MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let $f(x)=\int \frac{dx}{x^{1/4}(x^{1/4}+1)}$. If $f(0) = -6$, then $f(2)$ is

(1) $4\left[\frac{1}{\sqrt{2}}-2^{1/4}+\ln\left|1+2^{1/4}\right|\right]-6$

(2) $4\left[\frac{1}{\sqrt{2}}-2^{1/4}+\ln\left|1+2^{1/4}\right|\right]+6$

(3) $4\left[\frac{1}{\sqrt{2}}+2^{1/3}+\ln\left|2^{1/4}\right|\right]-6$

(4) $4\left[3+2^{1/3}-\ln 2^{1/4}\right]+6$

Answer (1)

Sol. $\int \frac{dx}{x^{1/4}(x^{1/4}+1)}$

$x^{1/4} = t \Rightarrow dx = 4t^3dt$

$\int \frac{4t^3dt}{t(t+1)} = 4\int \frac{t^2dt}{t+1} = 4\left[\int\left(\frac{t^2-1}{t+1}+\frac{1}{t+1}\right)dt\right]$

$=4\left[\int(t-1)dt+\ln|t+1|\right]+c$

$f(x)=4\left[\frac{x^{1/2}}{2}-x^{1/4}+\ln\left|x^{1/4}+1\right|\right]+c$

$f(0) = -6$

$-6 = 4(0) + c$

$\Rightarrow \quad c = -6$

$\Rightarrow \quad f(x)=4\left[\frac{\sqrt{x}}{2}-x^{1/4}+\ln\left|1+x^{1/4}\right|\right]-6$

$f(2)=4\left[\frac{1}{\sqrt{2}}-(2)^{1/4}+\ln\left|1+2^{1/4}\right|\right]-6$

2. Evaluate $\sum_{r=1}^{13}\frac{1}{\sin\left[\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right]\sin\left[\frac{\pi}{4}+\frac{r\pi}{6}\right]}$

(1) $2\sqrt{3}+2$

(2) $2\sqrt{3}-2$

(3) $3\sqrt{2}+2$

(4) $3\sqrt{2}-4$

Answer (2)

Sol. $\sum_{r=1}^{13}\frac{1}{\sin\left[\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right].\sin\left[\frac{\pi}{4}+r.\frac{\pi}{6}\right]}$

$=\sum_{r=1}^{13}\frac{\sin\left\{\left(\frac{\pi}{4}+r.\frac{\pi}{6}\right)-\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)\right\}}{\sin\frac{\pi}{6}.\sin\left[\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right].\sin\left[\frac{\pi}{4}+r.\frac{\pi}{6}\right]}$

$\boxed{\sin(A-B)=\sin A\cos B-\cos A\sin B}$

$=2.\sum_{r=1}^{13}\cot\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+r.\frac{\pi}{6}\right)$

$=2.\cot\left(\frac{\pi}{4}+0.\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{13\pi}{6}\right)$

$=2.\left\{\cot\frac{\pi}{4}-\cot\left(\frac{\pi}{4}+\frac{\pi}{6}\right)\right\}$

$=2[1-2+\sqrt{3}]$

$=2[\sqrt{3}-1]$

3. Area bounded between the curves $C_1 : x(1+y^2)-1=0$ and $C_2 : y^2-2x=0$ is (in sq. unit)

(1) $\frac{\pi}{2}-\frac{1}{3}$

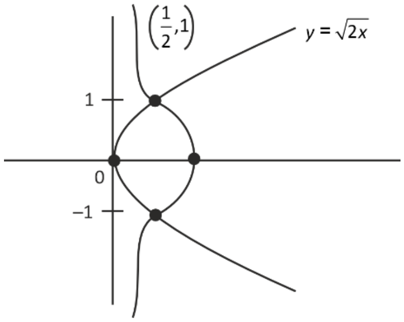
(2) $\frac{\pi}{4}-\frac{1}{6}$

(3) $2\left(\frac{\pi}{2}-\frac{1}{6}\right)$

(4) $\frac{\pi}{6}+\frac{1}{2}$

Answer (1)

