

NARAYANA'S UNSTOPPABLE DOMINANCE

IN JEE MAIN 2024

**ALL INDIA
OPEN CATEGORY RANKS
IN TOP 12**



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**JEE MAIN (JAN) 2025
22-01-2025 (9 AM-12 PM)**

Memory - Based Question Paper

MATHEMATICS



JEE-Main-22-01-2025 (Memory Based) [SESSION-1]

MATHEMATICS

Question: $f(x) = 7 \tan^8 x + 7 \tan^6 x - 3 \tan^4 x - 3 \tan^2 x$ $I_1 = \int_0^{\frac{\pi}{4}} f(x) dx$ $I_2 = \int_0^{\frac{\pi}{4}} x f(x) dx$ Find

$7I_1 + 12I_2$

Options:

(a) 1

(b) 2

(c) 3

(d) 4

Answer: (a)

$$f(x) = 7 \tan^8 x + 7 \tan^6 x - 3 \tan^4 x - 3 \tan^2 x$$

$$= 7 \tan^6 x \cdot \sec^2 x - 3 \tan^2 x \cdot \sec^2 x$$

$$I_1 = \int_0^{\frac{\pi}{4}} f(x) dx$$

$$= \int_0^{\frac{\pi}{4}} (7 \tan^6 x - 3 \tan^2 x) \sec^2 x dx$$

$$= \int_0^1 (7t^6 - 3t^2) dt$$

$$= t^7 - t^3 \Big|_0^1$$

$$= 0$$

$$\begin{aligned}
 I_2 &= \int_0^{\frac{\pi}{4}} x f(x) dx \\
 &= x \left[\tan^7 x - \tan^3 x \right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{2}} \tan^7 x + \tan^3 x \\
 &= 0 - \int_0^{\frac{\pi}{2}} \tan^3 x (\tan^2 x - 1) \tan^3 x \\
 &= - \int_0^{\frac{\pi}{2}} t^3 (t^2 - 1) dt \\
 &= - \int_0^{\frac{\pi}{2}} (t^5 - t^3) \\
 &= - \left[\frac{t^6}{6} - \frac{t^4}{4} \right]_0^1 \\
 &= \frac{1}{4} - \frac{1}{6} = \frac{1}{12}
 \end{aligned}$$

Hence, $7I_1 + 12I_2 = 1$

Question: The number of 5 letter words which can be formed in alphabetical order such that M is always at middle taking every alphabet

Options:

- (a) 5143
- (b) 5148
- (c) 5144
- (d) 5149

Answer: (b)

M is the middle

$${}^{12}C_2 \times 1 \times {}^{13}C_2 = \frac{12 \times 11}{2} \times 1 \times \frac{13 \times 12}{2}$$

$$= 6 \times 11 \times 6 \times 13$$

$$= 5148$$

Question: $e^{5(\log_e x)^2 + 3} = x^8$ find the product of solutions

Options:

- (a) $e^{2/5}$
- (b) $e^{3/5}$
- (c) $e^{8/5}$
- (d) $e^{1/5}$

Answer (c)

$$e^5 (\log_e x)^2 + 3 = x^8$$

$$\Rightarrow 5(\log_e x)^2 + 3 = 8 \ln_e x$$

$$\Rightarrow 5t^2 + 3 = 8t$$

$$\Rightarrow 5t^2 - 8t + 3 = 0$$

$$\Rightarrow 5t^2 - 5t - 3t + 3 = 0$$

$$\Rightarrow 5t(t-1) - 3(t-1) = 0$$

$$t = 1, t = \frac{3}{5}$$

$$\ln_e x = 21 \quad \ln x = \frac{3}{5}$$

$$x = e \quad x = e^{\frac{3}{5}}$$

$$p = e^1 \cdot e^{\frac{3}{5}}$$

$$= e^{\frac{3}{5}+1} = e^{\frac{8}{5}}$$

Question: Let the triangle PQR be the image of the triangle with vertices (1, 3), (3, 1), (2, 4) in the line $x + 2y = 2$. If the centroid of ΔPQR is the point (α, β) then $15(\alpha - \beta)$ is equation

Options:

