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MEMORY BASED QUESTIONS & TEXT SOLUTION

SHIFT-2

DATE & DAY: 28th January 2025 & Tuesday

PAPER-1

Duration: 3 Hrs.
Time: 03:00 PM – 06:00 PM

SUBJECT: MATHEMATICS

PART : MATHEMATICS

1. The value of $\sum_{k=1}^{13}\frac{1}{\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$ is equal to $a\sqrt{3}-b$ then a^2+b^2

(1) 2 (2) 4 (3) 6 (4) 8

Ans. **(4)**

Sol. $\sum_{k=1}^{13}\frac{\sin\left[\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)-\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)\right]}{\sin\frac{\pi}{6}\left(\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)\sin\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)\right)}=2\sum_{k=1}^{13}\left(\cot\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)\right)$

$=2\left(\cot\frac{\pi}{4}-\cot\left(\frac{\pi}{4}+\frac{13\pi}{6}\right)\right)=2\left(1-\cot\left(\frac{29\pi}{12}\right)\right)=2\left(1-\cot\left(\frac{5\pi}{12}\right)\right)=2\,(1-(2-\sqrt{3}))=2\,(-1+\sqrt{3})$

$=2\,(\sqrt{3}-1)$

$=2\sqrt{3}-2=a\sqrt{3}-b$

$a=2,\ b=2$

$a^2+b^2=4+4=8.$

2. Let $f(x)=\int\frac{dx}{x^{1/4}\left(x^{1/4}+1\right)}$. If $f(0)=-6$, then $f(1)$

(1) 4 (log 2 – 2) (2) 4 (log 2 + 2) (3) 4 (log 2 – 1) (4) 2 (log 2 – 2)

Ans. **(1)**

Sol. let $x^{1/4}=t$

$x=t^4$

$dx=4t^3\,dt$

$4\int\frac{t^3dt}{t(t+1)}$

$=4\int\frac{t^2dt}{(t+1)}$

$=4\int\left(\frac{t^2-1}{t+1}+\frac{1}{t+1}\right)dt$

$=4\left(\int(t-1)dt+\int\frac{1}{t+1}dt\right)$

$f(t)=4\left(\frac{t^2}{2}-t+\log(t+1)\right)+C$

$f(x)=2x^{1/2}-4x^{1/4}+4\log\left(x^{1/4}+1\right)+C$

$f(0)=-6$

$C=-6$

$f(1)=2-4+4\log 2-6=4\log 2-8$

$=4(\log 2-2)$

3. The area bounded by curve $x(1+y^2)=1$ & $y^2 = 2x$ is

(1) $\frac{\pi}{2}+\frac{1}{3}$ (2) $\frac{\pi}{2}-\frac{1}{3}$ (3) $\frac{\pi}{3}-\frac{1}{2}$ (4) $\frac{\pi}{3}+\frac{1}{2}$

