MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If $7 = 5 + \frac{1}{7}(5+\alpha) + \frac{1}{7^2}(5+2\alpha) + \ldots\ \infty$ terms, then α is equal to

(1) 6 (2) $\frac{6}{7}$

(3) $\frac{1}{7}$ (4) 1

Answer (1)

$S = a + (a + d)r + (a + 2d)r^2 + \ldots \infty$

Then $S = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

$$7 = \frac{S}{1-\frac{1}{7}} + \frac{\alpha \cdot \frac{1}{7}}{\left(1-\frac{1}{7}\right)^2}$$

$\Rightarrow\ \alpha = 6$

2. If A and B are binomial coefficients of 30[th] and 12[th] term of binomial expansion $(1 + x)^{2n-1}$. If $2A = 5B$, then the value of n is

(1) 20 (2) 21

(3) 14 (4) 20

Answer (2)

Sol. $T_{r+1} = {}^{2n-1}C_r x^r$

Coefficient of $T_{30} = {}^{2n-1}C_{29} = A$

Coefficient of $T_{12} = {}^{2n-1}C_{11} = B$

$\Rightarrow\ 2\left({}^{2n-1}C_{29}\right) = 5\left({}^{2n-1}C_{11}\right)$

$\Rightarrow$ Solving we get $n = 21$

3. The equation of chord of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ with (3, 1) as mid-point is

(1) $48x + 25y - 169 = 0$ (2) $25x + 5y - 125 = 0$

(3) $65x + 2y - 12 = 0$ (4) $45x + 4y - 135 = 0$

Answer (1)

Sol. Chord with given mid-point

$\Rightarrow\ T = S_1$

$$\Rightarrow \left(\frac{xx_1}{25} + \frac{yy_1}{16} - 1\right) = \frac{x_1^2}{25} + \frac{y_1^2}{16} - 1$$

$$\Rightarrow \frac{3x}{25} + \frac{y}{16} - 1 = \frac{9}{25} + \frac{1}{16} - 1$$

$\Rightarrow\ 48x + 25y - 400 = 144 + 25 - 400$

$\Rightarrow\ 48x + 25y = 169$

4. If system of equations

$x + 2y - 3z = 2$

$2x + \lambda y + 5z = 5$

$4x + 3y + \mu z = 33$ has infinite solutions, then $\lambda + \mu$ is equal to

(1) $\frac{1334}{5}$ (2) $\frac{1269}{5}$

(3) $\frac{261}{5}$ (4) $\frac{1063}{5}$

Answer (1)

Sol. $\Delta = \begin{vmatrix} 1 & 2 & -3 \\ 2 & \lambda & 5 \\ 4 & 3 & \mu \end{vmatrix} = 0$

$\Rightarrow\ 12\lambda + \lambda\mu - 4\mu + 7 = 0$

$\Delta z = 25\lambda - 95 = 0$

$\Rightarrow\ \lambda = \frac{95}{25} = \frac{19}{5}$

$\Delta y = 0$

$\Rightarrow \dfrac{263-\mu}{5}=0$

$\Rightarrow \mu = 263$

$\therefore \lambda+\mu=\dfrac{1334}{5}$

5. Let S_n denotes the sum of the first n terms of an arithmetic progression. If S_{40} = 1030 and S_{12} = 57, then the value of $S_{30} - S_{10}$ is

(1) 505 (2) 510

(3) 515 (4) 520

Answer (3)

Sol. $S_{40} = 1030 \Rightarrow \dfrac{40}{2}[2a+39d]=1030$ ...(i)

$S_{12} = 57 \Rightarrow \dfrac{12}{2}[2a+11d] = 57$ ...(ii)