- (a) 20
- (b) 21
- (c) 22
- (d) 23

Answer: (c)

$$\frac{1+3+2}{3}, \frac{3+1+4}{3} \quad x+2y-2=0$$

$$= \left(2, \frac{8}{3}\right) \quad \frac{x-2}{1} = \frac{y-\frac{8}{3}}{2} = -2 \left(1 + \frac{16}{3} - 2\right) = \frac{32}{15}$$

$$x = 2 - \frac{32}{15}, \quad y = \frac{8}{3} - \frac{64}{15}$$

$$= -\frac{2}{15} \quad = \frac{-24}{15} = \frac{-8}{5} = \frac{-24}{15}$$

$$15\alpha = -2$$

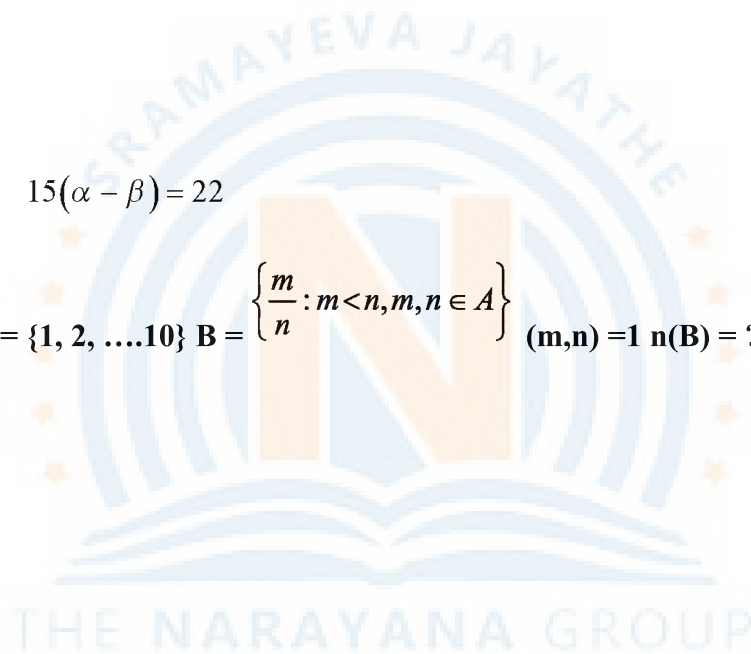
$$15\beta = -24 \quad 15(\alpha - \beta) = 22$$

Question: Set $A = \{1, 2, \dots, 10\}$ $B = \left\{ \frac{m}{n} : m < n, m, n \in A \right\}$ $(m, n) = 1$ $n(B) = ?$

Options:

- (a) 30
- (b) 31
- (c) 32
- (d) 33

Answer: (b)



$$A = \{1, 2, 3, \dots, 10\}$$

$$B = \left\{ \frac{m}{n} : m < n, m, n \in A \right\}$$

$$\frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{9}{10}$$

$$\frac{1}{9}, \frac{2}{9}, \frac{4}{9}, \frac{5}{9}, \frac{7}{9}, \frac{8}{9}$$

$$\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}$$

$$\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$$

$$\frac{1}{6}, \frac{5}{6}$$

$$\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$$

$$\frac{1}{4}, \frac{3}{4}$$

$$\frac{1}{3}, \frac{2}{3}$$

$$\frac{1}{2}$$

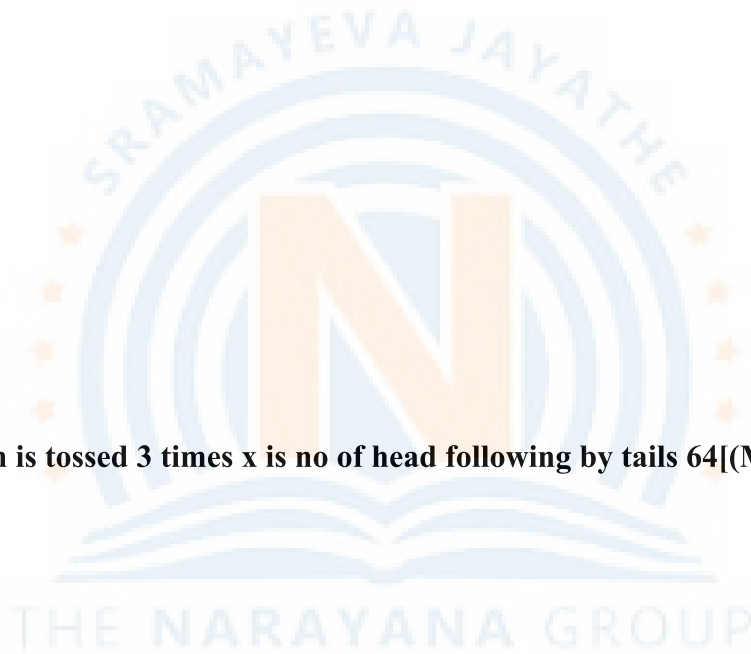
$$\text{Total} = 31$$

Question: A coin is tossed 3 times x is no of head following by tails $64[(M) + (\sigma^2)]$