Ans. **(2)**

Sol. $x(1+y^2)=1$ & $y^2=2x$

for point of intersection

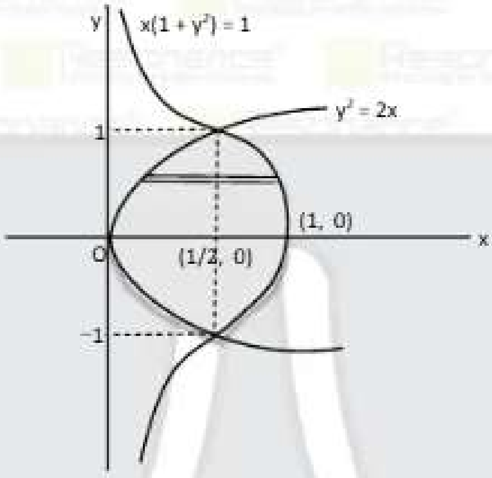
$x(1+2x)=1$

$2x^2+x-1=0$

$2x^2+2x-x-1=0$

$(2x-1)(x+1)=0$

$x=\frac{1}{2}, -1$



$$A = 2\int_0^1\left(\frac{1}{1+y^2}-\frac{y^2}{2}\right)dy$$

$$= 2\left[\tan^{-1}y-\frac{y^3}{6}\right]_0^1$$

$$= 2\left[\frac{\pi}{4}-\frac{1}{6}-0+0\right]=\frac{\pi}{2}-\frac{1}{3}$$

4. There are 3 Bags B_1, B_2, B_3 Which contain 6 black, 4 white and 4 black, 6 white and 5 black, 5 white balls respectively. If one ball is randomly selected from a bag and is found to be white. Find the probability that ball came from B_2 is:

(1) $\frac{1}{5}$ (2) $\frac{2}{5}$ (3) $\frac{3}{5}$ (4) $\frac{4}{5}$

Ans. **(2)**

Sol.

B_1	B_2	B_3
6 B, 4 W	4 B, 6 W	5 B, 5 W

Probability:- First select bag and then select Ball

$$\frac{\frac{1}{3}\times\frac{6}{10}}{\frac{1}{3}\times\frac{4}{10}+\frac{1}{3}\times\frac{6}{10}+\frac{1}{3}\times\frac{5}{10}} \qquad \frac{\text{favourable event}}{\text{total event}}$$

$$=\frac{6}{4+6+5}=\frac{6}{15}=\frac{2}{5}$$

5. $f(x) = 2x^3 - 15x^2 + 36x + 7$, $(f(x) : [0, 3] \to A)$ is onto function and

$g(x) = \frac{x^{2025}}{x^{2025} + 1}$, $((g : [0, \infty) \to B)$ is onto function). Then, number of integer in $(A \cup B)$:

Ans. 30

Sol. $f'(x) = 6x^2 - 30x + 36$

$f'(x) = 6(x - 2)(x - 3)$

$f(2) = 35$, $f(3) = 34$, $f(0) = 7$

$A \in [7, 35]$

Total number of integers in A = 29.

$g(x) = 1 - \frac{1}{x^{2025} + 1}$

$B \in [0, 1)$

Number of integers in B = 1.

So, total number of integers in the range of $(A \cup B) = 30$.

Because no elements are common in A and B.

6. $f(x) = \sum_{r=0}^{\infty} \frac{\tan\left(\frac{x}{2^{r+1}}\right) + \tan^3\left(\frac{x}{2^{r+1}}\right)}{1 - \tan^2\left(\frac{x}{2^{r+1}}\right)}$ then find the value of $\lim_{x \to 0} \frac{e^x - e^{f(x)}}{x - f(x)}$.

Ans. 1

Sol. $\because \quad \frac{\tan\theta + \tan^3\theta}{1 - \tan^2\theta} = \frac{\tan\theta}{\cos 2\theta} = \tan 2\theta - \tan\theta$

$\therefore \quad f(x) = \sum_{r=0}^{\infty} \tan\left(\frac{x}{2x}\right) - \tan\left(\frac{x}{2^{r+1}}\right)$

$\left(\lim_{n \to \infty} (\tan x - \tan(x/2)) + (\tan x/2 - \tan(x/2^2)) + \tan(x/2^2) - \tan(x/2^3) + \ldots + \left(\tan(x/2^n) - \tan(x/2^{n+1})\right)\right)$

$= \tan x - \tan 0 = \tan x$

$\lim_{x \to 0} \frac{e^x - e^{\tan x}}{x - \tan x} \quad \left(\frac{0}{0} \text{ form}\right)$

$\lim_{x \to 0} e^{\tan x}\left(\frac{e^{x - \tan x} - 1}{x - \tan x}\right) = e^0.1 = 1.$

7. If all natural numbers are listed from 212 to 999 then find number of natural numbers whose sum of digits is equal to 15.

