Institute of Actuaries of India

ACET December 2024 Indicative Solutions

Mathematics

1	C	For the function to be defined, the expression inside sqrt must be positive. In turn, the expression inside log, i.e. (x^2-2024) must be at least 1. This happens when $x^2 \geq 2025$, i.e. $\|x\| \geq 45$.
2	B	$f(g(x))$ is defined when $x^{2024} > 0$, i.e. $x \neq 0$. $g(f(x)$ is defined when log(x) is defined, i.e. x>0. So exactly one of them is defined when x<0. [When x=0, neither function is defined. When x>0, both functions are defined.]
3	A	The equation simplifies and factorizes to $x^{20}(x-1)(x+1)(x^2+1)=0$, which has three distinct real roots {0, 1, -1} and two non-real roots {i, -i}. So, 10m+n=32, a power of 2.
4	A	Note that $\alpha = \int_0^{\sqrt{2024}} \frac{1}{x^2+1} dx = \tan^{-1}\sqrt{2024}$. So $\tan\alpha = \sqrt{2024}$. So, $\sec^2\alpha = 1 + \tan^2\alpha = 2025$. Since α must be in first quadrant, sec(α) = 45, a positive integer.
5	B	We infer that: a+b+c = a => b+c = 0. Also, b = ab+bc+ca = bc + a(b+c) = bc + 0 = bc. So, b=0 or c=1. If b=0, then c=0 (contradiction). So c=1 and b = -1. Finally, abc = c. Substituting values of b and c gives a = -1. So the unique value of (a,b,c) = (-1, -1, 1).
6	D	$\lim_{x\to 0} \frac{e^{20x} - e^{24x}}{\sin(x)} = \lim_{x\to 0} \frac{e^{20x} - e^{24x}}{x} \cdot \lim_{x\to 0} \frac{x}{\sin(x)} = \left(\lim_{x\to 0} \frac{e^{20x}-1}{x} - \lim_{x\to 0} \frac{e^{24x}-1}{x}\right) \cdot 1$ $= (20 - 24) = -4$
7	A	Let a_i = a+(i-1)*d. Then, a+201d = 4 and a+ 3d = 202. Solving simultaneously yields d = -1 and a = 205. So, a_{2024} = 205-1*2023 = -1818
8	C	$M = \|\vec{a} + \vec{b}\| + \|\vec{a} - \vec{b}\| = a^2 + b^2 + 2ab\cos\theta + a^2 + b^2 - 2ab\cos\theta = 2m$
9	D	Cross product is proportional to sine of the angle, while dot product is proportional to its cosine. The angle between a vector and itself is 0, and sin(0)=0 while cos(0)=1.
10	C	Maximum value of |A| = 10, when $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ (for example). Minimum value of |A| = -10, when $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$ (for example). So difference = 20.
