

PART : MATHEMATICS

1. Evaluate :

$$\lim_{x \rightarrow 0} \operatorname{cosec} \left(\sqrt{2\cos^2 x + 3\cos x} - \sqrt{\cos^2 x + \sin x + 4} \right)$$

(1) 0

(2) 1

(3) $\frac{1}{2\sqrt{5}}$

(4) $-\frac{1}{2\sqrt{5}}$

Ans. (4)

Sol. $= \lim_{x \rightarrow 0} \frac{1}{\sin x} \frac{(2\cos^2 x + 3\cos x - \cos^2 x - \sin x - 4)}{\sqrt{2\cos^2 x + 3\cos x} + \sqrt{\cos^2 x + \sin x + 4}}$

$$= \frac{1}{2\sqrt{5}} \lim_{x \rightarrow 0} \frac{(\cos^2 x + 3\cos x - \sin x - 4)}{\sin x} \left(\frac{0}{0} \right)$$

By L' Hospital Rule

$$= \frac{1}{2\sqrt{5}} \lim_{x \rightarrow 0} \frac{2\cos x(-\sin x) - 3\sin x - \cos x}{\cos x}$$

$$= -\frac{1}{2\sqrt{5}}$$

2. Two students A & B throw two dices one by one. A wins the game when sum of numbers is 5 and B wins when the sum of numbers is 8. If A start the game then find probability that A wins the game

(1) $\frac{11}{19}$

(2) $\frac{12}{19}$

(3) $\frac{9}{19}$

(4) $\frac{10}{19}$

Ans. (3)

Sol. $P(\text{sum } 5) = \frac{4}{36}$

$$P(\text{sum } 8) = \frac{5}{36}$$

$$P(\text{A wins the game}) = P(A) + P(\bar{A}B\bar{A}) + P(\bar{A}B\bar{A}B\bar{A}) + \dots \infty$$

$$= \frac{4}{36} + \left(\frac{32}{36} \right) \left(\frac{31}{36} \right) \left(\frac{4}{36} \right) + \dots \infty$$

$$= \frac{4}{36}$$

$$1 - \left(\frac{32}{36} \right) \left(\frac{31}{36} \right)$$

$$= \frac{4}{36} \times \frac{36^2}{(1296 - 992)}$$

$$= \frac{144}{304} = \frac{36}{76} = \frac{9}{19}$$

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3. If the 5th, 6th and 7th term of the binomial expansion of $(1+x^2)^{n+4}$ are in A.P. Then greatest binomial coefficient in the expansion of $(1+x^2)^{n+4}$ is:
 (1) ${}^{16}C_8$ (2) ${}^{14}C_7$ (3) ${}^{12}C_6$ (4) ${}^{13}C_7$

Ans. (2)

Sol. ${}^{n+4}C_4, {}^{n+4}C_5, {}^{n+4}C_6$ are in A.P.

$$2 {}^{n+4}C_5 = {}^{n+4}C_4 + {}^{n+4}C_6$$

$$\frac{2 \binom{n+4}{5}}{\binom{n-1}{5}} = \frac{\binom{n+4}{4}}{\binom{n-1}{4}} + \frac{\binom{n+4}{6}}{\binom{n-2}{6}}$$

$$2(6)n = (6)(5) + n(n-1)$$

$$12n = 30 + n^2 - n$$

$$n^2 - 13n + 30 = 0$$

$$(n-3)(n-10) = 0$$

$$n = 3, 10$$

But $n = 3$ is not possible.

Now in expansion of $(1+x^2)^{14}$, greatest binomial coefficient = ${}^{14}C_7$

4. If A is 3 × 3 order matrix, such that $|A| = 2$ then value of $|\text{adj}(\text{adj}(\text{adj}(\text{adj} A)))|$ is equal to
 (1) 2^{32} (2) 2^{16} (3) 2^{20} (4) 2^8

Ans. (2)

Sol. $|A| = 2$

$$|\text{adj}(\text{adj}(\text{adj}(\text{adj} A)))|$$

$$|\text{adj}(\text{adj}(\text{adj} A))|^2$$

$$|\text{adj}(\text{adj} A)|^4$$

$$|\text{adj} A|^8$$

$$|A|^{16}$$

$$2^{16}$$

5. Let $S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots$, If $\sqrt{2026 \times S_{2025}}$ = sum of 6 terms of an A.P. whose first term is $-p$ and common difference is p then find difference of 20th term and 15th term of that A.P.