Ans. 64

Sol. Coefficients of x^{15} in $(1 + x + x^2 + x^3 + \ldots + x^9)^3$

Coefficients of x^{15} in $\left(\frac{1 - x^{10}}{1 - x}\right)^3$

Coefficients of x^{15} in $(1 - x^{10})^3 (1 - x)^{-3}$

Coefficients of x^{15} in $\left(1 - 3x^{10} + {}^3C_2 x^{20} + \ldots\right)(1 - x)^{-3}$

$=$(Coefficients of x^{15} in $(1 - x)^{-3}$) $-$ 3(Coefficients of x^5 in $(1 - x)^{-3}$)

$= {}^{3+15-1}C_{15} - 3.{}^{3+5-1}C_5$

$= {}^{17}C_{15} - 3.{}^7C_5 = \frac{17 \times 16}{2} - \frac{3 \times 7 \times 6}{2} = 17 \times 8 - 9 \times 7 = 73.$

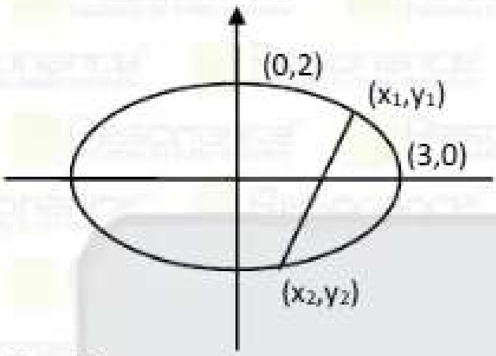
Number less than 212 whose sum of digits is (078, 087, 069, 096, 195, 159, 186, 177, 168).

Required number = 73 – 9 = 64.

8. $\frac{x^2}{9}+\frac{y^2}{4}=1$, $\left(\sqrt{2},\frac{4}{3}\right)$ is a mid point of chord and length of chord is $\frac{2\sqrt{\alpha}}{3}$ then the α is :

Ans. **(22)**

Sol.



$\frac{x^2}{9}+\frac{y^2}{4}=1$ ....(1)

$T = S_1$

$\frac{xx_1}{9}+\frac{yy_1}{4}=\frac{2}{9}+\frac{4}{9}$

$\frac{x\sqrt{2}}{9}+\frac{y}{3}=\frac{2}{3}$

$x\sqrt{2}+3y=6$ ....(2)

(x_1, y_1) and (x_2, y_2) both satisfy the equation

$x_1\sqrt{2}+3y_1=6$ $\quad$ $x_2\sqrt{2}+3y_2=6$

Subtracting both equations

$\sqrt{2}(x_1-x_2)+3(y_1-y_2)=0$

$(x_1-x_2)=\frac{-3}{\sqrt{2}}(y_1-y_2)$ ....(3)

Value of x from (2) in (1) in terms of y

$$\frac{\left(\frac{6-3y}{\sqrt{2}}\right)^2}{9}+\frac{y^2}{4}=1$$

$2(y-2)^2+y^2=4$

$3y^2-8y+4=0$

$(y-2)(3y-2)=0$

$y=2, \frac{2}{3}$

$|y_1-y_2|=|2-\frac{2}{3}|=\frac{4}{3}$

Length of the chord is distance between both points.

Distance formula

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

put the value of x_2-x_1 from equation (3)

Length of chord $=|y_2-y_1|\sqrt{\frac{11}{2}}=\frac{4}{3}\sqrt{\frac{11}{2}}=\frac{2\sqrt{22}}{3}$ $\Rightarrow$ $\alpha=22$

9. The interior angles of a polygon with n sides are in A.P. with common difference of 6º. If the biggest interior angle of polygon is 219º then n is equal to:

