) 5

(C) 0 (D) $2 + \sqrt{3}\,i$

25. If $f(x) = \begin{vmatrix} 1 & P & P \\ 2n & P^2 + P + 1 & P^2 + 1 \\ 2n-1 & P^2 & P^2 \end{vmatrix}$, then $\sum_{n=1}^{P} f(0) = ?$

(A) 0 (B) $P^2(P^2 + 1)$

(C) $P^2(P^2 - 1)$ (D) $P^3 + 1$

26. If a tetrahedron has vertices at O (0, 0, 0), A (1, 2, 1), B (2, 1, 3) and C (–1, 1, 2). Then the angle between the faces OAB and ABC will be

(A) $\cos^{-1}\left(\frac{19}{35}\right)$ (B) $\cos^{-1}\left(\frac{17}{31}\right)$

(C) 30 (D) 90

27. The value of n for which $\frac{x^{n+1}+y^{n+1}}{x^n+y^n}$ is the geometric mean of x and y is

(A) $n=-\frac{1}{2}$ (B) $n=\frac{1}{2}$

(C) $n=1$ (D) $n=-1$

28. If $a_1, a_2, .., a_n$ are in H.P., then, $a_1a_2+a_2a_3+...+a_{n-1}a_n=?$

(A) $n(a_1-a_n)$ (B) $(n-1)(a_1-a_n)$

(C) na_1a_n **(D)** $(n-1)a_1a_n$

29. The system of equations $x+y+z=6,\ x+2y+3z=10$ and $x+2y+\gamma z=\mu$ has no solution if

(A) $\gamma=3,\ \mu=10$ (B) $\gamma\neq 3,\ \mu=10$

(C) $\gamma\neq 3,\ \mu\neq 10$ **(D)** $\gamma=3,\ \mu\neq 10$

30. The third term of a G.P. is 4. The product of first five terms is

(A) 4^3 **(B)** 4^5

(C) 20 (D) 4^4

31. Let $\vec{a},\ \vec{b}$ and $\vec{c}$ be 3 non-zero vectors. If $\left|(\vec{a}\times\vec{b}).\vec{c}\right| = |\vec{a}|\,|\vec{b}|\,|\vec{c}|$ holds then

(A) $\vec{a}.\vec{b}\neq 0$ and $\vec{b}.\vec{c}=0$ (B) $\vec{b}.\vec{c}=0$ and $\vec{c}.\vec{a}\neq 0$

(C) $\vec{c}.\vec{a}\neq 0$ and $\vec{a}.\vec{b}\neq 0$ **(D)** $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0.$

32. A value of C for which the conclusion of mean value theorem holds for the function f (x) = $\log_e x$ on the interval [1,3] is

(A) $\log_C e$ (B) $\log_e 3$

(C) 2 $\log_3$ e (D) ½ $\log_e 3$

33. If the plane $3x+y+2z+6=0$ is parallel to the line $\frac{3x-1}{2b}=3-y=\frac{z-1}{a}$, then the value $3a+3b$ is

(A) $\frac{5}{2}$ **(B)** $\frac{3}{2}$

(C) $\frac{1}{2}$ (D) 2

34. The point on the curve $y-3x^2+4x-5=0$ where the tangent is parallel to the line $y+22x=7$

(A) $(-3,\ 44)$ (B) $(3,\ 44)$

(C) $(0,\ 5)$ (D) $(-3,\ -44)$

35. The sum of the infinite series $\log_4 2-\log_8 2+\log_{16} 2-\cdots to\ \infty$ is

(A) $1+\log 2$ (B) e^2

(C) $1-\log 2$ (D) $e-1$

36. If $x+\frac{1}{x}=2\cos\theta$, then $x^{10}+\frac{1}{x^{10}}$ is equal to

(A) $2\sin 5\theta$ **(B)** $2\cos 10\theta$
(C) $2^{10}\cos 10\theta$ (D) $2i\sin 10\theta$

37. There are 5 letters and 5 different envelops. The number of ways in which all the letters can be put in wrong envelop, is
(A) 119 (B) 40
(C) 59 **(D)** 44

38. The value of the limit $\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\ldots\left(1+\frac{n}{n}\right)\right]^{\frac{1}{n}}$ is

(A) $\frac{4}{e}$ (B) $\frac{e}{4}$

(C) $4e$ (D) $\frac{1}{e}$

39. If $f(x)$ is differentiable and $\int_0^{t^2} xf(x)dx=\frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ equals

(A) 2/5 (B) –5/2
(C) 1 (D) 5/2

40. If $x=\frac{e^t+e^{-t}}{2}$, $y=\frac{e^t-e^{-t}}{2}$, then $\frac{dy}{dx}$ is equal to
(A) xy **(B) x/y**
(C) y/x (D) None of these