Institute of Actuaries of India

ACET December 2024

Mathematics

1. The domain of the function $f(x) = sqrt(\ln(x^2 - 2024))$ is:
   A. (-∞,-√2024] U [√2024,∞)
   B. (-∞,-√2024) U (√2024,∞)
   C. (-∞,-45] U [45,∞)
   D. None of the above

1 mark

2. If $f(x) = \ln(x)$ and $g(x) = x^{2024}$, then the interval where exactly one of $f(g(x))$ and $g(f(x))$ is defined is:
   A. (-∞,0]
   B. (-∞,0)
   C. (0,∞)
   D. [0,∞)

1 mark

3. The equation $x^{20} = x^{24}$ has exactly m distinct real roots and n distinct non-real roots. Then 10m+n is:
   A. A power of 2
   B. A prime number
   C. Divisibly by 3
   D. None of the above

2 marks

4. If $\alpha = \int_0^{\sqrt{2024}} \frac{1}{x^2+1} dx$, then the value of $\sec(\alpha)$ is:
   A. A positive integer
   B. Not an integer, but a positive rational number
   C. A positive irrational number
   D. None of the above

1 mark

5. If a, b, c are the roots of $x^3 - ax^2 + bx - c = 0$ and c≠0, then the number of possible triples (a,b,c) are:
   A. 0
   B. 1
   C. More than 1, but finitely many
   D. Infinitely many

3 marks

6. Evaluate $\lim_{x \to 0} \frac{e^{20x} - e^{24x}}{\sin(x)}$:
   A. -1/120
   B. 1/120
   C. 4
   D. -4

1 mark

7. An arithmetic progression a_i is such that $a_{202} = 4$ and $a_4 = 202$. Then a_{2024} equals:

A. -1818
B. 1818
C. 0
D. None of the above

1 mark

8. Given two vectors $\vec{a}$, $\vec{b}$, let M be the sum of magnitudes of vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$. Let m be the sum of magnitudes of vectors $\vec{a}$ and $\vec{b}$. Then:
   A. M = m
   B. m < M < 2m
   C. M = 2m
   D. M > 2m

   1 mark

9. Which of the following statements is/are true?
   X: The dot product of a vector with itself is always 0.
   Y: The cross product of a vector with itself is always **0** (the zero vector).
   A. Both X and Y
   B. Neither X, nor Y
   C. X but not Y
   D. Y but not X

   1 mark

10. If A is a 2x2 matrix with entries from {1, 2, 3, 4} in some order and without repetition. The maximum and the minimum possible values of |A| (i.e. determinant of A) differ by:
    A. 4
    B. 10
    C. 20
    D. None of the above

    2 marks

11. For a square matrix M, let M^T denote the transpose of M. Also let $X = M + M^T$ and $Y = M - M^T$. Then:
    A. X is symmetric and Y is skew-symmetric.
    B. Y is symmetric and X is skew-symmetric.
    C. Both X and Y are symmetric.
    D. Both X and Y are skew-symmetric.

    1 mark

12. At x = 0, the function f(x) = x|x| is:
    A. Discontinuous
    B. Continuous but not differentiable
    C. Differentiable, but not infinitely differentiable
    D. Infinitely differentiable

    2 marks

13. If $1 + x + x^2 = xy$, then the value of $\frac{dy}{dx}$ at x = √2 equals:
    A. 0
    B. 1/2
    C. 1
    D. None of the above

    2 marks

14. Given that a,b,c,d are in arithmetic progression, a,b,d are in geometric progression and a<b, then compute $(c^2+d^2)/(a^2+b^2)$.
    A. 5
    B. 3.5
    C. 2
    D. None of the above

2 marks

15. The coefficient of x^{20} in $(x^2 - \frac{1}{x^2})^{24}$ is:
    A. $\binom{24}{17}$
    B. $-\binom{24}{17}$
    C. $\binom{24}{20}$
    D. None of the above

