# JEE-Main-22-01-2025 (Memory Based)
# [EVENING SHIFT]
## Maths

**Question: 4 boys and 3 girls are to be seated in a row such that all girls seat together and two particular boys $B_1$ and $B_2$ are not adjacent to each other. Then the number of ways in which this arrangement can be done.**

**Options:**

(a) 1002

(b) 516

(c) 430

(d) 432

**Answer: (d)**

$$3Girls, 4Boys. B_1, B_2, B_3, B_4, G_1, G_2, G_3$$

$$G_3\text{Total} - G_3B_1B_2$$

$$5! \cdot 3! - 4! \cdot 3! \cdot 2!$$

$$= 120 \times 6 - 24 \times 6 \times 2$$

$$= 432$$

**Question: If the mean deviation about median for the number 3, 5, 7, 2k, 12, 16, 21, 24 arranged in ascending order is 6 then the median is**

**Solution:**

$$\text{Median} = \frac{2k+12}{2} = 6 + k$$

$$48 = |k+3| + |k+1| + |k-1| + |6-k|$$

$$+ |6-k| + |10-k| + |15-k| + |18-k|$$

$$48 = k+3 + k+1 + k-1 + 6 - k$$

$$+6-k + 10-k + 15 - k + 18 - k$$

$$48 = -2k + 58$$

$$2k = 10$$

$$k = 5$$

$$\therefore \text{Median} = 11.$$

**Question: If $2x^2 + (\cos\theta)x - 1 = 0$, $\theta \in [0, 2\pi]$ has roots $\alpha$ and $\beta$. Hence the sum of maximum and minimum value of $\alpha^4 + \beta^4$ is**

**Options:**

**(a)** $\frac{25}{16}$

**(b)** $\frac{9}{16}$

**(c)** $\frac{41}{16}$

**(d)** $\frac{1}{17}$

**Answer: (a)**

$$2x^2 - 1(\cos\theta)x - 1 = 0$$

$$\alpha + \beta = \frac{-\cos\theta}{2}, \alpha\beta = \frac{-1}{2}$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \frac{\cos^2\theta}{2} + 1$$

$$\alpha^4 + \beta^4 = \left(\alpha^2 + \beta^2\right)^2 - 2\alpha^2\beta^2$$

$$= \left(\frac{\cos^4\theta}{4} + 1\right)^2 - 2\left(\frac{1}{4}\right)$$

$$Min \to \cos\theta = 0 \quad \text{Max} \to cos\theta = 1$$

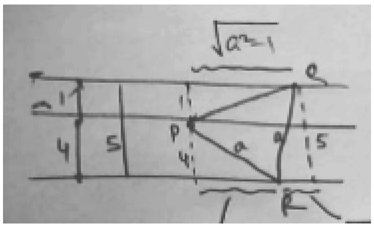
$$1 - \frac{1}{2} = \frac{1}{2} \qquad \left(\frac{1}{4} + 1\right)^2 \frac{1}{2}$$

$$\frac{25}{16} - \frac{1}{2} = \frac{17}{16}$$

$$\frac{17}{16} + \frac{1}{2} = \frac{25}{16}$$

**Question: Two parallel lines at distance of 5 units, a point p at distance 1 unit from one of the lines. let q be a point on one of the lines and r be the point another one. if the PQR is equilateral triangle, find$(qr)^2$**

**Solution :**



$\sqrt{a^2-1}=\sqrt{a^2-16}+\sqrt{a^2-r_5}$

$\cancel{a^2}-1=\cancel{a^2}-r_5+a^2-16+2\sqrt{a^2-16}\sqrt{a^2-r^5}$

$40-a^2=2\sqrt{a^2-16}\,\sqrt{a^2-r^5}$

$1600+a^4-80a^2=4\left[a^4-41a^2+400\right]$

$3a^4-84a^2=0$

$a^2=\frac{84}{3}$

**Question: Let A = {1, 2, 3, 4} and B = {1, 4, 9, 16}. If f : A → B, then number of many-one functions from A to B are**
**Options:**
**(a) 24**
**(b) 232**
**(c) 256**
**(d) 252**
**Answer: (b)**

