JEE-Main-22-01-2025 (Memory Based)
[MORNING SHIFT]
Maths

Question: f(x) = 7 $\tan^8$x + 7$\tan^6$x -3 $\tan^4$ x - 3$\tan^2$x $I_1 = \int_0^{\frac{\pi}{4}} f(x)dx I_2 = \int_0^{\frac{\pi}{4}} xf(x)dx$ **Find**

$7I_1 + 12I_2$

Options:

(a) **1**

(b) **2**

(c) **3**

(d) **4**

Answer: (a)

$$f(x) = 7\tan^{8x} + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$$

$$= 7\tan^6 .\sec^2 x - 3\tan^2 x..\sec^2 x$$

$$I_1 = \int_0^{\frac{\pi}{4}} f(a)dx$$

$$= \int_0^{\frac{\pi}{4}} \left(7\tan^6 x - 3\tan^2 x\right)\sec^2 xdx$$

$$= \int_0^1 \left(7t^6 - 3t^2\right)dt$$

$$= t^7 - t^3\Big|_0^1$$

$$= 0$$

$$I_2 = \int_0^{\frac{\pi}{4}} xf(x)dx$$

$$= x\left[\tan^7 x - \tan^3 x\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{2}} \tan^7 x + \tan^3 x$$

$$= 0 - \int_0^{\frac{\pi}{2}} \tan^3 x\left(\tan^2 x - 1\right)\tan^3 x$$

$$= -\int_0^{1\frac{\pi}{2}} t^3\left(t^2 - 1\right)dt$$

$$= -\int_0^{1\frac{\pi}{2}} \left(t^5 - t^3\right)$$

$$= -\left[\frac{t^6}{6} - \frac{t^4}{4}\right]_0^1$$

$$= \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$$

Hence, $7I_1 + 12I_2 = 1$

Question: The number of 5 letter words which can be formed in alphabetical order such that M is always at middle taking every alphabet

Options:

(a) 5143

(b) 5148

(c) 5144

(d) 5149

Answer: (b)

M is the middle

$$^{12}C_2 \times 1 \times {}^{13}c_2 = \frac{12 \times 11}{2} \times 1 \times \frac{13 \times 12}{2}$$

$$= 6 \times 11 \times 6 \times 13$$

$$= 5148$$

Question: $e^{5\left(log_e x\right)^2 + 3 = x^8}$ **find the product of solutions**

Options:

(a) $e^{2/5}$

(b) $e^{3/5}$

(c) $e^{8/5}$

(d) $e^{1/5}$

Answer (c)

$$e^5(\log_e x)^2+3=x^8$$

$$\Rightarrow 5(\log_e x)^2+3=8\ln_e x$$

$$\Rightarrow 5t^2+3=8t$$

$$\Rightarrow 5t^2-8t+3=0$$

$$\Rightarrow 5t^2-5t-3t+3=0$$

$$\Rightarrow 5t(t-1)-3(t-1)=0$$

$$t=1, t=\frac{3}{5}$$

$$\ln_e x=21 \quad \ln x=\frac{3}{5}$$

$$x=e \qquad x=e^{\frac{3}{5}}$$

$$p=e^l.e^{\frac{3}{5}}$$

$$=e^{\frac{3}{5}+1}=e^{\frac{8}{5}}$$

Question: Let the triangle PQR be the image of the triangle with vertices (1, 3), (3, 1), (2, 4) in the line x + 2y = 2. If the centroid of ΔPQR is the point (α, β) then 15(α - β) is equation
Options:
(a) 20
(b) 21
(c) 22
(d) 23
Answer: (c)

$\frac{1+3+2}{3}, \frac{3+1+4}{3}$ x+2y-2=0

$=\left(2, \frac{8}{3}\right)$ $\frac{x-2}{1}=\frac{y-\frac{8}{3}}{2}=-2\frac{\left(1+\frac{16}{3}-2\right)}{5}=\frac{32}{15}$

$x=2-\frac{32}{15}, \quad y=\frac{8}{3}-\frac{64}{15}$

$=-\frac{2}{15}$ $=\frac{-24}{15}=\frac{-8}{5}=\frac{-24}{15}$

$15\alpha=-2$

$15\beta=-24$ $15(\alpha-\beta)=22$

Question: Set A = {1, 2, ….10} B = $\left\{\frac{m}{n}: m<n, m, n \in A\right\}$ **(m,n) =1 n(B) = ?**
Options:
(a) 30
(b) 31
(c) 32
(d) 33
Answer: (b)