11	A	$M+M^T$ is symmetric, while $M-M^T$ is skew-symmetric.
12	C	At x=0, f(0)=LHL=RHL=0. Also f'(0) = 0. But, second derivative is undefined (-2 from left, 2 from right). So f is differentiable, but not infinitely differentiable.
13	B	Since $(1+2x)dx = xdy + ydx$, it follows that $\frac{dy}{dx} = \frac{1+2x-y}{x} = \frac{1+2x-x-1-\frac{1}{x}}{x} = \frac{x^2-1}{x^2}$ which is ½ at x=√2.
14	A	Let h > 0 be common difference. So, a = b-h = c-2h = d-3h. Also, $ad = b^2$. So $a(a+3h) = (a+h)^2$ which simplifies to h(h-a)=0. Since h>0, a=h, and so, b = 2a, c = 3a and d = 4a. So, $\frac{c^2+d^2}{a^2+b^2} = \frac{3^2+4^2}{1^2+2^2} = \frac{25}{5} = 5$.
15	B	The general n-th term will be $\binom{24}{n}(x^2)^{24-n}\left(-\frac{1}{x^2}\right)^n = \binom{24}{n}(-1)^n x^{48-4n}$. For coefficient of x^{20}, we need 48-4n = 20 or n = 7. So coefficient will be $\binom{24}{7}(-1)^7 = -\binom{24}{7}$. While it is not among the options, we know that $\binom{24}{7} = \binom{24}{17}$. So the correct answer can be $= -\binom{24}{17}$.
16	D	We simplify: $\sum_{n=1}^{\infty} \frac{240}{n^2+4n} = 60\sum_{n=1}^{\infty}(\frac{1}{n} - \frac{1}{n+4}) = 60 * \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) = 125$ which is a perfect cube.
17	D	True value: $\int_0^{\pi} \cos(x)dx = [\sin(x)]_0^{\pi} = 0$. Approximated value is 0 by all three approaches due to symmetry. For instance, for Simpson's 1/3rd rule, approx. value $= \frac{\frac{\pi}{3}}{3}\left(1 + 4 * \frac{\sqrt{3}}{2} + 2 * \frac{1}{2} + 4 * 0 + 2 * \left(-\frac{1}{2}\right) + 4 * (-\frac{\sqrt{3}}{2}) + (-1)\right) = 0$. Actual comparisons are unnecessary by observing the symmetry.
18	A	We have $f(x) = 2^x - x^3$. Initially, f(1) = 1 > 0, f(2) = -4 < 0. Then f(1.5) = 2.828 – 3.375 < 0. So interval narrows to [1,1.5]. The f(1.25) = 2.4 – 1.95 > 0. So interval narrows to [1.25,1.5]. After two instances, our estimate of root is (1.25+1.5)/2 = 1.375.

