Vedantu

JEE-Main-29-01-2025 (Memory Based)
[EVENING SHIFT]
Maths

Question: If the letters of the word "KANPUR" are arranged in dictionary, then the 440th word is
Options:
(a) PRKAUN
(b) PRKUAN
(c) PRKNAU
(d) PRKUNA
Answer: (a)

A K N P R U
A ---------⇒ 5! ⇒ 120
K ---------⇒ 5! ⇒ 120
N ---------⇒ 5! ⇒ 120
PA --------⇒ 4! ⇒ 2
PK --------⇒ 4! ⇒ 24
PN --------⇒ 4! ⇒ 24
PRA -------⇒ 3! ⇒ 6
PKKANU 2 ⇒ 1

PRRAUN ⇒ 1
440

Question: If 7^{103} is divided by 23, then remainder is
Options:
(a) 11
(b) 12
(c) 14
(d) 16
Answer: (c)

Question: Let $a_{ij} = \left(\sqrt{2}\right)^{t+j}, A = [a_{ij}]_{3\times 3}$ **If sum of third row of A^2 is** $\alpha + \beta\sqrt{2}$**, then $\alpha + \beta$ is**
Answer: (224)

Question: Let $f(x) = \int_0^x t\left(t^2 - 3t + 20\right)dt, x \in (1,3)$ **and range of f(x) is (α, β) then $\alpha + \beta$ is equal**
Options:

(a) $\frac{185}{4}$
(b) $\frac{185}{2}$
(c) $\frac{185}{3}$
(d) $\frac{37}{4}$

Answer: (b)

$$f(x) = \int_0^x t\left(t^2 - 3t + 20\right) dt$$

$$xt(1,3)$$

$$f'(x) = x^3 - 3x^2 + 20x$$

$$f(x) = \frac{x^4}{4} - x^3 + 10x^2$$

$$f'(x) > 0$$

$$f(x)\text{is } xt(1,3) function$$

$$\text{so } f(1) = \frac{37}{4} = \alpha$$

$$f(3) = \frac{333}{4} = \beta$$

$$\therefore \alpha + \beta = \frac{37+333}{4}$$

$$= \frac{185}{4}$$

Question: The value of the limit

$\lim\limits_{x \to 0} (\cos ecx)\left(\sqrt{2\cos^2 x + 3\cos x} - \sqrt{\cos^2 x + \sin x + 4}\right)$ **is**

Options:

(a) 0
(b) 1
(c) $\frac{1}{2\sqrt{5}}$
(d) $-\frac{1}{2\sqrt{5}}$

Answer: (d)

$$\lim_{x\to 0}\frac{1}{\sin x}\left(\sqrt{2\cos^2 x+3\cos x}-\sqrt{\cos^2+\sin x+4}\right)$$

$$\lim_{x\to 0}\frac{1}{\sin x}\left[\frac{2\cos^2 x+3\cos x-\cos^2-\sin x-4}{\sqrt{2\cos^2 x+5\cos x}+\sqrt{\cos^2 x+\sin x+4}}\right]$$

$$\Rightarrow \lim_{x\to 0}\frac{1}{\sin x}\left[\frac{\cos^2 x+3\cos x-\sin x-4}{\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}}\right]$$

$$\Rightarrow \lim_{x\to 0}\left[\frac{-2\cos x\sin x-3\sin x-\cos x}{\sqrt{5}+\sqrt{5}}\right]$$

$$\Rightarrow -\frac{1}{2\sqrt{5}}$$

Question: Let the line L be $\frac{x-1}{1}=\frac{y-4}{3}=\frac{z-7}{3}$ **and foot of perpendicular from (1, -2, -1) to L is (α, β, γ) then** $\alpha+\beta+\gamma$ **is**

Options:

(a) $-\frac{69}{35}$

(b) $\frac{102}{35}$

(c) $\frac{69}{35}$

(d) $-\frac{102}{35}$

Answer: (d)

$$A(A+1, 3\lambda+4, 5\lambda+7)$$

$$P(1,-2,-1),\ DR=(1,3,5)$$

$$\overrightarrow{AP}.\ (DR)\ =\ 0$$

$$\lambda.1\ +3(3N+6)\ +\ (5\lambda\ +2)\,5=0$$

$$\lambda=\frac{-58}{35}$$

$$A\Rightarrow\ \left(-\frac{58}{35}+1, 3\left(-\frac{58}{35}\right)+4, 5\left(\frac{-58}{35}\right)\right)$$

$$\Rightarrow\left(\frac{23}{35},\frac{-34}{35},\frac{-9}{7}\right)$$

$$\alpha+\beta+\gamma\ =\ \frac{23}{35}\ \frac{-34}{35}\ \frac{-9}{7}\ \Rightarrow\ \frac{-102}{35}$$

Question: If the exhaustive value of a for which the equation $2x^2 + (a - 5)x + 15 = 3a$ has no real root (α, β), then $|4(\alpha+\beta)|$ is equal to