Ans. (25)

Sol. T_n of $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots$ is $\frac{1}{n^2 + n} = \frac{1}{n(n+1)}$

$$S_{2025} = \sum_{n=1}^{2025} \left(\frac{1}{n} - \frac{1}{n+1} \right)$$

$$= \left(\frac{1}{1} - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \dots + \left(\frac{1}{2025} - \frac{1}{2026} \right)$$

$$= 1 - \frac{1}{2026} = \frac{2025}{2026}$$

Now,

$$\sqrt{2026 \times \frac{2025}{2026}} = \frac{6}{2} [-2p + (6-1)p]$$

$$45 = 3(3p)$$

$$p = 5$$

Now,

$$A_{20} - A_{15} = [-5 + (19)(5)] - [-5 + (14)(5)]$$






$$= 95 - 70 = 25$$

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6. If $(x+2)^2 f(x) + 3(x+2)^2 = \int_0^x (t+2)f(t)dt$ then $f(2) = ?$

(1) $\frac{9}{2}$

(2) $-\frac{9}{2}$

(3) $\frac{7}{2}$

(4) $-\frac{7}{2}$

Ans. (2)

Sol. $(x+2)^2 f(x) + 3(x+2)^2 = \int_0^x ((t+2)f(t)dt$

$$2(x+2)f(x) + (x+2)^2 f'(x) + 6(x+2) = (x+2)f(x)$$

$$2f(x) + (x+2)f'(x) + 6 = f(x)$$

$$(x+2)f'(x) = -f(x) - 6$$

$$(x+2) \frac{dy}{dx} = -y - 6$$

$$\frac{dy}{y+6} = -\int \frac{dx}{x+2}$$

$$\ln(y+6) = -\ln(x+2) + c \quad (1)$$

$$(x+2)^2 f(x) + 3(x+2)^2 = \int_0^x (t+2)f(t)dt$$

put $x=0$

$$4f(0) + 12 = 0$$

$$f(0) = -3 \quad (2)$$

put $x=0$ is equation (1)

$$\ln 3 = -\ln 2 + c$$

$$c = \ln 6$$

$$\ln(y+6) = -\ln(x+2) + \ln 6$$

$$(y+6) = \frac{6}{(x+2)}$$

then $x=2$

$$y+6 = \frac{6}{4}$$

$$y = \frac{3}{2} - 6$$

$$y = -\frac{9}{2}$$

$$\text{so } f(2) = -\frac{9}{2}$$

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7. If α and β are real numbers such that $\sec^2(\tan^{-1}(\alpha)) + \operatorname{cosec}^2(\cot^{-1}(\beta)) = 36$ and $\alpha + \beta = 8$, ($\alpha > \beta$) then $\alpha^3 + \beta^3$ is:
 (1) 150 (2) 152 (3) 148 (4) 146

Ans. (2)

Sol. $1 + \tan^2(\tan^{-1}\alpha) + 1 + \cot^2(\cot^{-1}\beta) = 36$

$$2 + \alpha^2 + \beta^2 = 36$$

$$\alpha^2 + \beta^2 = 34$$

$$(\alpha + \beta)^2 - 2\alpha\beta = 34$$

$$8^2 - 2\alpha\beta = 34 \quad \Rightarrow \quad 2\alpha\beta = 30 \quad \Rightarrow \quad \alpha\beta = 15$$

So, $\alpha + \beta = 8$ and $\alpha\beta = 15$

$$\alpha = 5 \text{ and } \beta = 3$$

$$\alpha^3 + \beta^3 = 125 + 27 = 152$$

8. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$, \vec{c} is a vector perpendicular to \vec{b} , also \vec{c} is coplanar with \vec{a} and \vec{b} . If $\vec{c} \cdot \vec{a} = 5$ then $|\vec{c}|$ is equal to

(1) $\sqrt{\frac{13}{6}}$

(2) $\sqrt{\frac{11}{6}}$

(3) $\sqrt{\frac{5}{6}}$

(4) $\sqrt{\frac{7}{6}}$

Ans. (2)