Sol.



$$\int_{-1}^{1}(x_2-x_1)dy$$

$$\Rightarrow \int_{-1}^{1}\left(\frac{1}{1+y^2}-\frac{y_2}{2}\right)dy$$

$$=\tan^{-1}y-\frac{y_3}{6}\Bigg|_{-1}^{1}$$

$$=\left(\tan^{-1}-\frac{1}{6}\right)-\left(\tan^{-1}(-1)-\left(\frac{-1}{6}\right)\right)$$

$$=2\tan^{-1}1-\frac{1}{3}=\frac{\pi}{2}-\frac{1}{3}$$

4. There are three bags such that bag 1 has 4 white, 6 blue, bag 2 has 6 white and 4 blue and bag 3 has 5 white and 5 blue balls. A bag is randomly selected and a ball is randomly picked out of it, it comes out to be white then probability that selected bag was bag 2.

(1) $\frac{2}{5}$ (2) $\frac{2}{15}$

(3) $\frac{1}{15}$ (4) $\frac{7}{15}$

Answer (1)

Sol.

4W + 6B	6W + 4B	5W + 5B
BAG 1	BAG 2	BAG 3

$$P\left(\frac{B_2}{W}\right)=\frac{P(B_2)\cdot P\left(\frac{W}{B_2}\right)}{P(W)}$$

$$P(W)=\sum_{i=1}^{3}P(Bi)P\left(\frac{W}{B_2}\right)$$

$$P(B_1)=\frac{1}{3}=P(B_2)=(B_3)$$

$$P\left(\frac{W}{B_1}\right)=\frac{4}{10},\ P\left(\frac{W}{B_2}\right)=\frac{6}{10},P\left(\frac{W}{B_3}\right)=\frac{5}{10}$$

$$\Rightarrow P\left(\frac{B_2}{W}\right)=\frac{\frac{1}{3}\times\frac{6}{10}}{\frac{1}{3}\times\frac{4}{10}+\frac{1}{3}\times\frac{6}{10}+\frac{1}{3}\times\frac{5}{10}}$$

$$=\frac{\frac{6}{10}}{\frac{4}{10}+\frac{6}{10}+\frac{5}{10}}=\left(\frac{6}{15}\right)=\frac{2}{5}$$

5. If S is a set of words formed by all the letters of word “GARDEN”, then find the probability that vowels are not in alphabetical order.

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$

(3) $\frac{1}{4}$ (4) $\frac{1}{5}$

Answer (1)

Sol. AE GRDN

Only 2 vowels are there.

∴ Only half of the cases will have A before E and vice-versa.

Required probability = $\frac{1}{2}$

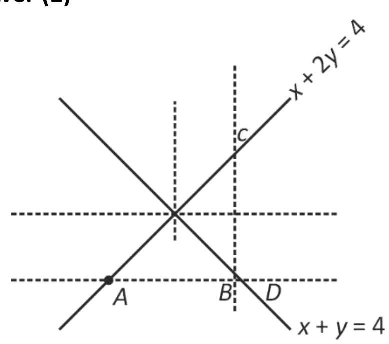
6. In isosceles triangle two sides are $x + 2y = 4$, $x + y = 4$ than the sum of all possible value of slope of third side of triangle is

(1) $\frac{3}{2}$ (2) $\frac{2}{3}$

(3) $\frac{-3}{2}$ (4) $\frac{-2}{3}$

Answer (2)

Sol.