$\text{Number of functions}=4^4=256$

$\text{Number of ono-one functions}=4!=24$

$\text{Number of ono-one functions}=256-24$

$=232$

**Question: If $\theta\in[0, 2\pi]$ satisfying the system of equations 2sin$^2\theta$ = cos 2$\theta$ and 2cos$^2\theta$ = 3 sin$\theta$. Then the solution of all real values of $\theta$ is**
**Options:**
**(a)** $\frac{3\pi}{2}$
**(b)** $\pi$
**(c)** $\frac{\pi}{2}$
**(d)** $\frac{5\pi}{6}$
**Answer: (d)**

$(i) 2\sin^2\theta = \cos 2\theta$

$2\sin^2\theta = 1 - 2\sin^2\theta$

$4\sin^2\theta = 1 \rightarrow \sin\theta = \pm\frac{1}{2}$

$(ii) 2\cos^2\theta = 3\sin\theta$

$2 - 2\sin^2\theta = 3\sin\theta$

$\rightarrow 2\sin^2\theta + 3\sin\theta - 2 = 0$

$2\sin\theta(\sin\theta + 2) - 1(\sin\theta + 2) = 0$

$\sin\theta = \frac{1}{2}$

$\therefore \sin\theta = \frac{1}{2} \quad \text{or } \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

**Question:** $\left(1+y^2\right) + \left(x - 2e^{\tan^{-1}y}\right)\frac{dy}{dx} = 0$

**Solution:**

$$\left(1+y^2\right)dx = \left(2e^{\tan^{-1}y} - x^2\right)dy$$

$$\frac{dx}{dy} = \frac{2e^{\tan^{-1}y}}{1+y^2} - \frac{x}{1+y^2}$$

$$\frac{dx}{dy} + \left(\frac{1}{1+y^2}\right)x = \frac{2e^{\tan^{-1}y}}{1+y^2}$$

$$I.F = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$$

$$x.\, e^{\tan^{-1}y} = \int \frac{2e^{\tan^{-1}y}e^{\tan^{-1}y}}{1+y^2}dy$$

$$x \cdot e^{\tan^{-1}} == \int 2e^{2t}.\, dt$$

$$x \cdot e^{\tan^{-1}y} = e^{2\tan^{-1}y} + C$$

**Question: Perpendicular distance from the point P(-2, 0, 2) to the line** $\frac{x+1}{2} = \frac{y-1}{-1} = \frac{z+3}{2}$

**Solution:**

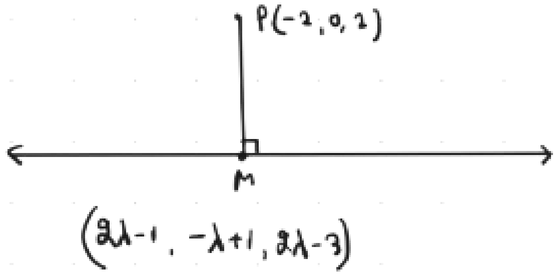
$\overline{PM} = (2\lambda + 1, 1 - \lambda, 2\lambda - 5)$

$\overline{PM} \cdot \bar{b} = 0$

$4\lambda + 12 - 1 + \lambda + 4\lambda - 10 = 0$

$7\lambda - 9 = 0$

$\lambda = \frac{9}{7}$




$M = \left(\frac{18}{7} - 1, 1 - \frac{9}{7}, \frac{18}{7} - 5\right)$

$M = \left(\frac{11}{7}, \frac{-2}{7}, \frac{-17}{7}\right)$

$PM = \sqrt{\left(-2 - \frac{11}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(2 + \frac{17}{7}\right)^2}$

$PM = \sqrt{\left(\frac{-25}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(\frac{31}{7}\right)^2}$

$PM = \sqrt{\frac{625+4+961}{49}} = \sqrt{\frac{1590}{7}}$

**Question: Let $\vec{a}\ and\ \vec{b}$ be two unit vectors such that angle between $\vec{a}\ and\ \vec{b}\ is\ \frac{\pi}{3}$.**

**If $\lambda\vec{a} + 3\vec{b}\ and\ 2\vec{a} + \lambda\vec{a}$ are perpendicular to each other, then the product of all possible values of λ is __________**