From (i) & (ii) $a = \dfrac{-7}{2}, d = \dfrac{3}{2}$

$\Rightarrow$

$S_{30} - S_{10} = \dfrac{30}{2}[2a+29d]-\dfrac{10}{2}[2a+9d] = 20a+390d$

$= 515$

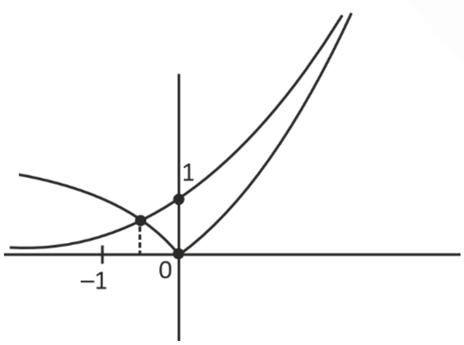
6. The area of region enclosed by the curves $y = e^x, y = |e^x - 1|$ and y – axis is (in sq. units)

(1) 1 (2) 1 – ln2

(3) 1 + ln2 (4) ln2

Answer (2)

Sol.



$e^x = 1 - e^x \Rightarrow 2e^x = 1$

$\Rightarrow e^x=\dfrac{1}{2}$

$\Rightarrow x=\ln\dfrac{1}{2}$

$\Rightarrow \int_{\ln\left(\frac{1}{2}\right)}^{0}\left[e^x-(1-e^x)\right]dx$

$= \int_{\ln 2}^{0}(2e^x-1)dx = 2e^x - x\Big|_{-\ln 2}^{0}$

= 2–(1 + ln2)

= 1 – ln2

7. Consider an event E such that a matrix of order 2 × 2 is invertible with entries 0 or 1. Then, $P(E)$ is (where $P(X)$ denotes the probability of event X)

(1) $\dfrac{5}{8}$ (2) $\dfrac{3}{8}$

(3) $\dfrac{1}{8}$ (4) $\dfrac{7}{8}$

Answer (2)

Sol. $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 0$

$\Rightarrow ad - bc = 0$

$\Rightarrow ad = bc$

Case-I $ad = bc = 1$

$a = b = c = d = 1$

Case-II $ad = bc = 0$

$a = 0, d = 0$	$b = 0, c = 0$
$a = 0, d = 1$	$b = 0, c = 1$
$a = 1, d = 0$	$b = 1, c = 0$

3 × 3 = 9 cases

$\therefore \dfrac{2^4-10}{2^4}=\dfrac{6}{16}=\dfrac{3}{8}$

8. The number of real roots of the equation

$x^2+3x+2=\min(|x+2|,|x-3|)$ is

(1) 0

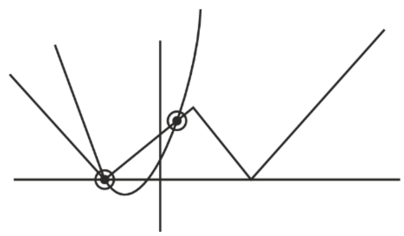
(2) 1

(3) 2

(4) 3

Answer (3)

Sol.



9. A function $f: R \to (-1, 1)$ such that $f(x) = \dfrac{2^x - 2^{-x}}{2^x + 2^{-x}}$. The function f is

(1) Both one-one and onto

(2) only one-one

(3) only onto

(4) Both many-one and onto

Answer (1)

Sol. $f(x)=\dfrac{2^{2x}-1}{2^{2x}+1}=1-\dfrac{2}{2^{2x}+1}$

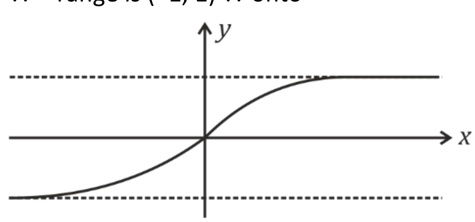
$f'(x)=\dfrac{2}{(2^{2x}+1)^2}\cdot 2^{2x}\cdot(\ln 2)\cdot 2>0$

$\therefore$ $f(x)$ is increasing $\therefore$ one-one

as $x \to -\infty$, $f(x) \to -1$

as $x \to \infty$, $f(x) \to 1$

$\therefore$ range is $(-1, 1)$ $\therefore$ onto



10. Let $\vec{a} = 3\hat{i}+2\hat{j}-\hat{k}$, $\vec{b} = \vec{a}\times(\hat{i}-2\hat{j})$ and $\vec{c} = \vec{b}\times\hat{k}$, then projection of $\vec{c}-2\hat{j}$ on $\vec{a}$ is equal to