Options:

(a) 56

(b) 52

(c) 54

(d) 18

Answer: (a)

Question: Area enclosed between the curves |y| = 1 - x^2 and $x^2 + y^2 = 1$ is (π - α)sq. units, then 9α is

Options:

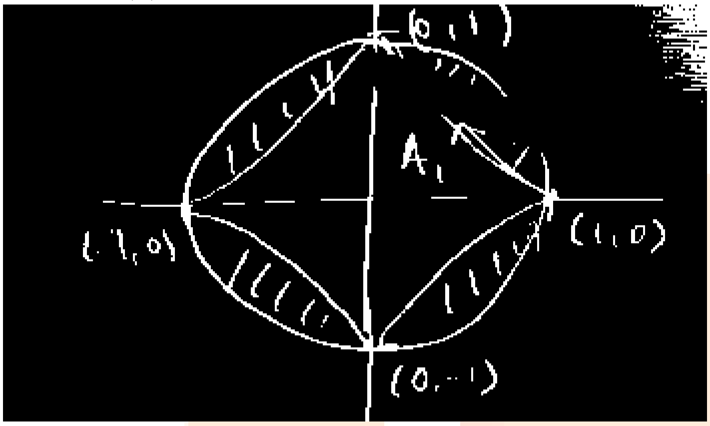
(a) 8

(b) 16

(c) 32

(d) 24

Answer: (d)



$|y| = 1 - x^2$

$x^2 + y^2 = 1$

$$\text{Required Area} = 4 \times \left(\frac{\pi}{4} - \int_0^1 \left(1 - x^2\right) dx\right)$$

$= \pi - 4\left[x - \frac{x^3}{3}\right]_0^1$

$\Rightarrow \pi - 4\left[1 - \frac{1}{3}\right]$

$\Rightarrow \pi - \frac{8}{3}$

$\alpha = \frac{8}{3}$

So $9\alpha = 9 \times \frac{8}{5} = 24$

Question: If $$\log y = x \log \frac{2}{5}, x \in N \cup \{0\}.$$ **Then sum of all values of y equals to**

Options:

(a) $\frac{5}{3}$

(b) $\frac{2}{3}$

(c) $\frac{5}{4}$

(d) $\frac{8}{3}$

Answer: (a)

$\log y = x\log\left(\frac{2}{5}\right), n \in NU\{0\}$

$y = \left(\frac{2}{5}\right)^x$

$y - 1, \frac{2}{5}, \left(\frac{2}{5}\right)^2, \ldots..$

$y = \frac{1}{1-\frac{2}{5}}$

$\Rightarrow \frac{5}{3}$

Question: There is an arithmetic progression $a_1, a_2, a_3, \ldots. a_{2024}$ and $a_1 + (a_5 + a_{10} + a_{15} \ldots a_{2020}) + a_{2024} = 2233$. Find the value of $a_1 + a_2 + a_3 + \ldots a_{2024}$.

Options:

(a) 11034

(b) 11132

(c) 10432

(d) 20462

Answer: (b)

$a_1, a_2, a_3 \ldots. a_{2024}$

$a_1 + a_{2024} = a_2 + a_{2023} = \ldots\ldots. = l$

$\therefore a_1 + (a_5 + a_{10} + a_{15} + \ldots\ldots.. a_{2020}) + a_{2024}$

$total = 1202$

$\therefore l + 202l = 2233$

$l = \frac{2233}{203}$

$\therefore a_1 + a_2 + a_3 \ldots\ldots a_{2024} \Rightarrow \frac{2024}{2} \times l$

$\Rightarrow \frac{2024}{2} \times \frac{2233}{203}$

$\Rightarrow 11132$

Question: Two points (4, 2) and (0, 2) lie on the circle whose centre lies on 3x + 2y + 2 = 0, then length of chord whose mid-point is (1, 2) is

Options:

(a) $\sqrt{3}$

(b) $\sqrt{5}$

(c) $2\sqrt{3}$

(d) $2\sqrt{5}$

Answer: (c)

Question: If α, β are the values of m where

x + y + 2z = 1

x + 2y + 4z = m

x + 4y + 8z = m^2 have infinitely many solutions.

Then $\sum_{n=1}^{10}\left(n^{\alpha}+n^{\beta}\right)$ is equal to

Answer: (440)

Question: The value of $\int_0^{\frac{\pi}{4}}\left(\sin\left|\left(4x-\frac{\pi}{2}\right)\right|+\sin[2x]\right)dx$ **is (where [.] denotes the greatest integer function)**

Options:

(a) $\frac{1}{2}+\left(\frac{\pi-2}{4}\right)\sin 1$

(b) $\frac{1}{4}+\left(\frac{\pi-2}{2}\right)\sin 1$

(c) $\frac{1}{2}-\left(\frac{\pi-2}{4}\right)\sin 1$

(d) $\frac{1}{4}-\left(\frac{\pi-2}{2}\right)\sin 1$

Answer: (a)