**Solution :**

$|\bar{a}| = |\bar{b}| = 1$

$Now\ \bar{a} \cdot \bar{b} = |\bar{a}||\bar{b}| as \frac{\pi}{3} = \frac{1}{2} \Rightarrow \bar{a} \cdot \bar{b} = \frac{1}{2}$

$\left(\lambda\bar{a} + 3\bar{b}\right) \cdot \left(2\bar{a} + \lambda\bar{b}\right) = 0$

$2\lambda + 3\lambda + \lambda^2\left(\bar{a} \cdot \bar{b}\right) + 6\left(a \cdot \bar{b}\right) = 0$

$5\lambda + \frac{\lambda^2}{2} + 3 = 0$

$\lambda^2 + 10\lambda + 6 = 0$

$\text{Produce} = 6$

**Question: The total number of terms in A.P are 2k. The sum of odd terms is 40 and the sum of even terms is 55 and last term of the A.P exceeds the first term by 27. Then find the value of k.**

**Solution :**

$T_1, T_2, T_3 \ldots\ldots., T_{2k}$

$T_1 + T_3 + T_5 + \ldots\ldots.T_{2k} - 1 = 40$

$T_2 + T_4 + T_6 + \ldots.. + T_{2k} = 55$

$T_{2k} = T_1 = 27$

$\frac{k}{2}[2a + (k-1)2d] = 40$

$k[a + (k-1)d] = 40 \ldots\ldots.(1)$

$\frac{k}{2}[2(a+d) + (k-1)2d] = 55$

$k[a + d + (k-1)d] = 55 \ldots\ldots..(2)$

$(2) - (1) \rightarrow dk = 15$

also $a + (2k-1)d - 1 = 27$

$(2k-1)d = 27$

$2kd - d = 27$

$30 - d = 27$

$\therefore d = 3$

$andk = 5$

**Question: Consider a function** $f(x) = \int_0^{x^2} \frac{t^2 - 8t + 15}{e^t} dt.$ **The number of points of extrema are**

**Options:**

**(a) 7**

**(b) 9**

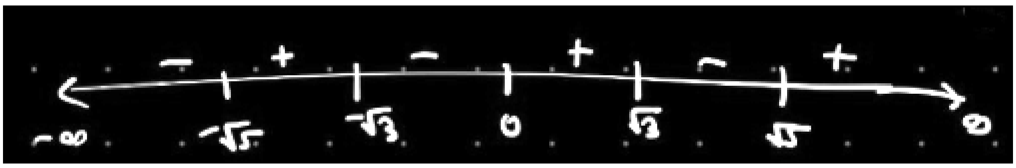
**(c) 5**

**(d) 3**

**Answer: (c)**

$$f'(x) = \frac{x^4 - 8x^2 + 15}{e^{x^2}} 2x$$

$$f'(x) \frac{(x^2-5)(x^2-3)2x}{e^{x^2}} = \frac{(x+\sqrt{5})(x+\sqrt{3})x(x-\sqrt{3})(x-\sqrt{5})}{e^{x^2}}$$




5 points and extrema

**Question: Let A = {1, 2, 3} then the number of relations on A which consist of ordered pair (1, 2) & (2, 3) and must be reflexive and transitive but not symmetric**

**Solution:**

$A=\{1,2,3\}$

$\text{Total relations} = 2$

$\text{Must in clude}$

$\{(1,1)(2,2)(3,3)(1,2)(2,3)(1,3)\}$

$(2,1);(3,2);(3,1)$

$R_1:\{(1,1)(2,2)(3,3)(1,2)(2,3)(1,3)\}$

$R_2:\{All,(2,1)\}$

$R_3:\{All(3,2)\}$

$R_4:\{All,(3,1)\}$

$R_5:\{All,(2,1),(3,2)\}$

$R_6:\{All(2,1)(3,1)\}$

$R_7:\{All,(3,2)(3,1)\}$

$R_8\{All(2,1)(3,2),(3,1)\}$

**Question:** $\lim_{x\to\infty}\left(\left(\frac{e}{1-e}\right)\left(\frac{1}{e}-\frac{x}{1+x}\right)\right)^x=\alpha,\ \text{find}\frac{\ln\alpha}{1+\ln\alpha}$

