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JEE (MAIN) 2025

MEMORY BASED QUESTIONS & TEXT SOLUTION

SHIFT-2

DATE & DAY: 29th January 2025 & Wednesday

PAPER-1

Duration: 3 Hrs.
Time: 03:00 PM – 06:00 PM

SUBJECT: MATHEMATICS

Selections in JEE (Advanced)/ IIT-JEE Since 2002	Selections in JEE (Main)/ AIEEE Since 2009	Selections in NEET (UG)/ AIPMT/AIIMS Since 2012
52395	257576	22494

Admission Open for 2025-26

Target: JEE (Advanced) | JEE (Main) | NEET (UG) | PCCP (Class V to X)

100% Scholarship on the basis of Class 10th & 12th & JEE (Main) 2025 %ile/ AIR

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PART : MATHEMATICS

1. If all the words formed by all the letters of word 'KANPUR' are arranged alphabetically, then 440^{th} word is ....

(1) PRKANU (2) PRKAUN (3) PRKUAN (4) PRKNUA

Ans. **(2)**

Sol. Firstly we arrange the word KANPUR alphabetically.

AKNPRU

A_ _ _ _ _ = 5! = 120

K_ _ _ _ _ = 5! = 120

N_ _ _ _ _ = 5! = 120

PA _ _ _ _ = 4! = 24

PK _ _ _ _ = 4! = 24

PN _ _ _ _ = 4! = 24

PRA _ _ _ = 3! = 6

PRKANU = 1

PRKAUN = 1

$120 \times 3 + 24 \times 3 + 6 + 1 + 1 = 440$

2. Let $a_{ij} = (\sqrt{2})^{i+j}$, $A = [a_{ij}]_{3\times3}$. If sum of third row of A^2 is $\alpha + \beta\sqrt{2}$ then $\alpha + \beta$ is

Ans. **(224)**

Sol. $a = \sqrt{2}$

$$A = \begin{bmatrix} a^2 & a^3 & a^4 \\ a^3 & a^4 & a^5 \\ a^4 & a^5 & a^6 \end{bmatrix} \begin{bmatrix} a^2 & a^3 & a^4 \\ a^3 & a^4 & a^5 \\ a^4 & a^5 & a^6 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} & & \\ & & \\ a^6 + a^8 + a^{10} & a^7 + a^9 + a^{11} & a^8 + a^{10} + a^{12} \end{bmatrix}$$

$\text{Sum} = a^6 + a^7 + 2a^8 + a^9 + 2a^{10} + a^{11} + a^{12}$

$= 4(14 + 14\sqrt{2} + 28)$

$= 168 + 56\sqrt{2} = \alpha + \beta\sqrt{2}$

$\alpha = 168, \beta = 56$

$\alpha + \beta = 224$

3. $\lim\limits_{x\to0} (\text{cosec } x) \left(\sqrt{2\cos^2 x + 3\cos x} - \sqrt{\cos^2 x + \sin x + 4}\right)$ is

(1) $\frac{1}{2\sqrt{2}}$ (2) $\frac{1}{2\sqrt{5}}$

(3) $-\frac{1}{2\sqrt{5}}$ (4) $-\frac{1}{2\sqrt{2}}$

Ans. **(3)**

Sol. $\lim_{x \to 0} \frac{(\sqrt{2\cos^2 x + 3\cos x} - \sqrt{\cos^2 x + \sin x + 4})}{\sin x(\sqrt{2\cos^2 x + 3\cos x} - \sqrt{\cos^2 x + \sin x + 4})}$

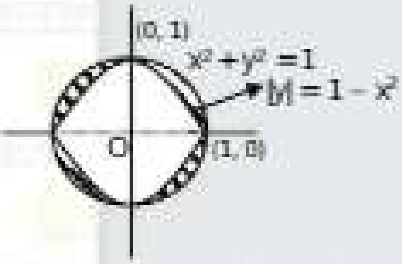
$= \lim_{x \to 0} \frac{\cos^2 x + 3\cos x - \sin x - 4}{\sin x(\sqrt{5} + \sqrt{5})}$

$= \frac{1}{2\sqrt{5}} \lim_{x \to 0} \frac{-2\cos x \sin x - 3\sin x - \cos x}{\cos x} = \frac{1}{2\sqrt{5}}\left(\frac{-1}{1}\right) = \frac{-1}{2\sqrt{5}}$

4. Let the area bounded by the curves $|y| = 1 - x^2$, $x^2 + y^2 = 1$ is α. If $9\alpha = \beta\pi - \gamma$, then, $|\beta - \gamma|$ is –

Ans. **(15)**

Sol.