(1) $2\sqrt{14}$

(2) $3\sqrt{7}$

(3) $2\sqrt{7}$

(4) $\dfrac{3\sqrt{14}}{14}$

Answer (4)

Sol. $\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & -1\\ 1 & -2 & 0\end{vmatrix} = -2\hat{i}-\hat{j}-8\hat{k}$

$\vec{c} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & -1 & -8\\ 0 & 0 & 1\end{vmatrix} = -\hat{i}+2\hat{j}$

$\Rightarrow \vec{c}-2\hat{j} = -\hat{i}$

$\Rightarrow$ Projection of $\vec{c}-2\hat{j}$ on $\vec{a}$ = $\left|(\vec{c}-2\hat{j})\cdot\hat{a}\right|$

$= \dfrac{3}{\sqrt{14}} = \dfrac{3\sqrt{14}}{14}$

11. If $\alpha > \beta > \gamma > 0$ then find

$\cot^{-1}\left(\dfrac{1+\alpha\beta}{\alpha-\beta}\right)+\cot^{-1}\left(\dfrac{1+\beta\gamma}{\beta-\gamma}\right)+\cot^{-1}\left(\dfrac{1+\gamma\alpha}{\gamma-\alpha}\right)$

(1) π

(2) 0

(3) $\dfrac{\pi}{2}-(\alpha+\beta+\gamma)$

(4) 3π

Answer (1)

Sol. $\cot^{-1}\left(\dfrac{1+\alpha\beta}{\alpha-\beta}\right)+\cot^{-1}\left(\dfrac{1+\beta\gamma}{\beta-\gamma}\right)+\cot^{-1}\left(\dfrac{1+\gamma\alpha}{\gamma-\alpha}\right)$

$= \tan^{-1}\left(\dfrac{\alpha-\beta}{1+\alpha\beta}\right)+\tan^{-1}\left(\dfrac{\beta-\gamma}{1+\beta\gamma}\right)+\pi+\tan^{-1}\left(\dfrac{\gamma-\alpha}{1+\gamma\alpha}\right)$

$\because\ \gamma-\alpha<0$

$= \tan^{-1}\left(\dfrac{\alpha-\beta}{1+\alpha\beta}\right)+\tan^{-1}\left(\dfrac{\beta-\gamma}{1+\beta\gamma}\right)+\tan^{-1}\left(\dfrac{\gamma-\alpha}{1+4\alpha}\right)+\pi$

$= \tan^{-1}\alpha - \tan^{-1}\beta + \tan^{-1}\beta - \tan^{-1}\gamma + \tan^{-1}\gamma - \tan^{-1}\alpha + \pi$

$= 0 + \pi$

$= \pi$

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If $\lim\limits_{x\to 0}\begin{vmatrix} a+\frac{\sin x}{x} & 1 & 1 \\ a & 1+\frac{\sin x}{x} & 1 \\ a & 1 & a+\frac{\sin x}{x} \end{vmatrix}$

$= \mu a^2 + \lambda a + \alpha$, then $\mu + \lambda + \alpha$ is

Answer (4)

Sol. $\begin{vmatrix} a+1 & 1 & 1 \\ a & 2 & 1 \\ a & 1 & a+1 \end{vmatrix} = a^2 + 2a + 1$

$\therefore\ \mu = 1$

$\lambda = 2$

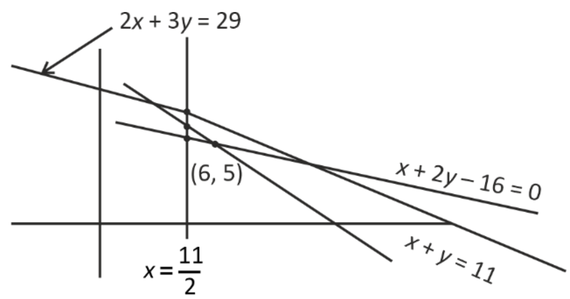
$\alpha = 1$

$\therefore\ \mu + \lambda + \alpha = 4$

22. The point $P\left(\frac{11}{2}, \alpha\right)$ lies on or inside the triangle formed by the lines $x + y = 11$, $x + 2y = 16$ and $2x + 3y = 29$, then minimum value of 10α is equal to

Answer (55)

Sol.