Now third side will be parallel to the bisector line of the two given sides.

$$\frac{x+2y-4}{\sqrt{5}}=\pm\left(\frac{x+y-4}{\sqrt{2}}\right)$$

$$L_1:\left(\sqrt{2}-\sqrt{5}\right)x+\left(2\sqrt{2}-\sqrt{5}\right)y-4\sqrt{2}+4\sqrt{5}=0$$

$$L_2:\left(\sqrt{2}+\sqrt{5}\right)x+\left(2\sqrt{2}+\sqrt{5}\right)y-4\sqrt{2}-4\sqrt{5}=0$$

$$M_{L_1}+M_{L_2}=-\left[\frac{\sqrt{2}-\sqrt{5}}{2\sqrt{2}-\sqrt{5}}+\frac{\sqrt{2}+\sqrt{5}}{2\sqrt{2}+\sqrt{5}}\right]$$

7. If α, β, γ, δ are real numbers such that $\alpha + i\beta$ and $\gamma + i\delta$ are roots of the equation $x^2 - (3 - 2i)x - (2i - 2) = 0$. (where $i=\sqrt{-1}$), then $(\alpha\gamma + \beta\delta)$ is

(1) –2 (2) 2

(3) 6 (4) –6

Answer (2)

Sol. $x^2-(3-2i)x-(2i-2)=0$

$\Rightarrow x^2-(3-2i)x+(2-2i)=0$

$\Rightarrow \alpha+\beta=3-2i$

$\alpha\beta=(2-2i)$ (i),

by observation

$(\alpha,\beta)=(1,2-2i)$

$\Rightarrow \alpha+i\beta=i+0i$

$\gamma+i\delta=2-2i$

$\Rightarrow \alpha\gamma+\beta\delta=(1)(2)+(0)(-2)$

$=2$

8. The domain of the function $f(x)=sec^{-1}(2[x]+1)$ is (where $[\cdot]$ represents greatest integer function))

(1) $(-\infty,\ \infty)$ (2) $(-\infty,\ -1]\ \cup[1,\ \infty)$

(3) $(-\infty,\ \infty)-\{0\}$ (4) $(-\infty,-1]\cup[0,\ \infty)$

Answer (1)

Sol. $2[x]+1\notin(-1,1)$

$2[x]\notin(-2,0)$

$[x]\notin(-1,0)$

But $[x]\notin(-1,0)$ for any x.

$\Rightarrow$ $x\in R$ is the domain.

9. If p is the number of possible values of r such that T_r, T_{r+1}, T_{r+2} are three terms of $(a + b)^{12}$ are in geometric progression and if q is the sum of rational terms in the expansion of $(3^{1/4} + 4^{1/3})^{12}$, then $(p + q)$ is

(1) 283

(2) 238

(3) 240

(4) 250

Answer (1)

Sol. Let $T_{r+1}={}^{12}C_r a^{12-r}b^r$

T_r, T_{r+1}, T_{r+2} in G.P.

$\Rightarrow$ $\left({}^{12}C_r\cdot a^{12-r}\cdot b^2\right)^2=\left({}^{12}C_{r-1}\cdot b^{r-1}\cdot a^{13-r}\right)\left({}^{12}C_{r+1}b^{r+1}\cdot a^{11-r}\right)$

$\Rightarrow$ $\left({}^{12}C_r\right)^2=\left({}^{12}C_{r-1}\right)\cdot\left({}^{12}C_{r+1}\right)$

but no three consecutive binomial coefficients are in G.P. or H.P. but A.P. is possible.

$\Rightarrow$ P = 0

$T_{k+1}={}^{12}C_k\cdot(4^{1/3})^k\cdot 3^{1/4(12-k)}$

$={}^{12}C_k\cdot 4^{k/3}\cdot 3^{\left(3-\frac{k}{4}\right)}$

for terms to be rational (4, 3) divides $k \Rightarrow$ 12 divides k

$\Rightarrow k = 0, 12$

$\Rightarrow$ Sum of rational terms

${}^{12}C_0 4^0\cdot 3^3+{}^{12}C_{12}\cdot 4^4\cdot 3^0={}^{12}C_0(3^3+4^4)$

= 27 + 256 = 283

$\Rightarrow$ $p + q = 283$

10. Let P_i be image of parabola $P : y^2 = 4x$ with respect to line $x + y + 1 = 0$. Let the line $y + 5 = 0$ intersect P_i at A and B. If a is the distance between A and B and d be the area of triangle SAB, where S is the focus of parabola P_i. Then $(a + d)$ is

(1) 10

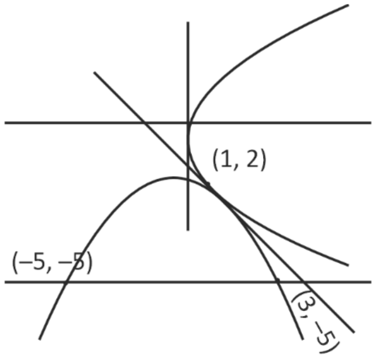
(2) 20

(3) 12

(4) 8

Answer (2)

Sol.