Required area $= 4\left(\frac{\pi\pi^2}{4} - \int_0^1 (1 - x^2)dx\right)$

$= \quad \pi_0 1^2 - 4\left[x - x^{3/2}\right]_0^1$

$= \quad \pi - 4\left[1 - \frac{1}{3}\right]$

$\alpha = \pi - \frac{8}{3}$

$9\alpha = 9\pi - 24 \quad \Rightarrow \beta = 9, \gamma = 24$

$|\beta - \gamma| = |9 - 24| = 15.$

5. The remainder when 7^{103} is divided by 23 is:

Ans. **(14)**

Sol. $7^{103} = 7(7^2)^{51} = 7(46 + 3)^{51}$

$= 23a + 7.3^{51}$

$= 23a + 7.27^{17}$

$= 23a + 7(23 + 4)^{17}$

$= 23b + 7 \times 4^{17}$

$= 23b + 7(4^3)^5 \times 4^2$

$= 23b + 7 \times 16 \times (69 - 5)^5$

$= 23b + 7 \times 16 \times (-5)^5$

$= 23b + (115 - 3)(-5)^5$

$= 23g + (-3)(-5)^5$

$= 23g + 3.5(25)^2$

$= 23d + 15 \times 2^2$

$= 23d + 60 = 23d + 46 + 14$

So remainder is 14.

6. $f(x) = \int_0^x t(t^2 - 9t + 20)dt$; $x \in [1, 5]$ and range of f(x) is $[\alpha, \beta]$ then $4(\alpha + \beta) =$

(1) 157 (2) 159 (3) 29 (4) 16

Ans. **(1)**

Sol. $f'(x) = x(x^2 - 9x + 20) = 0 \Rightarrow x = 0, 4, 5$

$$f(x) = \frac{x^4}{4} - 3x^2 + 10x^2$$

$$f(x) = x^2\left(\frac{x^2}{4} - 3x + 10\right)$$

$$f(1) = 1\left(\frac{1}{4} - 3 + 10\right) = \frac{29}{4}$$

$$F(4) = 16(4 - 12 + 10) = 32$$

$$f(5) = 25\left(\frac{24}{4} - 15 + 10\right) = \frac{125}{4}$$

Range $y \in \left[\frac{29}{4}, 32\right]$

$4(\alpha + \beta) = 29 + 128 = 157$

7. If mid point of chord of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is $\left(\frac{2}{5}, \frac{1}{2}\right)$, then the equation of chord is –

(1) $160x + 450y = 289$ (2) $160x + 450y = 161$

(3) $80x + 225y = 289$ (4) $80x + 225y = 161$

Ans. **(1)**

Sol. $T = S_1$

$$\frac{x(2/5)}{9} + \frac{y(1/2)}{4} = \frac{(2/5)^2}{9} + \frac{(1/2)^2}{4}$$

$$\frac{2x}{45} + \frac{y}{8} = \frac{4}{25 \times 9} + \frac{1}{16}$$

$160x + 450y = 289$

8. If $a_1, a_2, a_3, \ldots\ldots\ldots a_{2024}$ are in A.P. such that $a_1 + (a_5 + a_{10} + a_{15} + \ldots\ldots + a_{2020}) + a_{2024} = 2233$ then the value of $a_1 + a_2 + a_3 \ldots\ldots\ldots\ldots + a_{2024}$ is :

Ans. **11132**

Sol. $\because a_1 + a_{2024} = a_5 + a_{2020} = a_{10} + a_{2015} = \ldots\ldots\ldots\ldots$

