JEE-Main-24-01-2025 (Memory Based)
[EVENING SHIFT]
Maths

Question: If $7 = 5 + \frac{1}{7}(5+\alpha) + \frac{1}{7^2}(5+2\alpha) + ...\infty$ **terms, then** α **is equal to**

Options:
(a) 6
(b) 6/7
(c) 1/7
(d) 1

Answer: (a)

$S_\infty = a + (a+d)r + (a+2d)r^2 + ...\infty$

$S_\infty = \frac{a}{1-r} + \frac{rd}{(1-r)^2}$

$s = 7, a = 5, r = \frac{1}{17}, d = \alpha$

$7 = \frac{5}{1-\frac{1}{7}} + \frac{\frac{1}{7}ga}{\left(1-\frac{1}{7}\right)^2}$

$7 = \frac{35}{6} + \frac{7\alpha}{36}$

$1 = \frac{5}{6} + \frac{\alpha}{36}$

$36 = 30 + \alpha$

$\alpha = 6.$

Question: If A and B are binomial coefficients of 30^{th} and 12^{th} term of binomial expansion $(1 + x)^{2n-1}$. If 2A = 5B, then the value of n is

Options:
(a) 20
(b) 21
(c) 14
(d) 20

Answer: (b)

$(1+x)^{2n-1}$

$T_{30} = {}^{2n-1}C_{29}x^{29}$

$T_{12} = {}^{2n-1}C_{11}x^{11}$

$A = {}^{2n-1}C_{29}$

$B = {}^{2n-1}C_{11}$

$2 \times \frac{(2n-1)!}{29! \times (2n-30)!} = \frac{5 \times (2n-1)!}{11! \times (2n-12)!}$

$\frac{2}{29! \times 10!} = \frac{5}{11! \times 28!}$

$\frac{2}{29x} = \frac{5}{11}$

$n = 21$

$\frac{2}{29! \times 12!} = \frac{5}{11! \times 30!}$

$\frac{1}{6} = \frac{1}{6}$

$\therefore n = 21$

Question:The equation of chord of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ with (3,1) as mid-point is

Options:

(a) 48x + 25y - 169 = 0

(b) 25x + 5y - 125 = 0

(c) 65x + 2y - 12 = 0

(d) 45x + 4y - 135 = 0

Answer: (a)

$t = s$

$\frac{3x}{25} + \frac{y_1}{16} - 1 = \frac{9}{25} + \frac{1}{16} - 1$

$\frac{(3x)16 \times 25}{25-16} = \frac{(9)(16) \times (25)}{25-16}$

$48x + 25y - 169 = 0$

Question:If system of equations

x + 2y - 3z =2

2x + λy +5z = 5

4x + 3y + μz = 33 has infinite solutions, then $\lambda + \mu$ is equal to

Options:

(a) $\frac{1334}{5}$

(b) $\frac{1269}{5}$

(c) $\frac{261}{5}$

(d) $\frac{1063}{5}$

Answer: (a)

$$D_2 = 0\begin{vmatrix} 1 & 2 & -3 \\ 2 & 5 & 5 \\ 4 & 33 & \mu \end{vmatrix} = 0$$

$$4\,(25) - 33\,(11) + \mu\,(1) = 0$$

$$= 363 - 100\mu = 263$$

$$D_3 = 0\begin{vmatrix} 1 & 2 & 2 \\ 2 & \lambda & 5 \\ 4 & 3 & 33 \end{vmatrix} = 0$$

$$-\,2\,(46) + \lambda\,(25) - 3\,(1) = 0$$

$$25\lambda = 85$$

$$\lambda = \frac{85}{25} = \frac{19}{5} = \lambda$$

$$\lambda + \mu = \frac{19}{5} + 263$$

$$= \frac{1334}{5}$$

Question:Let S_n denotes the sum of first n terms of an arithmetic progression. If S_{40} = 1030 and S_{12} = 57, then the value of S_{30} - S_{10} is