Sol. Let $\vec{c} = \lambda \vec{b} \times (\vec{a} \times \vec{b})$

$$= \lambda [(\vec{b}\vec{b})\vec{a} - (\vec{b}\vec{a})\vec{b}]$$

$$= \lambda [1\vec{b} - 2\vec{b}]$$

$$= \lambda [1\hat{i} + 2\hat{j} + 3\hat{k} - 6\hat{i} - 2\hat{j} + 2\hat{k}]$$

$$= \lambda [5\hat{i} + 20\hat{j} + 35\hat{k}]$$

$$\vec{c} = 5\lambda [\hat{i} + 4\hat{j} + 7\hat{k}]$$

Now, $\vec{c} \cdot \vec{a} = 5$

$$5\lambda [1 + 8 + 21] = 5$$

$$\lambda = \frac{1}{30}$$

$$\vec{c} = \frac{1}{6} (\hat{i} + 4\hat{j} + 7\hat{k})$$

$$|\vec{c}| = \frac{1}{6} \sqrt{1 + 16 + 49}$$

$$|\vec{c}| = \frac{\sqrt{66}}{6} = \sqrt{\frac{11}{6}}$$

9. If system of equations
 $2x - y + z = 4$,
 $4x - \lambda y + 3z = 12$ and
 $100x - 41y + \mu z = 212$ has infinite solutions then $\mu - 2\lambda$ is
 (1) 59 (2) 55 (3) 56 (4) 57

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Ans. (2)

Sol. Let equations are P_1 , P_2 and P_3

Then

$$KP_1 + P_2 : x(2K+4) + y(-K-\lambda) + z(K+3) = 4K+12$$

$$P_3 : 100x - 41y + \mu z = 212$$

Now comparing coefficient

$$\frac{2K+4}{100} = \frac{-K-\lambda}{-41} = \frac{K+3}{\mu} = \frac{4K+12}{212}$$

Solving $\frac{2K+4}{100} = \frac{4K+12}{212}$

$$\frac{K+2}{50} = \frac{K+3}{53}$$

$$53K + 106 = 50K + 150$$

$$3K = 44$$

$$K = \frac{44}{3}$$

Now

$$\frac{\frac{88}{3} + 4}{100} = \frac{\frac{44}{3} - \lambda}{-41} = \frac{\frac{44}{3} + 3}{\mu} = \frac{\frac{176}{3} + 12}{212}$$

$$\frac{1}{3} = \frac{44+3\lambda}{123} \text{ and } \frac{53}{3\mu} = \frac{212}{3 \times 212} = \mu = 53$$

$$41 = 44 + 3\lambda$$

$$\lambda = -1$$

$$\text{Now } \mu - 2\lambda = 53 - 2(-1) = 55$$

10. Find the product of all rational roots of the equation $(x^2 - 9x + 11)^2 - (x-4)(x-5) = 3$.

Ans. (14)

Sol. $(x^2 - 9x + 11)^2 - (x^2 - 9x + 23) = 0$

Let $x^2 - 9x + 11 = t$

$$t^2 - (t+12) = 0 \quad \Rightarrow \quad (t-4)(t+3) = 0$$

$t = 4$

$$x^2 - 9x + 11 = 4$$

$$x^2 - 9x + 7 = 0$$

$$D = 81 - 28 = 53$$

D is not a perfect square \Rightarrow Irrational value of x.

$t = -3$

$$x^2 - 9x + 11 = -3$$

$$x^2 - 9x + 14 = 0$$

$$(x-7)(x-2) = 0$$

Product of roots $= 7 \times 2 = 14$.

