JEE-Main-23-01-2025 (Memory Based)
[EVENING SHIFT]
Math

Question: In AP with $cd = \frac{3}{2}$: $3x^2$ **- px +q was the equation and their roots are 10th and 11th terms and sum of 11 terms is 88 find q - 2p.**

Solution:

$\frac{11}{2}\left[2a + 10 \times \frac{3}{2}\right] = 88$

$2a + 15 = 16$

$2a = 1$

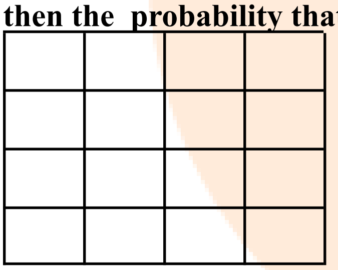
$T_{10} = \frac{1}{2} + \frac{27}{2} = 14 \quad \mathrm{T}_{11} = \frac{31}{2}$

$\frac{p}{3} = \frac{59}{2} \qquad \frac{q}{3} = \frac{14 \times 31}{2}$

$q - 2p = \frac{42 \times 31}{2} - 3 \times 59$

$= 474$

Question: f a square is divided in 4 × 4 squares. If two squares are chosen randomly then the probability that the squares doesn't share common side is



Options:

(a) $\frac{3}{5}$

(b) $\frac{4}{5}$

(c) $\frac{3}{20}$

(d) $\frac{7}{10}$

Answer: (b)

$\text{Total squares} = 16$

$\text{Choosen} =^{16} C_2 = \frac{16\times15}{2} = 120$

$\text{Adj square : Horizontal raw}$

$4 \times 3 = 12$

$\text{Vertical} : 4 \times 72$

$\therefore \text{Total adjacant} = 24$

$\therefore P(E) = \frac{24}{120} = \frac{1}{5}$

$\therefore P(E) = \frac{4}{5}$

No. of pair squares which share a common side = 24 = (12 × 12)

No. of squares which don't share = $^{16}C_2$ - 24

= 120 - 24 = 96

$\therefore \text{Prob} = \frac{96}{120} = \frac{8}{10} = \frac{4}{5}$

Question: There are 5 boys and 4 girls. The sum of number of ways to sit them such that all boys sit together and number of ways such that no boys sit together is equal to

Solution :

$(2) 5! \times 5! + 4! \times^5 C_5 \times 5!$

$= 120^2 + 24 \times 120 = 144 \times 120$

$= 17280$

Question: Find the variance of numbers 8, 21, 34, …, 320.

Solution :

$8, 21, 34, \ldots\ldots..320$

$T_n = (13n - 5)$

$\text{Variance of } 1, 2, \ldots\ldots.25$

$= \frac{\sum n^2}{25} - \left(\frac{\sum n}{25}\right)^2$

$= \frac{25\times26\times51}{6\times25} - \left(\frac{25\times26}{2\times25}\right)^2$

$= 13 \times 17 - 13^2 = 13 \times 14 = 52$

$\text{so reg var} = 13^2 \times 52$

$= 8788$

$\text{variance} = \left(\frac{n^2-1}{12}\right)d^2$

$= \frac{25^2-1}{12} \times 13^2$

$= 52 \times 169 = 8788$

Question: Let S be the region consisting of points (x,y) such that $-1 \leq x \leq 1 \ \& \ 0 \leq y \leq a + e^{|x|} - e^{-|x|}$ **if area bounded by region** $2\left(\frac{e^2 + 8e + 1}{e}\right)$ **is find "a".**

Solution :

$-1 \leq x \leq 1$

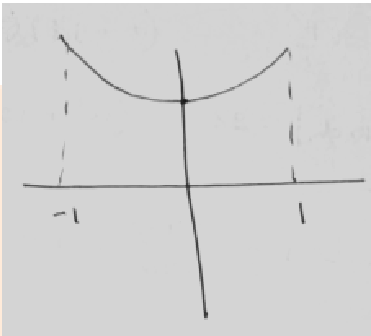
$y = a + e^{|x|} - e^{-|x|}$

$A = 2\int_0^1 a + e^x - e^{-x} dx$

$= 2\left[a + (e-1) + \left(e^{-1} - 1\right)\right]$

$= 2\left[\frac{e^2 + e(a-2) + 1}{e}\right]$

So $a = 10$



Question: Shortest distance between P(0, a) and parabola $y^2 = 4x$ is 4. Find the equation of circle whose center lies on axis of parabola and passing through P and focus of parabola.

Solution :

$y = mx - 2m - m^3$

$0 = ma - 2m - m^3$

$m=0 \ m^2 = (a-2)$

$(a-m^2)^2 + (2m)^2 = 16$

$4 + 4m^2 = 16$

$m^2 = 3$

$a = 5$

Eq of Circle

$(x-1)(x-5) + y^2 = 0$

$x^2 + y^2 - 6x + 5 = 0$

$y = mx - 2m - m^3$

$0 = ma - 2m - m^3$

$m = 0 \quad m^2 = (a-2)$

$\left(a - m^2\right)^2 + (2m)^2 = 16$

$4 + 2m^2 = 16$

$m^2 = 6$

$a = 8$

Eq of circle

$(x-1)(x-8) + y$

$x^2 + y^2 - 9x + 8$

Question: Find length of chord whose midpoint is $\left(1,\frac{1}{2}\right)$ of ellipse $\frac{x^2}{2}+\frac{y^2}{4}=1.$