Options:

(a) 505

(b) 510

(c) 515

(d) 520

Answer: (c)

$$S_{10} = 1030, S_{12} = 57$$

$$\frac{40}{2}(2a + 39d) = 103$$

$$2a + 39d = \frac{103}{2}$$

$$\frac{12}{2}(2a + 11d) = 57$$

$$2a + 11d = \frac{19}{2}$$

$$2a + 39d = \frac{103}{2}$$

$$2a + 11d = \frac{19}{2}$$

$$28d = 42$$

$$d = \frac{42}{28}$$

$$d = \frac{3}{2}$$

$$2a + \frac{33}{2} = \frac{19}{2}$$

$$2a = -7$$

$$a = \frac{-7}{2}$$

$$S_{30} - S_{10}$$

$$= 15(2a + 2ad) - 5(2a + 9d)$$

$$= 20a + 390d$$

$$= 20\left(\frac{-7}{2}\right) + 390\left(\frac{3}{2}\right)$$

$$= -70 + 585$$

$$= 515$$

Question: Consider an event E such that a matrix of order 2×2 is invertible with entries 0 or 1. Then, P(E) is (were P(X) denotes the probability of event X)

Options:

(a) $\frac{5}{8}$

(b) $\frac{3}{8}$

(c) $\frac{1}{8}$

(d) $\frac{7}{8}$

Answer: (b)

$2 \times 2 \rightarrow \text{Matrix}$

$|A| \neq 0$

$n(s) = 2 \times 2 \times 2 \times 2 \Rightarrow 16$

A can't be

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

So, total $\Rightarrow$ 10 types of matrix

we can't take

$$\therefore \text{P}\left(\bar{A}\right) = \frac{10}{16} = \frac{5}{8}$$

$$\therefore P(A) = 1 - P(\bar{n}) = 1 - \frac{5}{8} \Rightarrow \frac{3}{8}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0 \quad \text{ad} \neq \text{bc}$$

$ad = 1$ $\quad bc = 0 \quad$ 3ways

$ad = 0$ $\quad bc = 1 \quad$ 3ways

So $\frac{6}{16} = \frac{3}{8}$

Question: Two groups A consists of 5 boys and 3 girls and Bhave 4 boys and 2 girls, we need to select 4 boys and 4 girls in total such that there must be 5 members of A and 3 of B.

Solution :

A $\Rightarrow$ 5B, 3G

B $\Rightarrow$ 4B, 2G

4B, 4G □ Total

5 Members A, 3 Members of B

A	B
3B 2G	1B 2G
2B 3G	2B 1G

$^5C_3\,^3C_2\,^4C_1\,^2C_2 + {}^5C_2\,^3C_3\,^4C_2\,^2C_1$

$10 \cdot 3 \cdot 4 \cdot 1 + 10 \cdot 1 \cdot 6 \cdot 2$

$120 + 120$

$= 240$

Question: Solve $\int \frac{2x^2+5x+1}{\sqrt{x^2+x+1}}dx$

Solution :

$\int \sqrt{x^2+x+1}\,dx + \int \frac{3x-1}{\sqrt{x^2+x+1}}dx$

$\int 2\sqrt{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}\,dx + \frac{3}{2}\int \frac{2x+1}{\sqrt{x^2+x+1}}dx - \frac{5}{2}\int \frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}}dx$

$= 2\left[\frac{\left(x+\frac{1}{2}\right)}{2}\sqrt{x^2+x+1} + \frac{3}{8}\ell n\left(\sqrt{x^2+x+1}+x+\frac{1}{2}\right)\right]$

$+\frac{3}{2}\times 2\sqrt{x^2+x+1} - \frac{5}{2}\ell n\left(\sqrt{x^2+x+1}+x+\frac{1}{2}\right)$

$= \left(x+\frac{1}{2}\right)\sqrt{x^2 x+1} + 3\sqrt{x^2+x+1} - \frac{7}{2}\ell n\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2}+x+1+C\right|$

Question:A function f : R → (-1, 1) such that $f(x) = \frac{2^x - 2^{-x}}{2^x + 2^{-x}}$.