Ans. **20**

Sol. $a + (n-1)6 = 219 \quad \ldots(i)$

$\frac{n}{2}\left[2a + (n-1)6\right] = (n-2)180º \quad \ldots(ii)$

$\frac{n}{2}\left[a + 219\right] = (n-2) \times 180º$

$\frac{n}{2}\left[219 + 219 - (n-1)6\right] = (n-2) \times 180º$

$\frac{n}{2}\left[2 \times 219 - (n-1)6\right] = (n-2) \times 180º$

$n\,[219 - (n-1)3] = (n-2) \times 180º$

$n\,[222 - 3n] = (n-2) \times 180º$

$222n - 3n^2 = 180n - 360º$

$3n^2 - 222n + 180n - 360 = 0$

$3n^2 - 42n - 360 = 0$

$n^2 - 14n - 120 = 0$

$n = 20.$

10. If $x^2 - (3-2i)\,x - (2i-2) = 0$ has roots $\alpha + i\beta$ and $\gamma + i\delta$, where $\alpha, \beta, \gamma, \delta \in R$ and $i = \sqrt{-1}$ then the value of $\alpha\gamma + \beta\delta$.

Ans. **(2)**

Sol. $x^2 - (3-2i)\,x - (2i-2) = 0$

$x = \frac{(3-2i) \pm \sqrt{(3-2i)^2 + 4 \times 1(2i-2)}}{2}$

$= \frac{(3-2i) \pm \sqrt{9 + 4i^2 - 12i + 8i - 8}}{2}$

$= \frac{(3-2i) \pm \sqrt{-3-4i}}{2}$

$= \frac{(3-2i) \pm \sqrt{(1-2i)^2}}{2}$

$= \frac{3-2i+1-2i}{2}, \frac{3-2i-1+2i}{2} = \frac{4-4i}{2}, \frac{2}{2}$

$\alpha + i\beta = 2 - 2i \quad \gamma + i\delta = 1$

by comparing

$\alpha = 2 \qquad \gamma = 1$

$\beta = -2 \qquad \delta = 0$

$\alpha\gamma + \beta\delta$

$2 \times 1 + 0 = 2$

11. Consider a parabola $y^2 = 4x$ & lines $L_1 : x + y + 4 = 0$ & $L_2 : y + 5 = 0$, focus of parabola is S. If line L_2 cuts mirror image of parabola about L_1 at A & B such that AB = m & area of ΔSAB = n then m + n =

Ans. **(14)**

Sol. $P : y^2 = 4x$

$S(1, 0)$

$L_1 = x + y + 4 = 0$

Image of $(t^2, 2t)$ in $x + y + 4 = 0$ is

$$\frac{x-t^2}{1} = \frac{y-2t}{1} = \frac{-2(t^2+2t+4)}{2}$$

$x = -2t - 4,\ y = -t^2 - 4$

$\therefore$ Image of P is $(x+4)^2 = -4(y+4)$

at L_2 $\quad (x+4)^2 = -4(-5+4)$

$(x+4)^2 = 4 \qquad \therefore x + 4 = \pm 2$

$x = -2, -6$

$\therefore A(-2, -5), B(-6, -5)$

$\therefore \quad AB = 4 = m$

$$\text{area}\Delta SAB = \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ -2 & -5 & 1 \\ -6 & -5 & 1 \end{vmatrix} \qquad (R_3 \to R_3 - R_2)$$

$$= \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ -2 & -5 & 1 \\ -4 & 0 & 0 \end{vmatrix}$$

$$= \frac{1}{2}(-4)(0+5) = 10 = n$$

$\therefore \quad m + n = 4 + 10 = 14$

12. If $f(x) = \sec^{-1}(2[x]+1)$, then domain of f(x) is:

(1) $(-\infty, \infty)$ (2) $(-\infty, -1] \cup [1, \infty)$ (3) $(-\infty, \infty) - \{0\}$ (4) $(-\infty, -1] \cup [0, \infty)$

Ans. **(1)**

Sol. $\sec^{-1}(2[x]+1)$

$2[x] + 1 \geq 1$ or $2[x] + 1 \leq -1$

$2[x] \geq 0$ or $2[x] \leq -2$

$x \in [0, \infty)$ or $x \in (-\infty, 0)$

$x \in (-\infty, \infty)$

$D_f \in R$

13. For positive integer n, $4a_n = n^2 + 5n + 6$ and $S_n = \sum_{K=1}^{n}\left(\frac{1}{a_k}\right)$ then the value of 507 S_{2025} is