1 mark

16. The value of $\sum_{n=1}^{\infty} \frac{240}{n^2+4n}$ is:
    A. A perfect square
    B. A prime number
    C. A composite number, but not a perfect power
    D. A perfect cube

1 mark

17. Which of the below approximation methods, if used to evaluate the integral $\int_0^{\pi} \cos(x)dx$, will not give the correct answer?
    A. Trapezoidal rule, with h = π/4
    B. Simpson's 1/3rd rule, with h = π/3
    C. Simpson's 3/8th rule, with h = π/2
    D. None of the above

2 marks

18. For solving the equation $2^x = x^3$ using bisection method over the interval [1,2], the approximate root where we land after two iterations (instances of narrowing intervals) is
    A. 1.375
    B. 1.5
    C. 1.25
    D. None of the above

3 marks

19. If a complex number z satisfies the condition $|z - 20| = 24$, then z lies on:
    A. A circle with centre at (24,0) and radius 20
    B. A circle with centre at (20,0) and radius 24
    C. A line parallel to one of the axes
    D. None of the above

1 mark

20. If ω is the non-real cube-root of 1, then $1 + \omega + \omega^2 + \omega^3 + \cdots + \omega^{2024} =$
    A. 1
    B. 1+ω
    C. 0
    D. None of the above

1 mark

21. Two potentially biased (with probability of heads = p) are rolled and the number of heads is recorded. Let this be a random variable N. Given that the median of N is 1, the set of all possible values of p is:
    A. {0.5}
    B. [1/3,2/3]
    C. [3/8,5/8]
    D. None of the above

2 marks

22. The average of weight of five adults is 65 kg. We then take three (of the five) adults and record their average weight. We repeat this for all possible subsets of size 3. The minimum and maximum of these 3-person average weights is 58 kg and 70 kg respectively. The median weight of the five adults is:
    A. 65 kg
    B. 59 kg
    C. 64 kg
    D. Insufficient information / cannot be determined

1 mark

23. From a squad of 15 cricketers (6 batsmen, 2 wicket-keepers, 2 all-rounders and 5 bowlers), a playing team of 11 cricketers (with exactly five batsmen, 1 wicket-keeper, 1 all-rounder and 4 bowlers) needs to be selected. The number of possible selections is:
    A. 120
    B. 20
    C. 210
    D. None of the above

1 mark

24. The mode of marks of all boys in a class is 20, while the mode of marks of all girls in that class is 24. Then the mode marks of all students (boys and girls) in that class will be
    A. 22
    B. Indeterminate, but will surely be between 21 and 23.
    C. Indeterminate, but will surely be between 20 and 24.
    D. None of the above

1 mark

25. The probability of a random person contracting an infection X is 1%. The probability of a random person contracting an infection Y is 5%. However, if a person contracts infection X, their probability of contracting Y increases to 10%. Compute the probability of a person contracting X given they have contracted Y.
    A. 1%
    B. 2%
    C. 10%
    D. None of the above

1 mark

26. Five fair (= unbiased) coins are tossed leading to exactly two heads. The toss outcomes are laid out in a row. The probability that the two heads are not next to each other is:
    A. 40%
    B. 60%
    C. 50%
    D. None of the above

1 mark

27. If X follows an exponential distribution with mean 1, the interquartile range of X is:
    A. A rational number greater than 1
    B. A rational number less than 1
    C. An irrational number greater than 1
    D. An irrational number less than 1

2 marks

28. Two fair, six-sided dice are rolled together and their sum (say, S) recorded. The probability that S is a perfect square given that S is an even number is:
    A. 1/12
    B. 1/6
    C. 1/2
    D. None of the above

2 marks

29. If X and Y are two independent exponential distributions with means m_x and m_y, then X+Y follows:
    A. An exponential distribution with mean $m_x + m_y$.
    B. An exponential distribution with mean $(m_x + m_y)/2$.
    C. An exponential distribution with a mean other than the above.
    D. A distribution other than exponential distribution.