19	B	LHS denotes distance from (20,0) which must equal 24 for our point. This is the locus of a circle centred at (20,0) and radius 24.
20	C	We note that: $1 + \omega + \omega^2 = 0$. Likewise next three terms = $\omega^3(1 + \omega + \omega^2) = 0$. The total number of terms is 2025, which is a multiple of 3. So we can divide into groups of three terms each of which will be 0, leading to the overall sum being 0 too.

21	D	The median will be 2 if $p^2 > ½$ which happens if $p > 1/\sqrt{2}$. The median will be 0 if $(1-p)^2 > 1/2$ which happens if $p > 1-1/\sqrt{2}$. So the interval when median will be 1 is between $1-1/\sqrt{2}$ and $1/\sqrt{2}$.
22	B	Sum of weights of all five persons = 65*5 = 325 kg. Sum of weight of lightest three persons = 58*3 = 174 kg. Sum of weights of heaviest three persons = 70*3 = 210 kg. Sum of weights of all five plus the median weight = 174+210 = 384 kg. So median weight = 384 – 325 = 59 kg.
23	A	One each of batsmen, wicket-keeper, all-rounder and bowler is to be excluded. This can be done in 6*2*2*5 = 120 ways.
24	D	The overall mode could be anything, even less than 20 or more than 24. For example, consider that 3 boys had 20 marks and 3 girls had 24 marks (which was was the respective mode). However, it is possible that 2 boys and 2 girls had 26 marks, while no girl had 20 marks and no boy had 24 marks. Then, 26 will be the overall mode.
25	B	$P(X\|Y)=P(X\cap Y)/P(Y) = P(Y\|X)*P(X)/P(Y) = (10\%)(1\%)/(5\%) = 2\%$.
26	B	Total outcomes = $\binom{5}{2} = 10$. Unfavourable outcomes (i.e. heads next to each other) = $\binom{4}{1} = 4$. So favourable outcomes = 6 and probability = 6/10 = 0.6
27	C	For exp(1) distribution, Q_1 = -ln(0.75) and Q_3 = -ln(0.25). So IQR = $Q_3 - Q_1$ = ln(3) which is irrational and greater than 1.
28	B	The unconditional probabilities for each outcome are as follows: Sum: 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 Prob: 1/36 | 2/36 | 3/36 | 4/36 | 5/36 | 6/36 | 5/36 | 4/36 | 3/36 | 2/36 | 1/36 The probability of sum being even (E) = (1+3+5+5+3+1)/36 = ½. The only possible even perfect square is 4. Its probability (EP) = 3/36 = 1/12. The required probability = EP / E = 1/6.
29	D	X+Y will not follow an exponential distribution. If they are identically distributed, it will follow a Gamma distribution.
30	B	Poisson distribution can take arbitrarily large values.
31	C	We compute probability mass functions as: x: 1 | 2 | 3 | 4 | 5 P(x): 0.1 | 0.3 | k-0.4 | 0.9-k | 0.1 E(X) = 0.1*1+0.3*2+(k-0.4)*3+(0.9-k)*4+0.1*5 = 3.6-k < 3.2. So median and mean must be 3. So k = 0.6.
32	A	Among the 23 students who have passed (score 40 or above), the 12th largest is 66.
33	D	Regression line of x on y: (x-20) = r*(1.2/2)*(y-24) => x = 0.6ry+(20-24r). Comparing it with the given equation x = 0.3y + c/10, we get r = 0.5. Regression line of y on x: (y-24) = (0.5*2/1.2)*(x-20) => 6y = 5x + 44
34	B	Plugging values in definition of correlation coefficient gives: $\sqrt{\frac{5}{6}} = \frac{E(XY)-20*24}{\sqrt{24}\cdot\sqrt{20}}$. Simplifying gives E(XY) = 500.
35	C	For each n, probability = n/21. So, expected value = $(1^2+2^2+\ldots+6^2)/21$ = 13/3.
36	C	Let's tabulate the various cases of n as follows: n: 1 | 2 | 3 | 4 | 5 | 6 Pr(no tail): 1/2 | 1/4 | 1/8 | 1/16 | 1/32 | 1/64 The probability of no tail = 1/6*1/2 + … + 1/6*1/64 = 1/6*63/64 = 21/128. Probability of at least one tail = 107/128.
37	A	Map the stick to the interval [0,1]. If both points lie in [0,0.5], the probability of which is $(0.5)^2$ = 0.25, it'll be a favourable event. If both points like in [0.5,1], the probability of which is again $(0.5)^2$ = 0.25, it'll be a favourable event too. Moreover, there is also a possibility (with positive probability) of the two points lying in different halves, yet the event being favourable (e.g. cuts at 0.2 and 0.8). Given all these events are disjoint, the required probability exceeds ½.
38	D	Since A and B are independent, $P(A\|B) = P(A) > 0$. So they are definitely not disjoint.
39	C	For a binomial distribution with μ = 2.56, mode = [2.56] = 2. Likewise, standard deviation = $\sqrt{\mu}$ = 1.6
40	B	From each point, there are (n-3) diagonals. However, each diagonal will be counted twice. So total diagonals = n(n-3)/2. Equating it with 252 and solving for positive n gives n = 24.