To find image of $P(t^2, 2t)$

$$\frac{x-t^2}{1}=\frac{y-2t}{1}=\frac{-2(t^2+2t+1)}{1^2+1^2}=-(t+1)^2$$

$\Rightarrow$ $x = t^2 - (t+1)^2 = -2t - 1$

$y = 2t - (t+1)^2 = -t^2 - 1$

$$t=\left(\frac{-1-x}{2}\right)$$

$$\Rightarrow \quad (y+1)=-\left(\frac{-1-x}{2}\right)^2$$

$$(y+1)=-\frac{(x+1)^2}{4}$$

$\Rightarrow$ $(x+1)^2 = -4(y+1)$

$\Rightarrow$ $X^2 = -4y$, the vertex is

$X = 0, y = 0$

$\Rightarrow$ Focus $\Rightarrow$ $(-1, -2)$

Other method would have been to find image of $(1, 0)$ about $x + y + 1 = 0$

$\Rightarrow$ $y = -5$ intersect

$(-4)(-4) = (x+1)^2$

$\Rightarrow$ $(x + 1) = \pm 4$

$\Rightarrow$ $x = 3, -5$

$\Rightarrow$ $a = 8$

$$d=\frac{1}{2}\times a\times \text{height}=\frac{1}{2}\times 8\times(-2-(-5))$$

$= 4 . 3 = 12$ Sq. unit

$\Rightarrow$ $\boxed{a+d=20}$

11. For positive integer n, $4a_n = n^2 + 5n + 6$ and $S_n=\sum_{K=1}^{n}\frac{1}{a_K}$. Then the value of $507(S_{2025})$ is

(1) 675 (2) 540

(3) 1350 (4) 725

Answer (1)

Sol. $S_n=\sum_{K=1}^{n}\frac{4}{K^2+5K+6}$

$$=\sum_{K=1}^{n}\frac{4}{(K+2)(K+3)}=4\sum_{K=1}^{n}\left(\frac{1}{K+2}-\frac{1}{K+3}\right)$$

$$=4\left[\frac{1}{3}-\frac{1}{4}\right]$$

$$=4\left[\frac{1}{4}-\frac{1}{5}\right]$$

$$=4\left[\frac{1}{n+2}-\frac{1}{n+3}\right]$$

$$S_n=4\left[\frac{1}{3}-\frac{1}{n+3}\right]$$

$$S_{2025}=4\left[\frac{1}{3}-\frac{1}{2028}\right]$$

$$S_{2025}=4\left[\frac{675}{2028}\right]$$

$507\, S_{2025} = 675$

12. The number of natural numbers between 212 to 999 such that sum of their digit is 15 is equal to

(1) 63 (2) 61

(3) 62 (4) 65

Answer (2)

Sol.

717 $\rightarrow$ 3

726 $\rightarrow$ 3!

735 $\rightarrow$ 3!

744 $\rightarrow$ 3

636 $\rightarrow$ 3

645 $\rightarrow$ 3!

555 $\rightarrow$ 1

915 $\rightarrow$ 3!

924 $\rightarrow$ 3!