(1) 675 (2) 540 (3) 1350 (4) 135

Ans. **(1)**

Sol. $$\frac{1}{a_k} = \frac{4}{k^2+5k+6} = \frac{4}{(k+2)(k+3)}$$

$$\frac{1}{a_k} = \frac{4}{(k+2)} - \frac{4}{(k+3)}$$

$$S_n = \frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} \ldots\ldots \frac{1}{a_n}.$$

$$S_n = 4\left(\left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \ldots\ldots\left(\frac{1}{n+2} - \frac{1}{n+3}\right)\right)$$

$$S_n = 4\left(\frac{1}{3} - \frac{1}{n+3}\right) = 4\left(\frac{n+3-3}{3(n+3)}\right)$$

$$S_n = \frac{4n}{3(n+3)}$$

$$S_{2025} = \frac{4 \times 2025}{3 \times 2028}$$

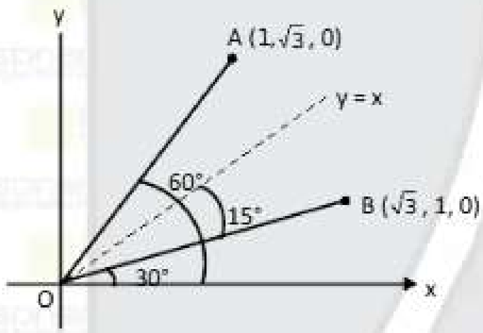
$$507 \times S_{2025} = 675$$

14. Let $\overrightarrow{OA} = \hat{i} + \sqrt{3}\hat{j}$ & $\overrightarrow{OB} = \sqrt{3}\hat{i} + \hat{j}$ are two vectors and distance of point C(a, 1 – a, 0) from internal angle bisector of $\overrightarrow{OA}$ & $\overrightarrow{OB}$ is $\left(\frac{9}{\sqrt{2}}\right)$, then sum of all possible values of a is

(1) 4 (2) 2 (3) 3 (4) 1

Ans. **(4)**

Sol.



A $(1, \sqrt{3}, 0)$ B – $(\sqrt{3}, 1, 0)$

Angle bisector ≡ y = x

⇒ y – x = 0

distance of point C(a, 1 – a, 0) from line y – x = 0 is $\frac{9}{\sqrt{2}}$

$$\left|\frac{1-a-a}{\sqrt{2}}\right| = \frac{9}{\sqrt{2}}$$

$$|1-2a| = 9$$

$1 - 2a = \pm 9 \Rightarrow a = 5$ & $a = -4$

sum = 1

15. $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & -2 \\ 0 & 1 \end{bmatrix}$ & $P = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, $\theta > 0$ if $B = PAP^T$. $C = (P^TBP)^{10}$, then sum of diagonal element of C is $\frac{m}{n}$ where gcd (m, n) = 1 then (m + n) is

Ans. **(65)**

Sol. $P = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} P^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$$PP^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I = P^TP$$

$C = (P^TBP)^{10}$

$A^2 = \begin{bmatrix} \frac{1}{\sqrt{2}} & -2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{\sqrt{2}}\right)^2 & -\sqrt{2}-2 \\ 0 & 1 \end{bmatrix}$

$= (P^TPAP^TP)^{10}$

$A^3 = \begin{bmatrix} \left(\frac{1}{\sqrt{2}}\right)^3 & .... \\ 0 & 1 \end{bmatrix}$

$= (IAI)^{10}$

from pattern $tr(A^n) = \left(\frac{1}{\sqrt{2}}\right)^n + 1$

$C = A^{10}$

$tr(A^{10}) = \left(\frac{1}{\sqrt{2}}\right)^{10} + 1$

$tr(C) = tr(A^{10}) = \frac{1}{32} + 1$

$\frac{m}{n} = \frac{33}{32}$

$m + n = 65$

16. Let the coefficient of three consecutive terms T_r, T_{r+1} & T_{r+2} in the Binomial expansion of $(a+b)^{12}$ be in a G.P. and let p be the number of all possible value of r. Let q be the sum of all rational terms in the expansion of $\left(\sqrt[4]{3} + \sqrt[3]{4}\right)^{12}$ then p + q is

(1) 299 (2) 287 (3) 295 (4) 283

Ans. **(4)**

Sol. ${}^nC_{r-1}$, ${}^nC_r$, ${}^nC_{r+1} \to$ G.P.