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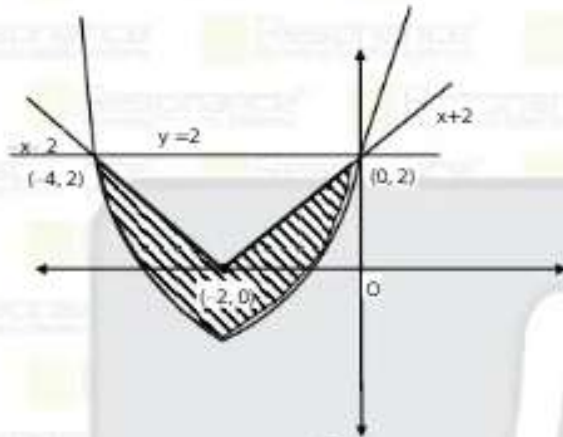
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11. The area of the region bounded by $S(x, y)$ such that $S = \{x, y : x^2 + 4x + 2 \leq y \leq |x + 2|\}$ is (in sq. units)

- (1) $\frac{24}{5}$ (2) 5 (3) $\frac{20}{3}$ (4) 7

Ans. (3)

Sol.



$$\begin{aligned} \text{Area of shaded region} &= \int_{-4}^0 2 - (x^2 + 4x + 2) dx - \frac{1}{2} \times 4 \times 2 \\ &= \int_{-4}^0 (-x^2 - 4x) dx - 4 \\ &= - \int_{-4}^0 (x^2 + 4x) dx - 4 \\ &= - \left[\frac{x^3}{3} + 2x^2 \right]_{-4}^0 - 4 \\ &= - \left[0 - \left(\frac{-64}{3} \right) - 32 \right] - 4 \\ &= -32 - \frac{64}{3} - 4 \\ &= -28 - \frac{64}{3} \\ &= \frac{84 - 64}{3} \\ &= \frac{20}{3} \text{ sq units} \end{aligned}$$

12. Find number of three digits numbers which are divisible by 2 & 3 both but not by 4 and 9.

Ans. (125)

Sol. Required numbers are which are divisible by 6

102, 114, 120, 132, 138, 150, 156, 168, ,996 (A.P.) (Total numbers are 150)

Now

Numbers which are divisible by 36

108, 120, 138, , 972 (A.P.) (Total numbers are 25)

Therefore, the required three digit numbers which are divisible by 6 and not by 36 are $150 - 25 = 125$.

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13. If $I(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx$, then $I(9,14) + I(10,13)$ is equal to

- (1) $I(7, 11)$ (2) $I(9, 13)$ (3) $I(11, 15)$ (4) $I(8, 11)$

Ans. (2)

Sol. Let $x = \sin^2\theta \Rightarrow dx = 2\sin\theta \cos\theta d\theta$

$$\therefore I(m, n) = \int_0^{\frac{\pi}{2}} \sin^{2m-2}\theta \cos^{2n-2}\theta \cdot 2\sin\theta \cos\theta d\theta$$

$$= 2 \int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2n-1}\theta d\theta$$

$$\therefore I(9,14) + I(10,13) = 2 \int_0^{\frac{\pi}{2}} \sin^{17}\theta \cos^{27}\theta d\theta + 2 \int_0^{\frac{\pi}{2}} \sin^{19}\theta \cos^{25}\theta d\theta$$

$$= 2 \int_0^{\frac{\pi}{2}} \sin^{17}\theta \cos^{25}\theta (\cos^2\theta + \sin^2\theta) d\theta$$

$$= 2 \int_0^{\frac{\pi}{2}} \sin^{17}\theta \cos^{25}\theta d\theta$$

$$= I(9,13)$$

14. Let $f(x) = \frac{2^{x+2} + 16}{2^{2x+1} + 2^{x+4} + 32}$ then $f\left(\frac{1}{15}\right) + f\left(\frac{2}{15}\right) + \dots + f\left(\frac{59}{15}\right)$ is equal to

- (1) 14.25 (2) 14.75 (3) 14 (4) 15

Ans. (2)

Sol. $f(x) = \frac{4(2^x + 4)}{2(2^{2x}) + 16(2^x) + 32}$

$$= \frac{2(2^x + 4)}{(2^x + 4)^2} = \frac{2}{2^x + 4}$$

$$\text{Now } f(x) + f(4-x) = \frac{2}{2^x + 4} + \frac{2}{2^{4-x} + 4} = \frac{2}{2^x + 4} + \frac{2 \times 2^x}{16 + 4 \times 2^x} = \frac{2}{2^x + 4} + \frac{2^x}{2(4 + 2^x)} = \frac{1}{2}$$

$$\text{So } f\left(\frac{1}{15}\right) + f\left(\frac{59}{15}\right) = \frac{1}{2}$$

$$f\left(\frac{2}{15}\right) + f\left(\frac{58}{15}\right) = \frac{1}{2}$$

$$\text{and } f\left(\frac{30}{15}\right) = \frac{2}{4+4} = \frac{1}{4} \text{ hence}$$

$$f\left(\frac{1}{15}\right) + f\left(\frac{12}{15}\right) + \dots + f\left(\frac{59}{15}\right) = \frac{29}{2} + \frac{1}{4} = 14.75$$