1 mark

30. Which of the following distributions can be used to model a discrete random variable that can take any (arbitrarily large) positive integer value?
    A. Binomial
    B. Poisson
    C. Bernoulli
    D. None of the above

1 mark

31. A discrete random variable X has the following cumulative distribution function:

x	0	1	2	3	4	5
F(X)	0	0.1	0.4	k	0.9	1

Find the value of k such that the expected value of X is the same as its median.
    A. 0.75
    B. 0.5
    C. 0.6
    D. None of the above

2 marks

32. The following stem-and-leaf plot depicts scores of students (out of 100) on a test where the passing score was 40.

Stem	Leaves
2	2,5
3	1,6,9
4	1,1,7,8,9,9
5	2,3,5,7
6	1,6,7,7
7	1,3,4,8
8	4,5,9

Find the median score among the set of students who have passed:

A. 66
B. 59
C. 56
D. None of the above

1 mark

33. The means of x and y are $\bar{x} = 20 \; and \; \bar{y} = 24$ respectively, and their standard deviations are s_x = 1.2 and s_y = 2 respectively. The regression line of x on y is 10x = 3y + c (for some constant c). Then the regression line of y on x will be:

A. y = (3/5)x + 12
B. y = (5/3)x – 28/3
C. y = (10/3)x – 72
D. y = (5/6)x + 22/3

3 marks

34. If X follows a normal distribution with mean 20 and variance 24, Y follows a normal distribution with mean 24 and variance 20, and the correlation coefficient between X and Y is $\sqrt{\frac{5}{6}}$, then the expected value of XY will be:

A. 480
B. 500
C. 520
D. None of the above

2 marks

35. If a six-faced die is loaded (made biased) such that the probability of each face is proportional to the number of it, the expected value of the outcome of its roll is:

A. 91/15
B. 7/2
C. 13/3
D. 9/2

2 marks

36. An actuarial student rolls a fair, six-sided die. If it shows n, the student then tosses a fair coin n times and records the number of tails. The probability of recording at least one tail:

A. 3/16
B. 63/64
C. 107/128
D. None of the above

3 marks

37. A child decides to make two cuts on a stick, at points chosen uniformly randomly and independently. This will break the stick into three pieces The probability that the largest of the three pieces will exceed half the length of the stick is:

A. Greater than 1/2
B. Exactly 1/2
C. Less than 1/2
D. Indeterminate

1 mark

38. If A and B are independent events, each with a positive probability, then they are:

A. Necessarily disjoint
B. May or may not be disjoint – it depends on their probability values.
C. May or may not be disjoint – it depends on the universal set.
D. Definitely not disjoint

1 mark

39. If the average number of goals scored in a soccer match is 2.56 and it is given that the Poisson distribution is appropriate to model these number of goals, then the mode and the standard deviation of the number of goals in a soccer match will be (respectively):
    A. 2 and 2.56
    B. 3 and 2.56
    C. 2 and 1.6
    D. 3 and 1.6

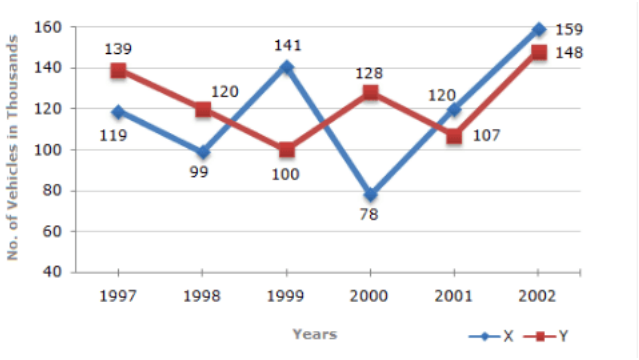
1 mark

40. All possible diagonals are drawn in a regular n-gon (n-sided polygon). There are 252 diagonals in all. Then n equals:
    A. 20
    B. 24
    C. 16
    D. None of the above

1 mark

Data Interpretation

The following chart shows the annual production of vehicles by two vehicle manufacturers - X and Y during the observation period (1997 to 2002 – both inclusive). Questions 41 to 44 are based on the information.