$n = 12$

$\frac{{}^{12}C_{r-1}}{{}^{12}C_r} = \frac{{}^{12}C_r}{{}^{12}C_{r+1}}$

$\frac{r}{13-r} = \frac{r+1}{12-r} \Rightarrow 12r - r^2 = 13r + 13 - r^2 - r$

$-r = 13 - r \Rightarrow 13 = 0$ (not possible)

$p = 0$

Now $\left(3^{\frac{1}{4}} + 4^{\frac{1}{3}}\right)^{12} = {}^{12}C_0\left(3^{\frac{1}{4}}\right)^{12}\left((4)^{\frac{1}{3}}\right)^0 + {}^{12}C_{12}\left(3^{\frac{1}{4}}\right)^0\left(4^{\frac{1}{3}}\right)^{12}$

$= 3^3 + 4^4 = 27 + 256$

$= 283 = q$

$p + q = 283$

17. If $y = y(x)$ satisfies differential equation $\sqrt{4-x^2}\frac{dy}{dx} = \left(\sin^{-1}\frac{x}{2}\right)\left(\left(\sin^{-1}\frac{x}{2}\right)^2 - y\right)$, such that $y(0) = \frac{\pi^2-8}{4}$

then $y(2) =$

(1) $\left(\frac{\pi^2-8}{4}\right) + \frac{\pi^2}{4} \cdot e^{-\frac{\pi^2}{8}}$ (2) $\left(\frac{\pi^2-2}{4}\right) + \frac{\pi^2}{4} \cdot e^{-\frac{\pi^2}{8}}$

(3) $\left(\frac{\pi^2-4}{8}\right) + \frac{\pi^2}{4} \cdot e^{-\frac{\pi^2}{8}}$ (4) $\left(\frac{\pi^2-8}{8}\right) + \frac{\pi^2}{4} \cdot e^{-\frac{\pi^2}{8}}$

Ans. **(1)**

Sol. Given $\Rightarrow \frac{dy}{dx}+\frac{\left(\sin^{-1}\frac{x}{2}\right)y}{\sqrt{4-x^2}}=\frac{\left(\sin^{-1}\frac{x}{2}\right)^3}{\sqrt{4-x^2}}$

$I.F=e^{\int\frac{\sin^{-1}\frac{x}{2}dx}{\sqrt{4-x^2}}}=e^{\frac{\left(\sin^{-1}\frac{x}{2}\right)^2}{2}}$

Solution is

$y.\ e^{\frac{\left(\sin^{-1}\frac{x}{2}\right)^2}{2}}=\int\frac{\left(\sin^{-1}\frac{x}{2}\right)^3}{\sqrt{4-x^2}}\,e^{\frac{\left(\sin^{-1}\frac{x}{2}\right)^2}{2}}dx+C$

Let $\left(\sin^{-1}\frac{x}{2}\right)^2=t \qquad \therefore \frac{2\sin^{-1}\frac{x}{2}}{\sqrt{4-x^2}}dx=dt$

$y.e^{\frac{\left(\sin^{-1}\frac{x}{2}\right)^2}{2}}=\int\frac{t}{2}e^{\frac{t}{2}}dt+C$

$y.\ IF=t\cdot e^{\frac{t}{2}}-2e^{\frac{t}{2}}+C$

$y.\ IF=(t-2)e^{\frac{t}{2}}$

$ye^{\frac{\left(\sin^{-1}\frac{x}{2}\right)^2}{2}}=\left(\left(\sin^{-1}\frac{x}{2}\right)^2-2\right)e^{\frac{\left(\sin^{-1}\frac{x}{2}\right)^2}{2}}+C$