Options:

- (a) 71
- (b) 40
- (c) 60
- (d) 48

Answer: (d)



$$x = 0 \{HHH, TTT, HTT, HHT\}$$

$$x = 1 \left\{ \begin{array}{c} THH, HTH, TTH \\ \text{ } \\ THT \end{array} \right\}$$

$$P(x = 0) = \frac{1}{2} = P(x = 1)$$

$$\mu = \frac{1}{2}$$

$$\sigma^2 = \sum p_i \left(x_i - \frac{1}{2} \right)^2$$

$$= \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{4}$$

$$64 \left(\frac{1}{2} + \frac{1}{4} \right) = 32 + 16 = 48$$

2T,1H

TTH, THT, HTT

2H1T

HHTHTH

THH

Question: Two balls are selected at random one by one without replacement from the bag containing 4 white and 6 black balls. If the probability that the first selected ball is black given that the second selected is also black, is m/n where $\gcd(m, n) = 1$, then $m + n = ?$

Options:

- (a) 11
- (b) 12
- (c) 13
- (d) 14

Answer: (d)

$$4W, 6B \quad P\left(\frac{\text{FirstBlack}}{\text{SecondBlack}}\right) = \frac{P(BB)}{P(WB)+P(BB)} = \frac{\frac{6}{10} \cdot \frac{5}{9}}{\frac{4}{10} \cdot \frac{6}{9} + \frac{6}{10} \cdot \frac{5}{9}} = \frac{30}{54} = \frac{5}{9} = \frac{m}{n} \quad m + n = 14$$

Question: $\sum_{r=1}^n T_r = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$, then $\lim_{n \rightarrow \infty} \sum_{r=1}^n \left(\frac{1}{T_r} \right) =$

Options:

- (a) 1
- (b) 0
- (c) 2
- (d) 3

Answer: (c)

$$T_r = S_n - S_{n-1}$$

$$= \frac{(2r-1)(2r-2)(2r+3)(2r+4)}{64} - \frac{(2r-3)(2r-1)(2r+1)(2r+3)}{64}$$

$$= 64(2r-1)(2r+1)(2r+3)[2r+5 - (2r-3)]$$

$$T_n = \frac{1}{\theta}(2r-1)(2r+1)(2r+3)$$

$$\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{\ln}$$

$$\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{\theta}{(2r-1)(2r+1)(2r+3)} \left[\frac{(2r+3) - (2r-1)}{4} \right]$$

$$= 2 \lim_{n \rightarrow \infty} \sum_{r=1}^n \left[\frac{1}{(2r-1)(2r+1)} - \frac{1}{(2r+1)(2r+3)} \right]$$

$$= 2 \left[1 - \frac{1}{\infty} \right] = 2$$

Question: Parabola of equation $y^2 = 2\sqrt{3}x$ and circle of equation $y^2 + (x - 2\sqrt{3})^2 = 12$. Find the area inside the circle but outside the parabola?

Options:

(a) $2\pi - 8$

(b) $\pi - 8$

(c) $3\pi - 8$

(d) $\pi - 4$

Answer: (c)