$A=\{1,2,3,.......10\}$

$B=\left\{\frac{m}{n}:m<n,m,n\in A\right\}$

$\frac{1}{10},\frac{3}{10},\frac{7}{10},\frac{9}{10}$

$\frac{1}{9},\frac{2}{9},\frac{4}{9},\frac{5}{9},\frac{7}{9},\frac{8}{9}$

$\frac{1}{8},\frac{3}{8},\frac{5}{8},\frac{7}{8}$

$\frac{1}{7},\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7},\frac{6}{7}$

$\frac{1}{6},\frac{5}{6}$

$\frac{1}{5},\frac{2}{5},\frac{3}{5},\frac{4}{5}$

$\frac{1}{4},\frac{3}{4}$

$\frac{1}{3},\frac{2}{3}$

$\frac{1}{2}$

$\text{Total}=31$

Question: A coin is tossed 3 times x is no of head following by tails 64[(M)+ (σ^2)]
Options:
(a) 71
(b) 40
(c) 60
(d) 48
Answer: (d)

$x=0\{HHH,TTT,HTT,HHT\}$

$x=1\left\{\begin{matrix}THH,HTH,TTH\\ \text{THT}\end{matrix}\right\}$

$P(x=0)=\frac{1}{2}=p(x=1)$

$\mu=\frac{1}{2}$

$\sigma^2=\sum pi\left(x_i-\frac{1}{2}\right)^2$

$=\frac{1}{2}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{1}{4}=\frac{1}{4}$

$64\left(\frac{1}{2}+\frac{1}{4}\right)=32+16=48$

$2T,1H$

TTH,THT,HTT

$2H1T$

$HHTHTH$

THH

Question: Two balls are selected at random one by one without replacement from the bag containing 4 white and 6 black balls. If the probability that the first selected ball is black given that the second selected is also black, is m/n where gcd(m, n) = 1, then m + n = ?

Options:

(a) 11

(b) 12

(c) 13

(d) 14

Answer: (d)

$4W.6B\ P\left(\frac{FirstBlock}{SecondBlock}\right)=\frac{P(BB)}{P(WB)+P(BB)}=\frac{\frac{6}{10}\cdot\frac{5}{9}}{\frac{4}{10}\cdot\frac{6}{9}+\frac{6}{10}\cdot\frac{5}{9}}=\frac{30}{54}=\frac{5}{9}=\frac{m}{n}\ m+n=14$

Question: $\sum_{r=1}^{n}T_r=\frac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$, **then** $\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{T_r}\right)=$

Options:

(a) 1

(b) 0

(c) 2

(d) 3

Answer: (c)

$$T_r = S_n - S_{n-1}$$

$$= \frac{(2r-1)(2r-2)(2r+3)(2r+4)}{64} - \frac{(2r-3)(2r-1)(2r+1)(2r+3)}{64}$$

$$= 64(2r-1)(2r+1)(2r+3)\left[2r+5-(2r-3)\right]$$

$$T_n = \frac{1}{\theta}(2r-1)(2r+1)(2r+3)$$

$$\lim_{n\to 0}\sum_{r-\infty}^{n}\frac{1}{\ln}$$

$$\lim_{n\to 0}\sum_{r-\infty}^{n}\frac{\theta}{(2r-1)(2r+1)(2r+3)}\frac{\left[(2r+3)-(2r-1)\right]}{4}$$

$$= 2\lim_{n\to b}\sum_{r-\infty}^{n}\left[\frac{1}{(2r-1)(2r+1)} - \frac{1}{(2r+1)(2r+3)}\right]$$

$$= 2\left[1-\frac{1}{\infty}\right] = 2$$

Question: Parabola of equation $y^2 = 2\sqrt{3}$ and circle of equation $y^2 + (x - 2\sqrt{3})^2 = 12$. Find the area inside the circle but outside the parabola?

Options:

(a) 2π - 8

(b) π - 8

(c) 3π - 8

(d) π - 4

Answer: (c)

$y^2 = 2\sqrt{3}x$

$y^2 + \left(x - 2\sqrt{3}\right)^2 = 12$

$2\sqrt{3}x + \left(2 - 2\sqrt{3}\right)^2 = 12$

$2\sqrt{3}x + x^2 + 12 - 4\sqrt{3}x$

$x^2 = 2\sqrt{3}x$

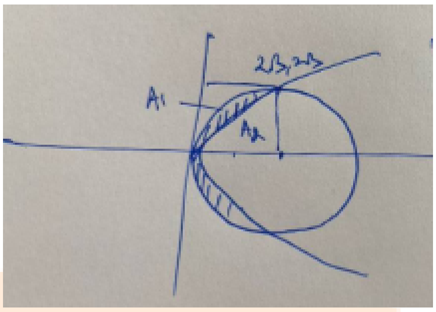
$x = 0, 2B$

$y^2 = 2B.2B = 12$

$A_1 + A_2 = \dfrac{\pi.12}{4} = 3\pi$

$A_2 = \dfrac{2}{3}.2\beta.2\beta = 8$

$A_1 = 3\pi - 8$



Question: a_1, a_2, a_3 …… are positive terms of GP if a_1 a_5 = 28 and a_2 + a_5 = 29. Then find a_6 = ?
Options:
(a) 780
(b) 782
(c) 784
(d) 786
Answer: (a)