Solution :

$\frac{x-1}{\cos\theta}=\frac{y-\frac{1}{2}}{\sin\theta}=r$

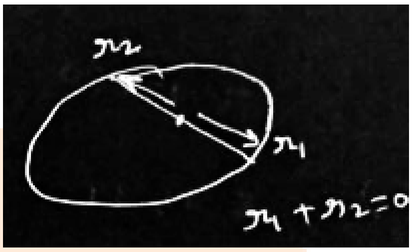
$\frac{(r\cos\theta+1)^2}{2}+\frac{\left(r\sin\theta+\frac{1}{2}\right)^2}{4}=1$

$sum=0\rightarrow\left[\cos\theta+\frac{\sin\theta}{4}\right]=0\rightarrow\tan\theta=-4$

$\Pr o.=\frac{\left|\frac{1}{2}+\frac{1}{16}-1\right|}{\frac{\cos^2\theta}{2}+\frac{\sin^2\theta}{4}}=\frac{7}{16\left[\frac{1}{2\cdot 17}+\frac{16}{17.4}\right]}$

$=\frac{7\times 17}{8+64}=\frac{7\times 17}{72}=r_1^2$

$\text{Length}=2r_1=2\sqrt{\frac{119}{72}}=\sqrt{\frac{119}{18}}$



Question: If the square of the shortest distance between the lines $\frac{x-2}{1}=\frac{y-1}{2}=\frac{x+3}{-3}$ $\frac{x+1}{2}=\frac{y+3}{4}=\frac{x+5}{-5}$ is $\frac{m}{n}$, and where m, n are coprime numbers then m + n is equal to

Options:

(a) 6

(b) 9

(c) 14

(d) 21

Answer: (b)

$$S.D.=\frac{\begin{vmatrix}3 & 4 & 2\\ 1 & 2 & -3\\ 2 & 4 & -5\end{vmatrix}}{\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -3\\ 2 & 4 & -5\end{vmatrix}}$$

$=\frac{16-41}{2\hat{i}-\hat{j}+0\hat{k}}=\frac{2}{\sqrt{5}}$

$\frac{m}{n}=\frac{4}{5}\rightarrow m+n=9$

Question: If $I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}x\,dx}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}$, then the value of definite integration $\int_0^{2I}\frac{x\sin x\cos x}{\sin^4x+\cos^4x}dx$ is

Options:

(a) $\frac{\pi}{16}$

(b) $\frac{\pi^2}{16}$

(c) $\frac{\pi}{8}$

(d) $\frac{\pi^2}{8}$

Answer: (b)

$$I=\int_0^{\pi/2}$$

$$2I=\int_0^{\pi/2}1.dx \quad I=\frac{\pi}{4}$$

$$I_1=\int_0^{\pi/2}\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx$$

$$I_1=\int_0^{\pi/2}\frac{\frac{\pi}{2}\sin x\cos x}{\sin^4 x+\cos^4 x}dx$$

$$I_1=\frac{\pi}{2}\int_0^{\pi/4}\frac{\tan x.\sec^2 x}{1+\tan^4 x}dx$$

$$=\frac{\pi}{2}\left[\frac{\tan^{-1}(\tan^2 x)}{2}\right]_0^{\pi/4}=\frac{\pi}{2}\left[\frac{\pi}{8}\right]$$

$$=\frac{\pi^2}{16}$$

Question: In expansion o f $(1 + x)^p$ $(1 - x)^Q$ coefficient of x and x^2 is 1 and -2 then find $p^2 + q^2$.

Solution :

$$(1+x)^p(1-x)^q=\left(1+px+{}^pC_2x^2\right)\left(1-qx+{}^qC_2x^2\right)$$

$$p-q=1$$

$$-pq+\frac{p(p-1)}{2}+\frac{q(q-1)}{2}=-2$$

$$\left(p^2+q^2-2pq\right)-p-q=-4$$

$$p+q=5$$

$$p=3 \quad q=2$$

$$p^2+q^2=13$$

Question: If $y=\left(x-y\frac{dy}{dx}\right)\sin\left(\frac{x}{y}\right)$ if $x(1)=\frac{\pi}{2}$ **then find cos(x(2)).**

Solution :

$x = vy$

$y=\left[vy-y\left(v+y\cdot\frac{dv}{dy}\right)\right]\sin v$

$1\cdot\cos ecv = v - v - y\frac{dv}{dy}$

$\frac{dy}{y}+\sin vdv$

$\ln y-\cos v=c$

$\ln y-\cos\frac{x}{y}=c$

$\ln 1-\cos\frac{\pi}{2}=c=0$

$\ln 2-\cos\frac{x}{2}=0$

$\cos\left(\frac{x}{2}\right)=\ln 2$

$\cos(x)=2\cos^2\frac{x}{2}-1$

$=2(\ln 2)^2-1$

Question: A = {(x, y)| |x + y| ≥ 3};
B = {(x, y)| |x| + |y| ≤ 3}
Let C = A ∩ B. Find the sum of x + y ∀ x, y ∈ C.
Solution :

$A=|x+y|\geq 3\rightarrow x+y\geq 3$ or $x+y\leq -3$

$B=|x|+|y|\leq 3$