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15. Let $y(x)$ be the solution of differential equation $\frac{dy}{dx} + \left(\frac{x}{1+x^2}\right)y = \frac{\sqrt{x}}{\sqrt{1+x^2}}$

If $y(0) = 0$ then $y(1)$ is equal to

- (1) $\frac{2}{3}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{\sqrt{2}}{3}$ (4) $\sqrt{\frac{2}{3}}$

Ans. (3)

Sol. I.F. = $\int \frac{xdx}{1+x^2} = e^{\frac{1}{2}\ln(1+x^2)} = \sqrt{1+x^2}$

Solution of differential equation

$$y \cdot \sqrt{1+x^2} = \int \sqrt{1+x^2} \cdot \frac{\sqrt{x}}{\sqrt{1+x^2}} dx + C$$

$$y\sqrt{1+x^2} = \frac{2}{3}x^{3/2} + C$$

when $x=0, y=0$ then $C=0$

so $y = \frac{2x^{3/2}}{3\sqrt{1+x^2}}$ now at $x=1$

$$y = \frac{2}{3\sqrt{2}} = \frac{\sqrt{2}}{3}$$

16. Let $f(x) - 6f\left(\frac{1}{x}\right) = \frac{35}{3x} - \frac{5}{2}$, if $\lim_{x \rightarrow 0} \left(\frac{1}{\alpha x} + f(x)\right) = \beta$

Then value of $\alpha + 2\beta$ is

- (1) 0 (2) 1 (3) 3 (4) 4

Ans. (4)

Sol. $f(x) - 6f\left(\frac{1}{x}\right) = \frac{35}{3x} - \frac{5}{2}$ (1)

Replace $x \rightarrow \frac{1}{x}$

$$f\left(\frac{1}{x}\right) - 6f(x) = \frac{35x}{3} - \frac{5}{2}$$
(2)

from (1) and (2)

$$f(x) = -2x - \frac{1}{3x} + \frac{1}{2}$$

Now

$$\beta = \lim_{x \rightarrow 0} \left(\frac{1}{\alpha x} - 2x - \frac{1}{3x} + \frac{1}{2}\right)$$

$$\Rightarrow \alpha = 3, \beta = \frac{1}{2}$$

So, $\alpha + 2\beta = 3 + 1 = 4$

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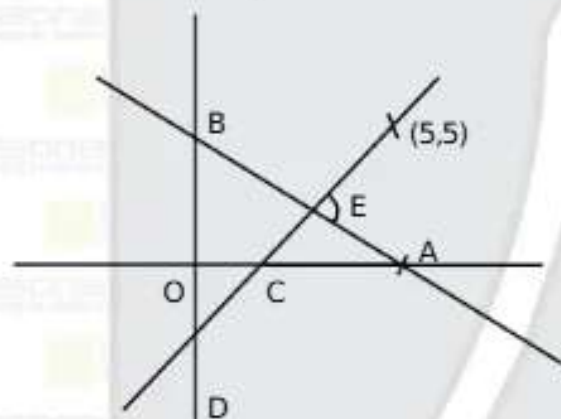
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17. The line $2x+3y=12$ meets the x-axis at A and y-axis at B. The line through (5, 5), perpendicular to AB meets the x-axis, y-axis and AB at C, D and E respectively. If O is origin of coordinate axes then area of OCEB is
- (1) $\frac{16}{3}$ (2) $\frac{14}{3}$ (3) $\frac{23}{3}$ (4) $\frac{17}{2}$

Ans. (3)