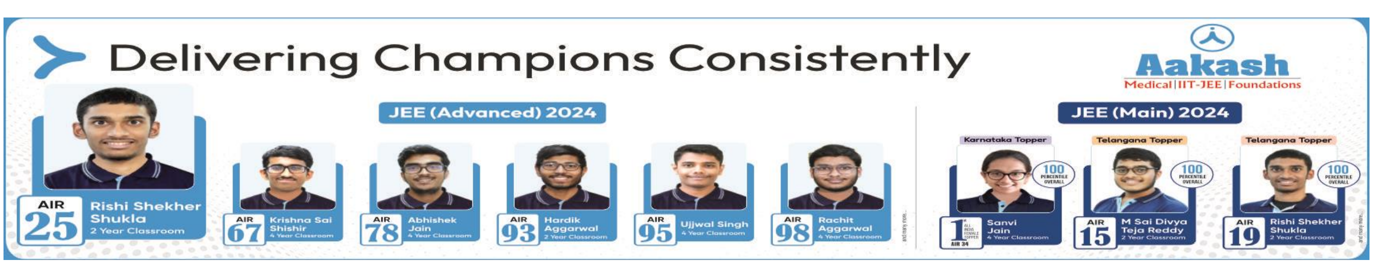
933 $\rightarrow$ 3

816 $\rightarrow$ 3!

825 $\rightarrow$ 3!

834 $\rightarrow$ 3!

Total : 61



13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If $Q=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$, $B = QPQ^T$ and matrix A is defined as $A = Q^TB^{10}Q$ (where $P=\begin{bmatrix}\sqrt{2} & -2\\ 0 & 1\end{bmatrix}$), then trace of matrix A is

Answer (33)

Sol. $Q=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

$B = QPQ^T$

$A = Q^TB^{10}Q$

$Q^TQ=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

$=\begin{bmatrix}\cos^2\theta+\sin^2\theta & 0\\ 0 & \sin^2\theta+\cos^2\theta\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$

$\Rightarrow$ $Q^TQ = QQ^T$

$A = Q^T(QPQ^T)B^9Q$

$= IPQ^TB^9Q = PQ^TB^9Q$

$\Rightarrow$ $A = PQ^T(QPQ^T)B^8Q$

$= P^2Q^TB^8Q = P^9Q^TB^1Q$

$= P^9Q^T(QPQ^T)Q = P^{10}$

Trace (P^{10}) = $\left(d_1^{10}+d_2^{10}\right)$ where d_1 and d_2 are diagonal elements

$\Rightarrow$ Trace (A) = $(\sqrt{2})^{10}+1^{10}$

$= 2^{10/2} + 1 = 2^5 + 1 = 33$

22. $f:[0, 3]\to b, f(x) = 2x^3 - 15x^2 + 36x + 7$ is an onto function $g:[0, \infty)\to d,\ g(x)=\frac{x^{2025}}{x^{2025}+1}$ is also an onto function. Find the number of elements in the set $S = \{x : x \in Z, x \in b \text{ or } x \in d\}$

Answer (30)

Sol. $f(x) = 2x^3 - 15x^2 + 36x + 7$

$f'(x)=6x^2-30x+36=0$

$\Rightarrow$ $x^2 - 5x + 6 = 0$

$\therefore$ $x = 1, 5$

$f(0) = 7, f(2) = 35, f(3) = 34$

$\therefore$ $b = [7, 35]$

$g(x)=\dfrac{x^{2025}}{1+x^{2025}}$

$d = [0, 1)$

$\therefore$ $S = [0, 7, 8, 9, \ldots, 35]$

Number of elements = 30

23. The maximum interior angle of a polygon is 171° with n sides such that its angles are in Arithmetic progression with common difference of 6°. Then n is equal to

Answer (10)

Sol. Sum of interior angle

$\Rightarrow\frac{n}{2}\left(2a+(n-1)d\right)=180°(n-2)$

$\Rightarrow 171=a+(n-1)d$

$\Rightarrow\frac{n}{2}\left(171+a\right)=180(n-2)$

$a = 171 - 6(n-1) = 177 - 6n$

$\Rightarrow\frac{n}{2}\left(171+177-6n\right)=180(n-2)$

$\Rightarrow n\left(174-3n\right)=180n-360$

$\Rightarrow 3n^2+6n-360=0$

$n^2+2n-120=0$

$(n+12)(n-10)=0$

$\Rightarrow n=10$

24.

25.

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