The function f is

Options:

(a) Both one-one and onto

(b) Only one-one

(c) Only onto

(d) Both many - one and onto

Answer: (a)

$f(x) = \frac{2^x-2^{-x}}{2^x+2^{-x}} f_R \rightarrow (1,1)$

$f(2) = \frac{2^{2x}-1}{2^{2x}+1}$

$\Rightarrow \frac{1-2^{2x}}{1+2^{2x}}$

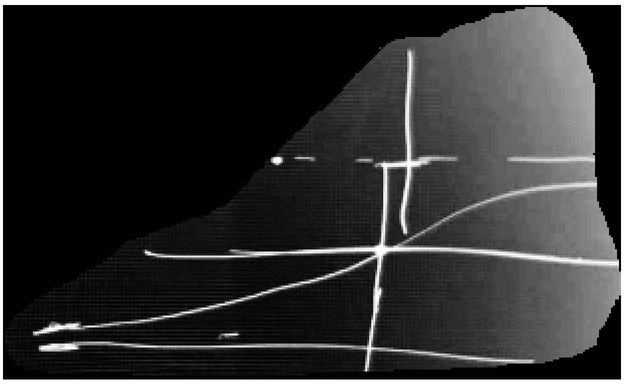
$\frac{1-\infty}{1+\infty}$

$f'(x) = \frac{2^{2x}\ln 4\left(2^{2x}+1\right)-2^{2x}\ln 4}{\left(2^{2x}+1\right)^2}$

$=> \frac{2\times 2^{2x}\ln 4}{1}$

$\therefore one - one$

$onto$



Question: **The area of region enclosed by the curves y = e^x, y = |e^x - 1| and y - axis is (in sq. units)**

Options:

(a) 1

(b) 1-ℓn2

(c) 1+ℓn2

(d) ℓn2

Answer: (b)

$y = e^x, y = |e^x - 1|$

$$\therefore \text{ Area} = \int_{-\ln 2}^{0} e^x - (-e^x + 1)dx$$

$$= 2 \int_{-\ln 2}^{2} e^x - \int_{-\ln 2}^{0} 1dx$$

$$= 2[e^x]_{-\ln 2}^{0} - [x]_{-\ln 2}^{0}$$

$$\Rightarrow 2\left(1 - \frac{1}{2}\right) - [0 + \ln 2]$$

$$\Rightarrow 1 - \ln 2$$

$$e^x = -e^x + 1$$

$$2e^n = 1$$

$$e^x = \frac{1}{2}$$

$$x = \ln\left(\frac{1}{2}\right)$$

Question: **The number of real roots of the equation x^2 + 3x + 2 = min (|x+2|,|x+3|)**

Options:

(a) 0

(b) 1

(c) 2

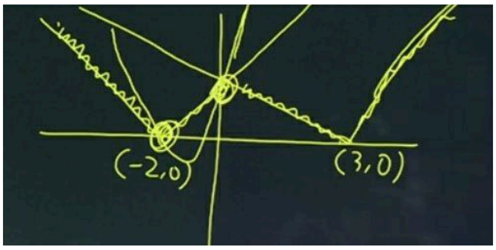
(d) 3

Answer: (2)

$$x^2 + 3x + 2 = \min(|x + 2|, |x - 3|)$$

Point of intersection $= 2$ points

$\therefore$ no. of soln $= 2$



$$f(x)=\begin{vmatrix} a+\frac{\sin x}{x} & 1 & b \\ a & 1+\frac{\sin x}{x} & b \\ a & 1 & b+\frac{\sin x}{x} \end{vmatrix},$$