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$$y^2 = 2\sqrt{3}x$$

$$y^2 + (x - 2\sqrt{3})^2 = 12$$

$$2\sqrt{3}x + (2 - 2\sqrt{3})^2 = 12$$

$$2\sqrt{3}x + x^2 + 12 - 4\sqrt{3}x$$

$$x^2 = 2\sqrt{3}x$$

$$x = 0, 2B$$

$$y^2 = 2B \cdot 2B = 12$$

$$A_1 + A_2 = \frac{\pi \cdot 12}{4} = 3\pi$$

$$A_2 = \frac{2}{3} \cdot 2\beta \cdot 2\beta = 8$$

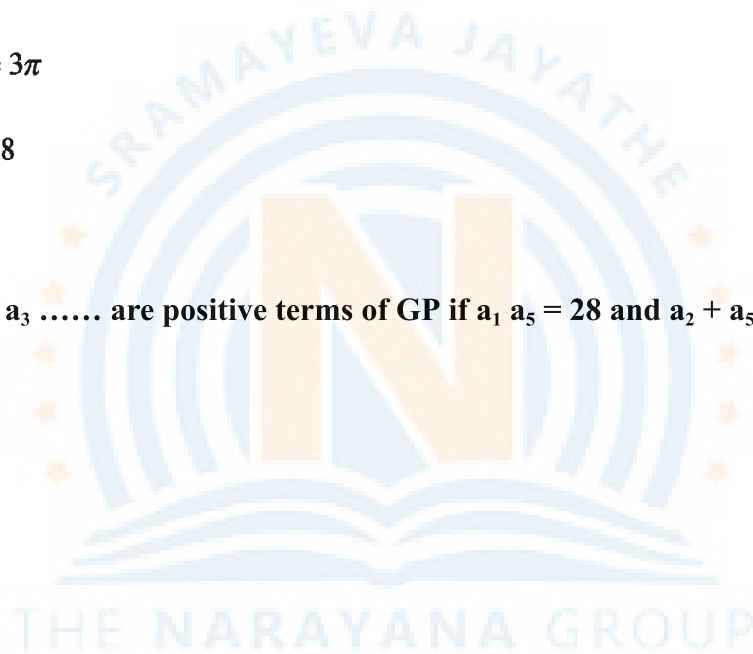
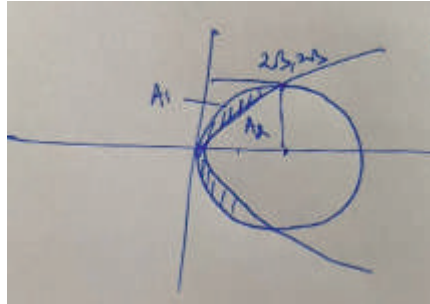
$$A_1 = 3\pi - 8$$

Question: a_1, a_2, a_3, \dots are positive terms of GP if $a_1 a_5 = 28$ and $a_2 + a_5 = 29$. Then find $a_6 = ?$

Options:

- (a) 780
- (b) 782
- (c) 784
- (d) 786

Answer: (a)



$$a \cdot ar^4 = 28 \quad a_1 a_5 = 29$$

$$a_2 + a_4 = 25$$

$$a \cdot ar_4 = 28 \quad ar + ar_3 = 29$$

$$ar \cdot ar^3 = 28 \quad ar + \frac{28}{ar} = 29$$

$$(ar)^2 \cdot r^2 = 28 \quad (ar)^2 + 28 = 2r^2$$

$$ar = 28 \Rightarrow r^2 = \frac{1}{28} \quad (ar)^2 - 29(ar) + 29$$

$$ar = 1 \Rightarrow r^2 = 28 \quad (ar - 28)(ar - 1) = 0$$

$$ar=28 \quad ar=1$$

$$a_6 = ar_5 = (ar \cdot r^2 - r^2)$$

$$= 1 \times 28 \times 28$$

$$= 784$$

Question: $16[(\sec^{-1} x)^2 + (\operatorname{cosec}^{-1} x)^2]$ Find $m + M$ where m and M are the min and max values respectively .

Options:

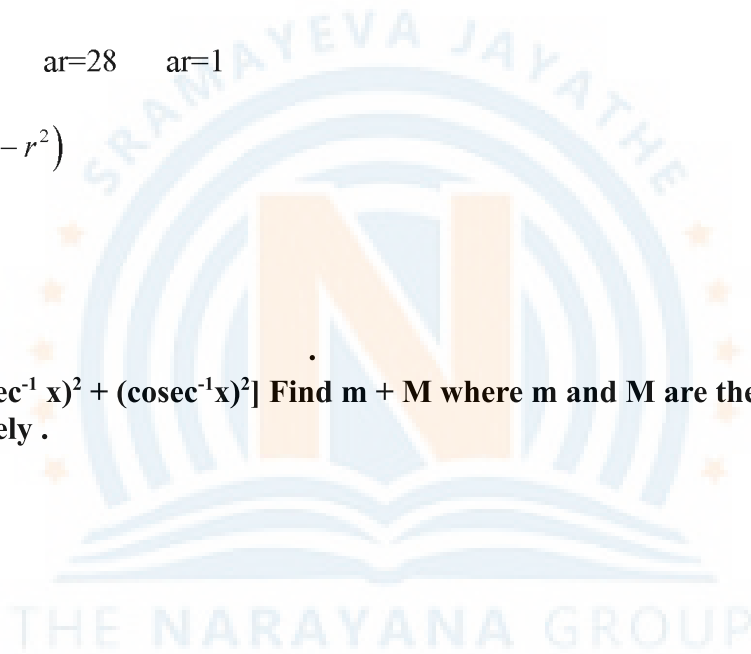
(a) $20\pi^2$

(b) $22\pi^2$

(c) $2\pi^2$

(d) π^2

Answer: (a)