41. The median annual combined production (of X and Y) of vehicles during the observation period is between:
    A. 230,000 and 240,000
    B. 220,000 and 230,000
    C. 240,000 and 250,000
    D. None of the above

1 mark

42. The highest year-on-year increase in production by a manufacturer was:
    A. By X from 1998 to 1999
    B. By X from 2000 to 2001
    C. By Y from 2001 to 2002
    D. By X from 2001 to 2002

1 mark

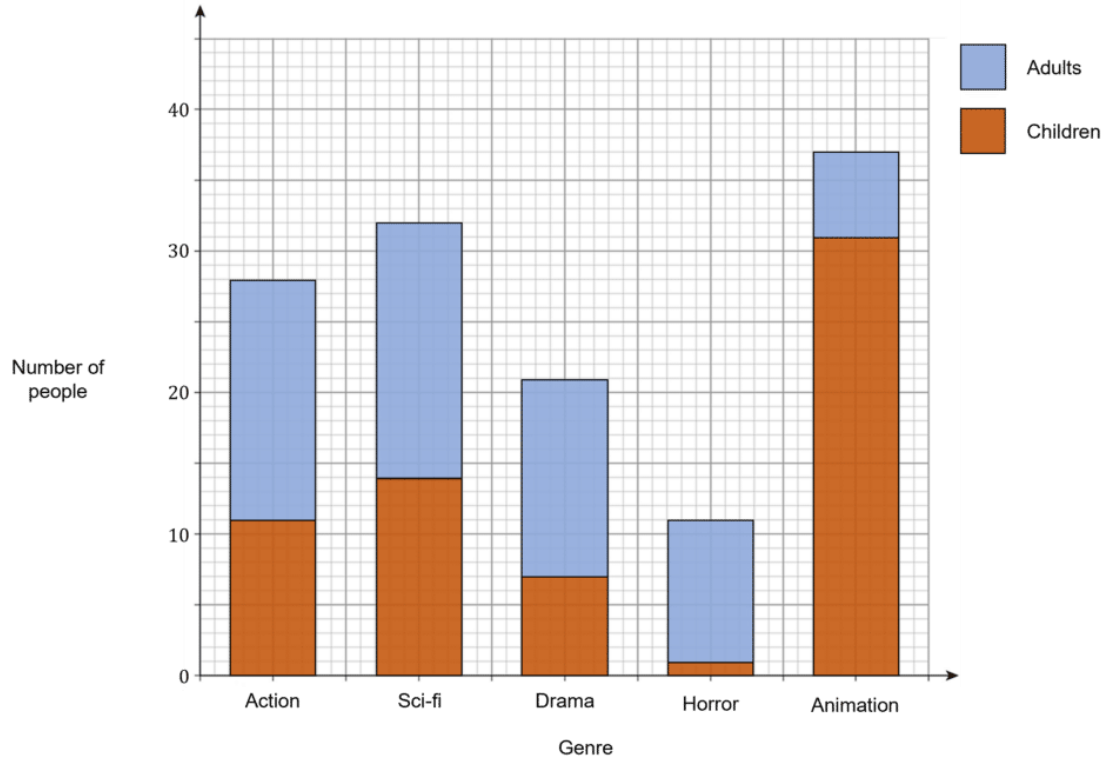
43. During the entire observation period, relative to the total production by Y, the total production by X was:
    A. More by around 3-5%
    B. Nearly the same (i.e. within 2% on either side)
    C. Less by around 3-5%
    D. None of the above

2 marks

44. For a particular year, the market share of a company is defined as the number vehicles manufactured by that company divided by the total number of vehicles manufactured by both companies. During the observation period, the market share was maximum of:
    A. Company Y in 1999
    B. Company Y in 2000
    C. Company X in 1999
    D. Company X in 2000

1 mark

The chart below contains the results of a survey asking people to specify their favourite genre of films. Questions 45 to 47 are based on this information.