Question:

$\lim_{x\to 0} f(x)=\lambda+\alpha a+\beta b$ then $(\lambda+\alpha+\beta)^2=$

Solution :

$$\lim_{x\to 0} f(x)=\begin{vmatrix} a+1 & 1 & b \\ a & 2 & b \\ a & 1 & b+1 \end{vmatrix}$$

$$=\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ a & 1 & b+1 \end{vmatrix}$$

$$=(b+1)+a+1=2+a+b$$

$$(\lambda+\alpha+\beta)^2=4^2=16$$

Question: Let $\vec{a}=3\hat{i}+2\hat{j}-\hat{k}, \vec{b}=\vec{a}\times(\hat{i}-2\hat{j})$ **and** $\vec{c}=\vec{b}\times\hat{k},$ **then projection of** $\vec{c}-2\hat{i}$ on $\vec{a}$ **is equal to**

Options:

(a) $2\sqrt{14}$

(b) $3\sqrt{7}$

(c) $2\sqrt{7}$

(d) $\frac{3\sqrt{14}}{14}$

Answer: (d)

$$\bar{c}=\left(\bar{a}\times\left(\hat{i}-2\hat{j}\right)\right)\times\hat{k}$$

$$=\left(\bar{a}\cdot\hat{k}\right)\cdot\left(\hat{i}-2\hat{j}\right)-0$$

$$=-i+2\hat{j}$$

$$\bar{c}-2\hat{j}=-\hat{j}$$

$$\text{Proj}=\frac{-\hat{j}\cdot\bar{a}}{|\bar{a}|}$$

$$=\frac{-3}{\sqrt{14}}$$

Question:If $\alpha>\beta>\gamma>0$**, then find** $\cot^{-1}\left(\frac{1+\alpha\beta}{\alpha-\beta}\right)+\cot^{-1}\left(\frac{1+\beta\gamma}{\beta-\gamma}\right)+\cot^{-1}\left(\frac{1+\gamma\alpha}{\gamma-\alpha}\right)$

Options:

(a) π

(b) zero

(c) $\frac{\pi}{2}-(\alpha+\beta+\gamma)$

(d) 3π

Answer: (a)

$$\tan^{-1}\frac{\alpha-\beta}{1+\alpha\beta}+\tan^{-1}\frac{\beta-\gamma}{1+\beta\gamma}+\pi+\tan^{-1}\frac{\gamma-\alpha}{1+\gamma\alpha}$$
$$=\tan^{-1}\alpha-\tan^{-1}\beta+\tan^{-1}\beta-\tan^{-1}\gamma+\pi$$
$$+\tan^{-1}\gamma-\tan^{-1}\alpha=\pi$$

Question: The point $P\left(\frac{11}{2},\alpha\right)$ lies on or inside the triangle formed by the lines x + y = 11, x + 2y = 16 and 2x + 3y = 29, then minimum value of 10α is equal to

Solution :

$y=\frac{11}{2}=3.5$

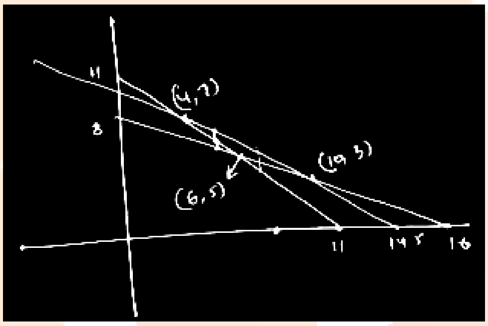
$2y=16-\frac{11}{2}$

$y=\frac{21}{2}$

$y=\frac{21}{4}=5.25$

$3y=29-1=18$

$y=6$



$$\left.\begin{aligned}x+y&=11\\x+2y&=16\\2x+3y&=29\end{aligned}\right\}$$

$(6,5)$

$(10,3)$

$(4,7)$