$$16 \left[\left(\sec^{-1} x \right)^2 + \left(\operatorname{cosec}^{-1} x \right)^2 \right]$$

$$16 \left(t^2 + \left(\frac{n}{2} - t \right)^2 \right), t = \sec^{-1} x \in [0, n] - \left\{ \frac{n}{2} \right\}$$

$$b(t) = 16 \left(2t^2 - \pi t + \frac{\pi^2}{4} \right)$$

$$= 32t^2 - 16\pi t + 4\pi^2$$

$$b'(t) = 64t - 16\pi = 0 \Rightarrow t = \frac{\pi}{4}$$

$$b(0) = 4\pi^2, b(\pi) = 20\pi^2$$

$$b\left(\frac{\pi}{4}\right) = 2\pi^2 - 4\pi^2 + 4\pi^2 = 2\pi^2$$

$$m = 2\pi^2 M = 20\pi^2$$

Question: If A be a 3×3 square matrix such that $\det(A) = -2$. If $\det(3\operatorname{adj}(-6\operatorname{adj}(3A))) = 2^n \times 3^m$, where $m \geq n$, then $4m + 2n$ is equal to

Options:

(a) 104

(b) 100

(c) 114

(d) 124

Answer: (a)

$$|A| = -2$$

$$|3\operatorname{adj}(-6\operatorname{adj}3A)| = 27|-6\operatorname{adj}3A|^2$$

$$= 27 \times (6^3)^2 |3A|^4$$

$$= 27 \times 6^6 \times (3^3)^4 \cdot |A|^4$$

$$= 27 \times 2^6 \times 3^6 \times 3^{12} \cdot 2^4 = 2^{10} \cdot 3^{21}$$

$$4m + 2n = 84 + 20 = 104.$$

Question: Let $f(x)$ be a real differentiable function such that $f(0) = 1$ and $f(x+y) = f(x)f'(y) + f(y)f'(x)$ for all $x, y \in \mathbb{R}$. Then

Options:

(a) 2525

(b) 1224

(c) 2500

(d) 1000

Answer: (a)

$$f(x+y) = f(x)f'(y) + f(y)f'(x)$$

$$y=0 \quad f'(x) + f'(0) \cdot f(x) = f(x)$$

$$\frac{dy}{y} = (1 - f'(0)) dx$$

$$\ln y = (1 - f'(0))x + c$$

$$0 = 0(1 - f'(0)) + c \rightarrow c = 0$$

$$\ln y = (1 - f'(0))x \rightarrow \frac{y^1}{y} = 1 - f'(0) \rightarrow f'(0) = \frac{1}{2}$$

$$\sum_{n=1}^{100} \ln f(x) = (1 - f'(0)) \frac{100(101)}{2}$$

$$= \frac{100 \times 101}{4} = 2525$$

Question: Hyperbola \rightarrow F(1, 12) F1 (1,-14) passes through (1,6) Find Latus Rectum**Options:**

(a) 20/7

(b) 30/7

(c) 40/7

(d) 50/7

Answer: (b)

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$$Foci = F(1,12), F'(1,-14)$$

$$P(1,6)$$

$$PF = 6, PF' = 20 \Rightarrow PF' - PF = 14$$

$$\Rightarrow 2a=14 \Rightarrow a=7$$

$$\text{Also, } FF' = 16 \Rightarrow 2ae = 16 \Rightarrow e = \frac{8}{7}$$

$$b^2 = a^2(e^2 - 1) = 49\left(\frac{64}{49} - 1\right) = 15$$

$$LR = \frac{2b^2}{a} = \frac{30}{7}$$

Question: Number of equivalence relation on set $A = \{1, 2, 3\}$

Options:

- (a) 5
- (b) 8
- (c) 2
- (d) 0

Answer: (a)

Number of relations

$$\left\{ (1,1), (2,2), (3,3) \right\} \left\{ (1,1), (2,2), (3,3), (1,2), (2,1) \right\}$$

$$\left\{ (1,1), (2,2), (3,3), (1,2), (2,1), (1,3), (3,1), (2,3), (3,2) \right\}$$

$$= 5$$

Question: Z_1, Z_2, Z_3 lies on $|z| = 1$, arg are $\frac{\pi}{4}, -\frac{15}{4}, 0$.