$a.ar^4 = 28 \quad a_1 a_5 = 29$

$a_2 + a_4 = 25$

$a.ar_4 = 28 \quad ar + ar_3 = 29$

$ar.ar^3 = 28 \quad ar + \frac{28}{ar} = 29$

$(ar)^2 \cdot r^2 = 28 \quad (ar)^2 + 28 = 2r9r$

$ar = 28 \Rightarrow r^2 = \frac{1}{28} \quad (ar)^2 - 29(ar) + 29$

$ar = 1 \Rightarrow r^2 = 2\theta \quad (ar - 28)(ar - 1) = 0$

ar=28 ar=1

$a_6 = ar_5 = (ar.r^2 - r^2)$

$= 1 \times 28 \times 28$

$= 780$

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Question: 16[(sec^{-1} x)^2 + (cosec^{-1}x)^2] Find m + M where m and M are the min and max values respectively .
Options:
(a) 20π^2
(b) 22π^2
(c) 2π^2
(d) π^2
Answer: (a)

$$16\left[\left(\sec^{-1}x\right)^2+\left(\cos ec^{-1}x\right)^2\right]$$

$$16\left(t^2+\left(\frac{n}{2}-t\right)^2\right)^2, t=\sec^{-1}x\in[0,n]-\left\{\frac{n}{2}\right\}$$

$$b(t)=16\left(2t^2-\pi t+\frac{\pi^2}{4}\right)$$

$$=32t^2-16\pi+4\pi''$$

$$b'(t)=64t-16\pi=0\Rightarrow t=\frac{\pi}{4}$$

$$b(0)=4\pi^2, b(\pi)=20\pi^2$$

$$b\left(\frac{\pi}{4}\right)=2m^2-4\pi^2+4\pi^2=2\pi^2$$

$$m=2\pi^2 M=20\pi^2$$

Question: If A be a 3 × 3 square matrix such that det(A) = -2. If det(3adj(-6adj(3A))) = $2^n \times 3^m$, where m ≥ n, then 4m + 2n is equal to

Options:

(a) 104

(b) 100

(c) 114

(d) 124

Answer: (a)

$$|A|=-2$$

$$|3adj(-6adj3A)|=27|-6adj3A|^2$$

$$=27\times\left(6^3\right)^2|3A|^4$$

$$=27\times6^6\times\left(3^3\right)^4\cdot|A|^4$$

$$=27\times2^6\times3^6\times3^{12}\cdot2^4=2^{10}\cdot3^{21}$$

$$4m+2m=84+20=104.$$

Question: Let f(x) be a real differentiable function such that f(0) = 1 and f(x + y) = f(x) f’(y) + f(y)f’(x) for all x, y ∈ R. Then

Options:

(a) 2525

(b) 1224

(c) 2500
(d) 1000
Answer: (a)

$$f(x+y)=f(x)f'(y)+f(y)f'(x)$$

$$y=0 \quad f'(x)+f'(0)\cdot f(x)=f(x)$$

$$\frac{dy}{y}=(1-f'(0))dx$$

$$liny=(1-f'(0))x+c$$

$$0=0(1-f'(0))+c\rightarrow c=0$$

$$liny=(1-f'(0))x\rightarrow\frac{y^1}{y}=1-f'(0)\rightarrow f'(0)=\frac{1}{2}$$

$$\sum_{n=1}^{100} lin f(x)=(1-f'(0))\frac{100(101)}{2}$$

$$=\frac{100\times 101}{4}=2525$$

Question: Hyperbola → F(1, 12) F1 (1,-14) passes through (1,6) Find Latus Rectum
Options:
(a) 20/7
(b) 30/7
(c) 40/7
(d) 50/7
Answer: (b)

$Foci = F(1,12), F'(1,-14)$

$P(1,6)$

$PF = 6, PF' = 20 \Rightarrow PF' - PF = 14$

$\Rightarrow 2a=14 \Rightarrow a=7$

Also, $FF' = 16 \Rightarrow 2ae = 16 \Rightarrow e = \frac{8}{7}$

$b^2 = a^2(e^2-1) = 49\left(\frac{64}{49}-1\right) = 15$

$LR = \frac{2b^2}{a} = \frac{30}{7}$

Question: Number of equivalence relation on set A = {1, 2, 3}
Options:
(a) 5
(b) 8
(c) 2
(d) 0
Answer: (a)

Number of relations

$\underset{(1)}{\{(1,1),(2,2),(3,3)\}}\underset{(3)}{\{(1,1),(2,2),(3,3),(1,2),(2,1)\}}$

$\underset{(1)}{\{(1,1),(2,2),(3,3),(1,2),(2,1),(1,3),(3,1),(2,3),(3,2)\}}$

$= 5$

Question: Z_1, Z_2, Z_3 lies on |z| = 1, arg are $\frac{\pi}{4}, -\frac{15}{4}, 0$.