$\therefore y(0)=\frac{\pi^2-8}{4}$

$\frac{\pi^2-8}{4}=-2+C$

$C=\frac{\pi^2}{4}$

$ye^{\frac{\left(\sin^{-1}\frac{x}{2}\right)^2}{2}}=\left(\left(\sin^{-1}\frac{x}{2}\right)^2-2\right)e^{\frac{\left(\sin^{-1}\frac{x}{2}\right)^2}{2}}+\frac{\pi^2}{4}$

Put x =2

$y(2)=\left(\left(\frac{\pi^2-8}{4}\right)e^{\frac{\pi^2}{8}}+\frac{\pi^2}{4}\right)e^{-\frac{\pi^2}{8}}$

$\Rightarrow y(2)=\left(\frac{\pi^2-8}{4}\right)+\frac{\pi^2}{4}\cdot e^{-\frac{\pi^2}{8}}$

18. A hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ cuts parabola $y^2=2x$ at two points A and B. If P is point on $2x+3y=4$ then locus of centroid of ΔPAB is

(1) $9x+6y=28$ (2) $6x+9y=28$ (3) $9x-6y=28$ (4) $6x-9y=28$

Ans. **(2)**

Sol. hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$

parabola $y^2=2x$ $\therefore \frac{x^2}{9}-\frac{2x}{4}=1$

$2x^2-9x-18=0$

$2x^2-12x+3x-18=0$

$(2x+3)(x-6)=0$

$x=6, -\frac{3}{2}$

$\therefore x=6 \therefore y^2=12 \Rightarrow y \pm 2\sqrt{3}$

$X=-\frac{3}{2}$ $y^2=-3$ (not possible)

$A\left(6, 2\sqrt{3}\right), \quad B\left(6, -2\sqrt{3}\right)$

Let P (α,β) s.t. $2\alpha+3\beta=4$

Centroid of ΔPAB is $\left(\frac{\alpha+6+6}{3},\frac{\beta}{3}\right)=(h,k)$

$\therefore \alpha=3h-12, \beta=3k$

$\therefore$ locus of centroid is

$2(3h-12)+3(3k)=4$

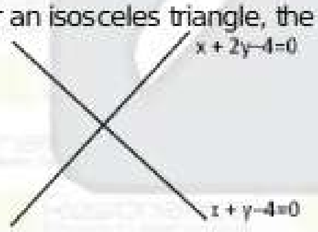
$6x+9y=28$ Ans.

19. In an isosceles triangle two sides are $x+2y=4$ and $x+y=4$, then the sum of all possible values of slope of third side of triangle is

(1) $\frac{3}{2}$ (2) $\frac{2}{3}$ (3) $\frac{-3}{2}$ (4) $\frac{-2}{3}$

Ans. **(2)**

Sol. For an isosceles triangle, the third side is parallel to the angle bisectors of two sides.



Equation of angle bisectors

$$\frac{x+2y-4}{\sqrt{5}}=\pm\frac{x+y-4}{\sqrt{2}}$$

$$L_1=\left(\sqrt{2}-\sqrt{5}\right)x+\left(2\sqrt{2}-\sqrt{5}\right)y-4\sqrt{2}+4\sqrt{5}=0$$

$$L_2=\left(\sqrt{2}+\sqrt{5}\right)x+\left(2\sqrt{2}+\sqrt{5}\right)y-4\sqrt{2}-4\sqrt{5}=0$$

$$\Rightarrow m_1=-\frac{\left(\sqrt{2}-\sqrt{5}\right)}{2\sqrt{2}-\sqrt{5}} \qquad m_2=-\frac{\left(\sqrt{2}+\sqrt{5}\right)}{2\sqrt{2}+\sqrt{5}}$$

$m_1+m_2=\frac{2}{3}$ Ans.