**Solution:**

$\alpha=\lim_{x\to\infty}\left(\left(\frac{e}{1-e}\right)\left(\frac{1}{e}-\frac{x}{1+x}\right)\right)^x l^{\infty form}$

$\log\alpha=\lim_{x\to\infty}x\log\left(\left(\frac{e}{1-e}\right)\left(\frac{1}{e}-\frac{x}{1+x}\right)\right)$

$\log\alpha=\lim_{x\to\infty}\left[\left(\frac{e}{1-e}\right)\left(\frac{1}{e}-\frac{x}{1+x}\right)-1\right]$

$\text{if x}\to o$

$\text{then}\log(1+x)\simeq x$

$\log\alpha=\lim_{x\to\infty}\frac{ex}{1-e}\left(\frac{1}{e}-\frac{x}{1+x}\right)-x$

$\log^a=\lim_{x\to\infty}\frac{ex}{1-e}\left(\frac{1+x-ex}{e(1+x)}-x\right)$

$\log\alpha=\lim_{x\to\infty}\frac{x}{1+x}\frac{[1+x-ex]}{(1-e)}-x$

$$\log\alpha = \lim_{x\to\infty}\frac{x+x^2-ex^2-x(1+x)(1-e)}{(1+x)(1-e)}$$

$$\log\alpha = \lim_{x\to\infty}\frac{x-x^2-ex^2-(x+x^2)(1-e)}{(1+x)(1-e)}$$

$$\log\alpha = \lim_{x\to\infty}\frac{x+x^2-ex^2-x-x^2+ex+ex^2}{(1+x)(1-e)}$$

$$\log\alpha = \frac{e}{e} \Rightarrow \frac{\log\alpha}{1+\log\alpha} = \frac{\frac{e}{1-e}}{1+\frac{e}{1-e}}$$

$$= \frac{e}{1-e+e} = e.$$

**Question: Let $\alpha$, $\beta$, $\gamma$ and $\delta$ be the coefficient of $x^7$, $x^5$, $x^3$ and x respectively in the expansion of $\left(x+\sqrt{x^2-1}\right)^5+\left(x+\sqrt{x^3-1}\right)^5$, x > 1. If u and v satisfy the equations $\alpha$u + $\beta$v = 18 $\gamma$u + $\delta$v = 20 then u + v equals**

**Options:**

**(a) 4**

**(b) 5**

**(c) 6**

**(d) 3**

**Answer: (b)**

$$\left(x+\sqrt{x^3-1}\right)^5+\left(x+\sqrt{x^3-1}\right)^5$$

$$^5C_0\cdot x^5+^5C_1\cdot x^4(y)+^5C_2\cdot x^3y^2+^5C_3x^2y^3+^5C_4x^1y^4+^5C_5y^5$$

$$2\left[^5C_0x^5+^5C_2x^3\cdot\left(x^3-1\right)+^5C_4x\left(x^3-1\right)^2\right]$$

$$2\left[^5C_0x^5+^5C_2x^6-^5C_2x^3+^5C_4x\left(x^6+1-2x^3\right)\right]$$

$$2\left[^5C_0x^5+^5C_2x^6-^5C_2x^3+^5C_4x^7+^5C_4x-2^5C_4x^2\right]$$

$$\alpha=2^5C_4=10$$

$$\beta=2\left(^5C_0\right)=20$$

$$\gamma=2\left(^5C_2\right)=-20$$

$$\delta=2\left(^5C_4\right)=10$$

$$10u+2v=18$$

$$-20u+10v=20$$

$$14v=46v=56$$

$$v=4$$

$$\text{and } 4=1$$

**Question: Let A and B are two events such that P(A ∩ B)= $\frac{1}{10}$ and P(A/B) and P(B/A) are the roots of equation $12x^2 - 7x + 1 = 0$, then $\frac{P\left(\bar{A}\cup\bar{B}\right)}{P\left(\bar{A}\cap\bar{B}\right)}$ is equal to**

**Options:**

**(a) 4/9**

**(b) 9/4**

**(c) 3/2**

**(d) 2/3**

**Answer: (b)**