$\left(\left|z\cdot\bar{z}_2 + z_2, \bar{z}_3 + z_3 \cdot \bar{z}_1\right|\right)^2 = (\alpha + \beta)\sqrt{2}$ **Find $\alpha^2 + \beta^2$.**
Options:
(a) 29
(b) 2
(c) 4
(d) 25
Answer: (a)

$z_1 = e^{i\pi/4}, z_2 = \overline{z}_1, z_3 = 1$

$\left(z_1\overline{z}_2 + z_2\overline{z}_3 + z_3\overline{z}_1\right)^2 = \left(i + e^{-1\pi/4} + e^{-i\pi/4}\right)^2$

$= \left[i + \sqrt{2} - \sqrt{2}i\right]^2 = 2 + \left(1 - \sqrt{2}\right)^2$

$= 5 - 2\sqrt{2}$

$\alpha = 5\beta = -2$

$\alpha^2 + \beta^2 = 29$

Question: $\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^2}$ *then* $x(0) = 1$ *and* $x\left(\frac{1}{2}\right) = ?$

Options:
(a) 1-e
(b) 2-e
(c) 3-e
(d) 4-e
Answer: (c)

$x.e^{\frac{-1}{y}} = \int \frac{2}{y^3} e^{\frac{-1}{y}} dy$

$= \int te^t\, dt \quad \frac{-1}{y} = t$

$= \int te^t\, dt$

$= e^t(1-t) + c$

$= e^{\frac{-1}{y}}\left(1 + \frac{1}{y}\right) + ce^{\frac{1}{y}}$

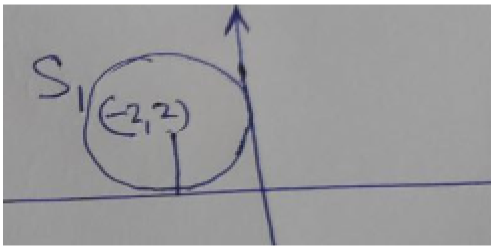
$y = +1 . x = 1 \quad 1 = 1 + 1 + c.e^1$

$c = -1/e$

$x\left(\frac{1}{2}\right) = 3 - \frac{1}{e} \cdot e^2$

$= 3 - e$

Question: S_2 : (2, 5) is centre red = r, r ∈ [α, β] such that both cut at different point.
Options:
(a) $r \in [3, 7]$
(b) $r \in [2, 7]$
(c) $r \in [3, 5]$
(d) None



Answer: (a)

$$C_1C_2 = \sqrt{(2+2)^2 + (5-2)^2} = 5$$

$$|r-2| < 5 < r+2$$

$r > 3$ and $r < 7$ or

$$r \in [3, 7]$$

Question: Shortest distance between $\frac{x-1}{2} = \frac{y-2}{3} = \frac{2-1}{4}$ **and** $\frac{x+2}{7} = \frac{y-2}{8} = \frac{2+1}{2}$

Options:

(a) $\frac{80}{\sqrt{1277}}$

(b) $\frac{88}{\sqrt{1277}}$

(c) $\frac{8}{\sqrt{1277}}$

(d) $\frac{87}{\sqrt{1277}}$

Answer: (b)

$$S.D \frac{|3\ 0\ 2\ 2\ 3\ 4\ 7\ 8\ 2\ |}{|\hat{i}\ \hat{j}\ \hat{k}\ 2\ 3\ 4\ 7\ 8\ 2\ |} = \frac{|-78-10|}{-26\hat{i}+24\hat{j}-5\hat{j}} = \frac{88}{\sqrt{1277}}$$

Question: $\sum_{r=0}^{5} \frac{^{11}C_{2r-1}}{2r+2} = ?$

Options:

(a) 2048/6

(b) 2048/12

(c) 2047/12

(d) 2047/6

Answer: (d)

$$\sum_{r=0}^{5} \frac{^{11}C_{2r-1}}{2r+2} = \int_0^1 (1+x)^{11} + (1-x)^{n1}\, dx$$

$$= \frac{(1+x)^{12}}{12} + \frac{(1-x)^{12}}{12}\Bigg|_0^1$$

$$= \frac{12^{12}}{12} - \frac{1}{12} + 0 - \frac{1}{12}$$

$$= \frac{12^{12} - 2}{12} = \frac{2047}{6}$$