$\therefore a_1 + (a_5 + a_{10} + a_{15} + \ldots\ldots\ldots + a_{2020}) + a_{2024} = 2233$

$202[a_1 + a_{2024}] + [a_1 + a_{2024}] = 2233$

$203[a_1 + a_{2024}] = 2233$

$[a_1 + a_{2024}] = 11$

Now $a_1 + a_2 + \ldots\ldots a_{2024} = \frac{2024}{2} \times (a_1 + a_{2024})$

$= 1012 \times 11$

$= 11132$

9. If the exhaustive values of a for which the equation $2x^2 + (a-5)x + 15 = 3a$ has no real root is (α, β) then $|4\alpha + \beta|$ is equal to

(1) 75 (2) 71 (3) 69 (4) 42

Ans. **(2)**

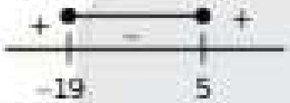
Sol. $D < 0$

$(a-5)^2 - 4 \times 2 \times (15 - 3a) < 0$

$(a-5)^2 - 8 \times 3 \times (5 - a) < 0$

$(a-5)[a - 5 + 24] < 0$

$(a-5)(a+19) < 0$



$a \in (-19, 5) = (\alpha, \beta)$

$\alpha = -19$

$\beta = 5$

Now, $|4\alpha + \beta|$

$= |-76 + 5|$

$= 71$

10. If $\log y = x \log \frac{2}{5}$, $x \in N \cup \{0\}$. Then sum of all values of y equals to

(1) $\frac{5}{3}$ (2) $\frac{2}{3}$ (3) $\frac{5}{4}$ (4) $\frac{8}{3}$

Ans. **(1)**

Sol. $\log y = \log \left(\frac{2}{5}\right)^x$

$\Rightarrow y = \left(\frac{2}{5}\right)^x$

$x = 0, 1, 2, 3, \ldots\ldots \infty$

$\Sigma y = \left(\frac{2}{5}\right)^0 + \left(\frac{2}{5}\right)^1 + \left(\frac{2}{5}\right)^2 + \ldots\ldots \infty$

$= \frac{1}{1 - \frac{2}{5}} = \frac{5}{3}$

11. A circle passes through (0,2) and (4,2). The line $3x + 2y + 6 = 0$ passes through centre of the circle. Find the length of shortest chord passes through (1, 2).

Ans. $2PR = 2\sqrt{3}$

Sol. Let centre of the circle C(a,b)

Circle passes through A(0,2) and B(4,2)

AC = BC

$\sqrt{(a-0)^2 + (b-2)^2} = \sqrt{(a-4)^2 + (b-2)^2}$

$\Rightarrow \quad a^2 + (b-2)^2 = a^2 - 8a + (b-2)^2 + 16$

$a = 2$

Line $3x + 4y + 6 = 0$ is passing through C(a,b)

$3a + 4b + 6 = 0$

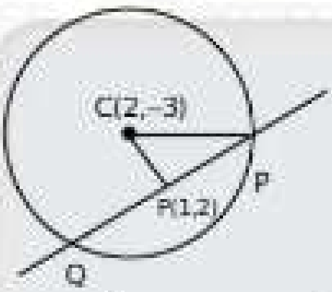
$6 + 4b + 6 = 0$

$b = -3$

$C(2, -3)$

$CR = \sqrt{(2-1)^2 + (-3-2)^2}$

$CR = \sqrt{1+25} = \sqrt{26}$



Radius $AC = r = \sqrt{(2-0)^2 + (-3-2)}$

$r = \sqrt{29}$

From right angle ΔPCR

$CP^2 = CR^2 + PR^2$

$29 = 26 + PR^2$

$3 = PR^2$

$PR = \sqrt{3}$

So, length of the chord with mid-point (1, 2) = 2PR = $2\sqrt{3}$.