Data Interpretation

<table>
<tr><td>41</td><td>A</td><td>For the 6 years, the 4th largest production is in 2001 (120+107=227 thousand) and 3rd largest production is in 1999 (141+100=241 thousand). So the median is average of 227 and 241 which is 234 thousand.</td></tr>
<tr><td>42</td><td>B</td><td>The increase by X from 2000 to 2001 was approx. 53% which was the highest.</td></tr>
<tr><td>43</td><td>C</td><td>Total production by X = 716. Total production by Y = 742. So, X's production was less by ~3.5%.</td></tr>
<tr><td>44</td><td>B</td><td>Company Y's market share in 2000 was 128/(128+78) = 62% which was the highest.</td></tr>
<tr><td>45</td><td>C</td><td>Respondents: Adult = 65, Children = 64, Total = 129; Adult % = 65/129 = 50.4%</td></tr>
<tr><td>46</td><td>A</td><td>% children preferring other than Animation = (64-31)/64 = 51.6%</td></tr>
<tr><td>47</td><td>D</td><td>Sci-fi has 43.75% (14/32) and 56.25% (18/32) split between children and adults respectively, which is the most even among the genres.</td></tr>
<tr><td>48</td><td>B</td><td rowspan="4">The given information can be analysed to produce the following table:
<table><tr><td></td><td></td><td colspan="2">Points for</td><td colspan="2">Cumulative points</td></tr><tr><td>Game</td><td>Winner</td><td>DC</td><td>C</td><td>DC</td><td>C</td></tr><tr><td>1</td><td>DC</td><td>1</td><td>0</td><td>1</td><td>0</td></tr><tr><td>2</td><td>Draw</td><td>0.5</td><td>0.5</td><td>1.5</td><td>0.5</td></tr><tr><td>3</td><td>Draw</td><td>0.5</td><td>0.5</td><td>2</td><td>1</td></tr><tr><td>4</td><td>DC</td><td>1</td><td>0</td><td>3</td><td>1</td></tr><tr><td>5</td><td>Draw</td><td>0.5</td><td>0.5</td><td>3.5</td><td>1.5</td></tr><tr><td>6</td><td>C</td><td>0</td><td>1</td><td>3.5</td><td>2.5</td></tr><tr><td>7</td><td>Draw</td><td>0.5</td><td>0.5</td><td>4</td><td>3</td></tr><tr><td>8</td><td>C</td><td>0</td><td>1</td><td>4</td><td>4</td></tr><tr><td>9</td><td>DC</td><td>1</td><td>0</td><td>5</td><td>4</td></tr><tr><td>10</td><td>C</td><td>0</td><td>1</td><td>5</td><td>5</td></tr><tr><td>11</td><td>Draw</td><td>0.5</td><td>0.5</td><td>5.5</td><td>5.5</td></tr><tr><td>12</td><td>C</td><td>0</td><td>1</td><td>5.5</td><td>6.5</td></tr><tr><td>13</td><td>Draw</td><td>0.5</td><td>0.5</td><td>6</td><td>7</td></tr><tr><td>14</td><td>C</td><td>0</td><td>1</td><td>6</td><td>8</td></tr><tr><td>Total</td><td>C</td><td>6</td><td>8</td><td></td><td></td></tr></table>
From these, the answers follow.</td></tr>
<tr><td>49</td><td>C</td></tr>
<tr><td>50</td><td>D</td></tr>
<tr><td>51</td><td>B</td></tr>
</table>

52	D	
53	B	
54	B	
55	C	
56	B	The past tense of ‘cost’ is ‘cost’.
57	D	
58	D	‘Noxious’ means ‘harmful’ or ‘poisonous’; others mean ‘angry’.
59	B	
60	C	“*Precision and correctness are like opposing forces.*” But they are not necessarily mutually exclusive. In fact, useful writing needs to be precise and bold, but correct.
61	D	
62	D	Ram was married **to** Sita. Sita was sitting **beside** Ram. Although Ram was a prince, ~~**but**~~ he had to spend fourteen years in the forest.

63	D	21st December's day of week: 2025: Sun, 2026: Mon, 2027: Tue, 2028: Thu (leap year), 2029: Fri, 2030: Sat
64	A	T1 = 10:30 and T2 = 13:30 are mirror images and 3 hours apart.
65	B	X = 27, Y = 54, Z = 36
66	C	X's mother and Y's father are siblings.
67	D	A is necessarily false due to statement about W's. B is necessarily false due to statement about Z's. C is necessarily false due to statement about W's and Z's taken together.
68	B	X follows as dancers are subset of singers which is disjoint from painters. Y need not be true as some singer need not be a dancer.
69	B	Crime happens, followed by arrest, leading to judgement and then punishment.
70	C	n-th term is sum of squares of first n positive integers. 6th term should have been 91 (not 90).

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