$$\left(|z_1 \bar{z}_2 + z_2 \bar{z}_3 + z_3 \bar{z}_1| \right)^2 = (\alpha + \beta)\sqrt{2} \text{ Find } \alpha^2 + \beta^2.$$

Options:

- (a) 29
- (b) 2
- (c) 4
- (d) 25

Answer: (a)

$$z_1 = e^{i\pi/4}, z_2 = \bar{z}_1, z_3 = 1$$

$$(z_1 \bar{z}_2 + z_2 \bar{z}_3 + z_3 \bar{z}_1)^2 = (i + e^{-i\pi/4} + e^{-i\pi/4})^2$$

$$= [i + \sqrt{2} - \sqrt{2}i]^2 = 2 + (1 - \sqrt{2})^2$$

$$= 5 - 2\sqrt{2}$$

$$\alpha = 5\beta = -2$$

$$\alpha^2 + \beta^2 = 29$$

Question: $\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^2}$ then $x(0) = 1$ and $x\left(\frac{1}{2}\right) = ?$

Options:

- (a) 1-e
- (b) 2-e
- (c) 3-e
- (d) 4-e

Answer: (c)

$$x.e^{\frac{1}{y}} = \int \frac{2}{y^3} e^{\frac{1}{y}} dy$$

$$= \int te^t dt \quad \frac{-1}{y} = t$$

$$= \int te^t dt$$

$$= e^t(1-t) + c$$

$$= e^{\frac{1}{y}} \left(1 + \frac{1}{y}\right) + ce^{\frac{1}{y}}$$

$$y = +1, x = 1 \quad 1 = 1 + 1 + c.e^1$$

$$c = -1/e$$

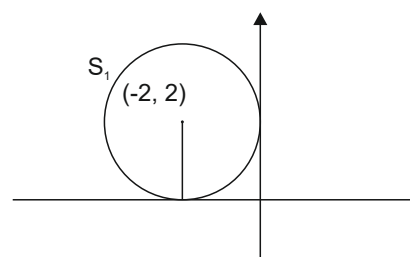
$$x\left(\frac{1}{2}\right) = 3 - \frac{1}{e} \cdot e^2$$

$$= 3 - e$$

Question: $S_2 : (2, 5)$ is centre red = r, $r \in [\alpha, \beta]$ such that both cut at different point.

Options:

- (a) $r \in [3, 7]$
- (b) $r \in [2, 7]$
- (c) $r \in [3, 5]$
- (d) None



Answer: (a)

$$C_1 C_2 = \sqrt{(2+2)^2 + (5-2)^2} = 5$$

$$|r-2| < 5 < r+2$$

$r > 3$ and $r < 7$ or

$$r \in [3, 7]$$

Question: Shortest distance between $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-1}{4}$ **and** $\frac{x+2}{7} = \frac{y-2}{8} = \frac{z+1}{2}$

Options:

(a) $\frac{80}{\sqrt{1277}}$

(b) $\frac{88}{\sqrt{1277}}$

(c) $\frac{8}{\sqrt{1277}}$

(d) $\frac{87}{\sqrt{1277}}$

Answer: (b)

$$S.D \frac{|\begin{matrix} 3 & 0 & 2 & 2 & 3 & 4 & 7 & 8 & 2 \\ \hat{i} & \hat{j} & \hat{k} & 2 & 3 & 4 & 7 & 8 & 2 \end{matrix}|}{|-26\hat{i} + 24\hat{j} - 5\hat{k}|} = \frac{|-78-10|}{\sqrt{1277}} = \frac{88}{\sqrt{1277}}$$

Question: $\sum_{r=0}^5 \frac{{}^{11}C_{2r-1}}{2r+2} = ?$

Options:

(a) 2048/6

(b) 2048/12

(c) 2047/12

(d) 2047/6

Answer: (d)

$$\sum_{r=0}^5 \frac{{}^{11}C_{2r-1}}{2r+2} = \int_0^1 (1+x)^{11} + (1-x)^{11} dx$$

$$= \left. \frac{(1+x)^{12}}{12} + \frac{(1-x)^{12}}{12} \right|_0^1$$

$$= \frac{12^{12}}{12} - \frac{1}{12} + 0 - \frac{1}{12}$$

$$= \frac{12^{12} - 2}{12} = \frac{2047}{6}$$