45. Of the total survey respondents, the proportion of adults was close to:
    A. 49.6%
    B. 50%
    C. 50.4%
    D. None of the above

1 mark

46. The proportion of children who prefer a genre other than Animation is:
    A. 51.6%
    B. 50%
    C. 48.4%
    D. None of the above

1 mark

47. The genre where the spread of people (between adults and children) who prefer it is most even (closest to a 50% split) is:
    A. Action
    B. Drama
    C. Animation
    D. Sci-fi

2 marks

The International Chess Championship is a tournament between two chess players – the Defending Champion (D) and the Challenger (C). It is a series of 14 games (numbered 1 to 14) where the winner of each game gets a point and the loser gets none. If a game ends as a draw, both players get half a point each. The winner of the tournament is the one with more points at the end of the 14 games. If both players end with the same number of points, the tournament ends as a tie. The following additional facts are known:

- All prime numbered games ended as a draw.
- Moreover, all games ending in a draw were prime numbered.

- The Defending Champion won the games that were numbered a perfect square and no others.

Questions 48 to 51 are dependent on the above information.

48. Who won the tournament?
    A. Defending Champion
    B. Challenger
    C. Neither – it ended as a tie
    D. Insufficient information to determine

1 mark

49. The difference in total points of the two players was
    A. 0
    B. 1
    C. 2
    D. Insufficient information to determine

1 mark

50. The first instance in the tournament where the Challenger had more points than the Defending Champion could have happened in Game numbered:
    A. 1
    B. 6
    C. 8
    D. 12

2 marks

51. The number of matches after which the scores were tied AFTER the start of the tournament was:
    A. 2
    B. 3
    C. 1
    D. None of the above / cannot be determined

2 marks

52. Something that is ‘ancillary’ is:
    A. Grand or magnificent
    B. Relating to a certain blood vessel (i.e. artery) in the body
    C. Old or ancient
    D. Subordinate or supplementary

1 mark

53. Which of the following is incorrect?
    A. A hat
    B. A hour
    C. A home
    D. A hand

1 mark

54. Someone who is ill or unwell can be said to be ‘under the _____’.
    A. Blanket
    B. Weather
    C. Fever
    D. Infection

1 mark

55. Poet : Poem :: Cartographer : ?
    A. Diamond jewelry
    B. Letters
    C. Maps
    D. Horse carts

1 mark

56. “This phone costed me a lot of money.” Identify the erroneous portion in these sentences.
    A. “This phone”
    B. “costed me”
    C. “a lot of money”
    D. None of the above

1 mark

57. “Ram and Sita are going _____ for _____ wedding.” Fill in the blanks.
    A. their, there
    B. there, they’re
    C. their, they’re
    D. there, their

1 mark

58. Which of the words is NOT similar in meaning to the others?
    A. Wrathful
    B. Enraged
    C. Furious
    D. Noxious

1 mark

59. “To avoid disappointment, one should not _____ too much from others.” Fill in the blanks.
    A. except
    B. expect

C. accept
D. ascept

1 mark

60. Which of these is NOT a valid inference from the passage below?

"What should an essay be? Many people would say persuasive. That's what a lot of us were taught essays should be. But I think we can aim for something more ambitious: that an essay should be useful.

To start with, that means it should be correct. But it's not enough merely to be correct. It's easy to make a statement correct by making it vague. That's a common flaw in academic writing, for example. If you know nothing at all about an issue, you can't go wrong by saying that the issue is a complex one, that there are many factors to be considered, that it's a mistake to take too simplistic a view of it, and so on.

Though no doubt correct, such statements tell the reader nothing. Useful writing makes claims that are as strong as they can be made without becoming false.