Clearly, $x = \frac{11}{2}$ intersect

$x + y - 11 = 0$ at $\left(\frac{11}{2}, \frac{11}{2}\right)$ and

$2x + 3y - 29 = 0$ at $\left(\frac{11}{2}, 6\right)$.

$\Rightarrow\quad \alpha \in \left[\frac{11}{2}, 6\right]$

$\Rightarrow$ minimum value of $10\alpha = 55$

23. If $\int \frac{2x^2+5x+9}{x^2+x+1}dx = x\sqrt{x^2+x+1} + \alpha\sqrt{x^2+x+1}$

$+\beta\ln\left(x + \frac{1}{2} + \sqrt{x^2+x+1}\right) + C$, then $\alpha + 2\beta$ equals to

Answer (16)

Sol. $I = \int \frac{2x^2+5x+9}{x^2+x+1}dx.$

Let $\frac{2x^2+5x+9}{\sqrt{x^2+x+1}} = \frac{A(x^2+x+1)+B(2X+1)+C}{\sqrt{x^2+x+1}}$

Then, $A = 2$, $B = \frac{3}{2}$ and $C = \frac{11}{2}$

$\therefore\ I = \int \frac{2(x^2+x+1)+\frac{3}{2}(2x+1)+\frac{11}{2}}{\sqrt{x^2+x+1}}dx$

$= 2\int\sqrt{x^2+x+1}\,dx + \frac{3}{2}\cdot 2\sqrt{x^2+x+1} + \frac{11}{2}\int\frac{1}{\sqrt{x^2+x+1}}dx$

$$=2\int\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\,dx+3\sqrt{x^2+x+1}$$

$$+\frac{11}{2}\int\frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$=2\left\{\frac{x+\frac{1}{2}}{2}\sqrt{x^2+x+1}+\frac{3}{8}\ln\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+1}\right|\right\}$$

$$+3\sqrt{x^2+x+1}+\frac{11}{2}\ln\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|+C$$

$$=\left(\frac{2x+1}{2}\right)\sqrt{x^2+x+1}+\frac{3}{4}\ln\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|$$

$$+3\sqrt{x^2+x+1}+\frac{22}{7}\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|+C$$

$$=\frac{2x+7}{2}\sqrt{x^2+x+1}+\frac{25}{4}\ln\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|+C$$

$$=x\sqrt{x^2+x+1}+\frac{7}{2}\sqrt{x^2+x+1}$$

$$+\frac{25}{4}\ln\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|+C$$

$$\therefore\ \alpha=\frac{7}{2},\ \beta=\frac{25}{4}$$

Then $\alpha+2\beta=\frac{7}{2}+\frac{25}{2}-16$

24. In group *A,* there are 7 boys and 3 girls and in group *B*, there are 4 boys and 5 girls. For a picnic trip 4 boys and 4 girls are required such that 5 are selected from group A and 3 are selected from group *B*. Then the total number of ways to select the team for picnic trip is

Answer (5880)

Sol. Group A (7B + 3G), Group B(4B + 5G)

Number of required ways = $({}^7C_4\cdot{}^3C_1)\cdot({}^4C_0\cdot{}^5C_3)$

$$+({}^7C_3\cdot{}^3C_2)\cdot({}^4C_1\cdot{}^5C_2)$$

$$+({}^7C_2\cdot{}^3C_3)\cdot({}^4C_2\cdot{}^5C_1)$$

= 5880

25.

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