Sol. A(6,0), B(0,4)
AB is $2x+3y=12$
ED is $3x-y=5$

Solving E (3,2), C($\frac{5}{3}$,0) O(0,0)



$$\text{Area} = \frac{1}{2} \begin{vmatrix} 0 & 5 & 3 & 0 & 0 \\ 0 & 3 & 2 & 4 & 0 \end{vmatrix} = \frac{1}{2} \left(\frac{10}{3} + 12 \right) = \frac{23}{3}$$

18. Let $P\left(\sqrt{3}, \frac{1}{2}\right)$ lies on ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, also product of focal distance of point P is $\frac{7}{4}$ there are two ellipse satisfying given conditions then product of eccentricities is:

- (1) $\left(\frac{1}{\sqrt{2}}\right)$ (2) $\left(\frac{1}{2}\right)$ (3) $\left(\frac{\sqrt{3}}{\sqrt{2}}\right)$ (4) $\left(\frac{\sqrt{3}}{4}\right)$

Ans. (1)

Sol. $PS_1 + PS_2 = 2a$ _____ (i) $P\left(\sqrt{3}, \frac{1}{2}\right)$

$$PS_1 PS_2 = \frac{7}{4}$$

Squaring (i) $PS_1^2 + PS_2^2 + 2PS_1 PS_2 = 4a^2$

$$(ae - \sqrt{3})^2 + \frac{1}{4} + (ae + \sqrt{3})^2 + \frac{1}{4} + 2 \cdot \frac{7}{4} = 4a^2$$

$$2a^2 e^2 + 10 = 4a^2$$

$$a^2 e^2 + 5 = 2a^2 \text{ _____ (ii)}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ satisfying } \left(\sqrt{3}, \frac{1}{2}\right) \text{ in the equation}$$

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$$\frac{3}{a^2} + \frac{1}{4b^2} = 1 \quad \{a^2 - a^2e^2 = b^2\}$$

$$\frac{3}{a^2} + \frac{1}{4a^2(1-e^2)} = 1$$

$$3 + \frac{1}{4(1-e^2)} = a^2 \quad \text{from (ii) } a^2 = \frac{5}{2-e^2}$$

$$3 + \frac{1}{4(1-e^2)} = \frac{5}{2-e^2}$$

$$12(1-e^2)(2-e^2) + (2-e^2) = 20(1-e^2)$$

Let $1 - e^2 = t$

$$12t(1+t) + (1+t) = 20t$$

$$12t^2 + 12t + t + 1 = 20t$$

$$12t^2 - 7t + 1 = 0$$

$$(4t-1)(3t-1) = 0$$

$$t = \frac{1}{4} \text{ or } t = \frac{1}{3}$$

$$1 - e^2 = \frac{1}{4} \quad \text{or} \quad 1 - e^2 = \frac{1}{3}$$

$$e^2 = \frac{3}{4} \quad \text{or} \quad e^2 = \frac{2}{3}$$

$$\text{so } e_1^2 \times e_2^2 = \frac{1}{2}$$

$$e_1 \times e_2 = \frac{1}{\sqrt{2}}$$

19. For ten observations $x_1, x_2, x_3, \dots, x_{10}$, mean is 5.5, $\sum_{i=1}^{10} x_i^2 = 377$, in which observations 4 and 5 are wrongly included. Replacing these wrong observations 4 and 5 by correct observations 6 and 8, then correct variance is:

(1) 6.7

(2) 8.7

(3) 7.5

(4) 7.6

Ans. (4)

Sol. $x_1 + x_2 + \dots + x_8 + 4 + 5 = 55 \quad \dots\dots\dots(1)$

$$x_1^2 + x_2^2 + \dots + x_8^2 + 4^2 + 5^2 = 377 \quad \dots\dots\dots(2)$$

For corrected data mean is

$$\frac{x_1 + x_2 + \dots + x_8 + 6 + 8}{10} = \frac{55 - 9 + 6 + 8}{10} = 6$$

$$\sigma^2 + \bar{x}^2 = \frac{1}{n} \sum x_i^2$$

$$\sigma^2 + 6^2 = \frac{1}{10} (377 - 4^2 - 5^2 + 36 + 64)$$






$$= \frac{1}{10} (477 - 41) = \frac{436}{10} \quad = \sigma^2 = 7.6$$

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