For example, it's more useful to say that Pike's Peak is near the middle of Colorado than merely somewhere in Colorado. But if I say it's in the exact middle of Colorado, I've now gone too far, because it's a bit east of the middle.

Precision and correctness are like opposing forces. It's easy to satisfy one if you ignore the other. The converse of vaporous academic writing is the bold, but false, rhetoric of demagogues. Useful writing is bold, but true."

A. Academic writing is usually correct but not always bold.
B. If one were to draw a vertical line passing through the middle of Colorado on a map, Pike's Peak is to the east of that line.
C. Any piece of writing cannot be both precise and correct at the same time.
D. The fact that an essay is persuasive does not imply that it is useful.

3 marks

61. The following sentences are jumbled up. Choose the correct sequence from the options below.

I. There are 12 year olds who work amazingly hard.
II. It might not seem there's much to learn about how to work hard.
III. And yet when I ask if I know more about working hard now than when I was in school, the answer is definitely yes.
IV. Anyone who's been to school knows what it entails, even if they chose not to do it.

A. II, IV, III, I
B. II, I, III, IV
C. II, I, IV, III
D. II, IV, I, III

2 marks

62. Consider the following set of sentences:

I. Ram was married with Sita.
II. Sita was sitting besides Ram.
III. Although Ram was a prince, but he had to spend fourteen years in the forest.

Of these, the sentences that are erroneous are:

A. I and II
B. I and III
C. II and III
D. I, II and III

2 marks

63. Today is 21st December 2024 and it falls on a Saturday. The next time that 21st December will fall on a Saturday will happen in:
    A. 2029
    B. 2035
    C. 2031
    D. 2030

1 mark

64. A three hour exam starts in the morning at time T1 and ends in the afternoon at time T2. Interestingly, the clock at time T2 is a mirror image of that at time T1. Then, T1 equals:
    A. 10:30
    B. 10:00
    C. 11:00
    D. None of the above

1 mark

65. All sides of a 5x5x5 cube are painted red. The cube is then divided into 125 unit cubes (i.e. of dimension 1x1x1). Let X be the number of unit cubes with no red sides, Y be the number of unit cubes with exactly one red side and Z be the number of unit cubes with exactly two red sides. Then which of the following statements are false?
    A. X+Y+Z is divisible by 9.
    B. Y+Z = 3X
    C. LCM of X, Y and Z is 108.
    D. Y > Z > X

2 marks

66. If X's mother's father is married to Y's father's mother, then X's father is Y's mother's:
    A. Husband's brother
    B. Sister's husband
    C. Husband's sister's husband
    D. None of the above

1 mark

67. Four couples (W, X, Y, Z) – each comprising a lady and a gentleman – are seated on a round table such that:
    - Mr. and Mrs. X are NOT sitting next to each other.
    - Mr. and Mrs. Z are sitting next to each other.
    - Mr. and Mrs. W are sitting exactly opposite each other.

    Which of the following statements is NOT necessarily false?

    A. Each gentleman is surrounded by a lady on both side (to his left and to his right).
    B. None of the Y's is seated next to a X.
    C. None of the Z's is seated next to a W.
    D. There are an even number of individuals seated between Mr. and Mrs. X.

2 marks

68. If all dancers are singers, and no singers are painters, then which of the following conclusions are true?
    Conclusion X: No dancer is a painter.
    Conclusion Y: All singers are dancers.

A. Both X and Y
B. Only X
C. Only Y
D. Neither X nor Y

1 mark

69. Arrange the following words in a meaningful sequence:
P = Punishment, Q = Crime, R = Judgement, S = Arrest
A. Q,S,P,R
B. Q,S,R,P
C. P,Q,S,R
D. None of the above

1 mark

70. Find the wrong number in the series: 1, 5, 14, 30, 55, 90, 140, …
A. 14
B. 55
C. 90
D. None of the above

1 mark

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