12. If Domain of $y = \log_3 (18x - x^2 - 77)$ is (α, β) and Domain of $y = \log_{(x-1)} \left[\frac{(2x+1)(x-2)}{(x+2)(x-1)}\right]$ is (γ, δ) then $\alpha^2 + \beta^2 + \gamma^2 = ?$

Ans. **(174)**

Sol. $18x - x^2 - 77 > 0$

$x^2 - 18x + 77 < 0$

$x^2 - 11x - 7x + 77 < 0$

$(x - 11)(x - 7) < 0$

$x \in (7, 11)$

Now, $\frac{(2x+1)(x-2)}{(x+2)(x-1)} > 0$; $x - 1 > 0$, $x \neq 2$

$x \in (2, \infty)$

$x \in (7, 11) = (\alpha, \beta)$

$x \in (2, \infty) = (\gamma, \delta)$ then

$\alpha = 7, \beta = 11, \gamma = 2, \delta \to \infty$

$49 + 121 + 4 = 174$

13. If $x + y + z = 1$; $x + 2y + 4z = m$ & $x + 4y + 10z = m^2$ have infinitely many solutions and m take 2 values α & β then find $\sum_{r=1}^{10}\left((r)^{\alpha} + (r)^{\beta}\right)$

Ans. (440)

Sol. For infinitely many solutions $\Delta = 0$, $\Delta_x = 0$, $\Delta_y = 0$, $\Delta_z = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 3 \\ 1 & 3 & 9 \end{vmatrix}$$

$= 1 \times [1 \times 6 - 2 \times 3] = 0$

$$D_z = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & m \\ 1 & 4 & m^2 \end{vmatrix} = 0$$

$1(2m^2 - 4m) - 1(m^2 - m) + 1(4 - 2) = 0$

$m^2 - 3m + 2 = 0$

$(m - 1)(m - 2) = 0$

$m = 1, 2$

$$D_x = \begin{vmatrix} 1 & 1 & 1 \\ m & 2 & 4 \\ m^2 & 4 & 10 \end{vmatrix} = 0 \quad \Rightarrow m = 1, 2$$

$$D_y = \begin{vmatrix} 1 & 1 & 1 \\ 1 & m & 4 \\ 1 & m^2 & 10 \end{vmatrix} = 0 \Rightarrow m = 1, 2$$

$$\sum_{r=1}^{10}\left((r)^1 + (r)^2\right)$$

$$= \frac{n(n+1)}{2} + \frac{n(n+1)(2n+1)}{6} = 55 + 385 = 440$$

14. There are two bags B_1 and B_2 such that B_1 contains 4 white and 5 black balls and B_2 contains "n" white and 3 black balls. One ball is transferred from bag B_1 to B_2 and now one ball is randomly drawn from bag B_2, the probability of the drawn ball being white is $\frac{29}{45}$ then "n" is equals to:

Ans. (6)

Sol. P (white ball) $= \left(\frac{n+1}{n+4}\right)\left(\frac{4}{9}\right) + \left(\frac{5}{9}\right)\left(\frac{n}{n+4}\right) = \frac{29}{45}$

$\frac{4}{9}\left(\frac{n+1}{n+4}\right) + \frac{5n}{9(n+4)} = \frac{29}{45}$

$\frac{4(n+1) + 5n}{9(n+4)} = \frac{29}{45}$ ....(1)

$(9n + 4) \,.\, 45 = 29.9(n + 4)$

$405n + 180 = 261n + 1044$

$(405 - 261)n = 864$

$144\, n = 864$

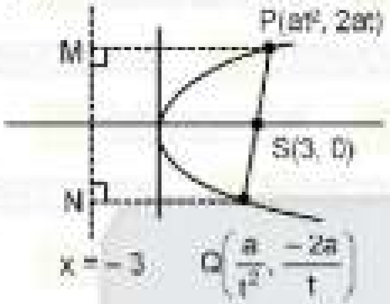
$n = \frac{864}{144}$

$n = 6.$

15. S is the focus and PQ is focal chord of parabola $y^2 = 12x$ and $(PS)(QS) = \frac{147}{4}$, then the equation of circle for which PQ is diameter

Ans. (0)

Sol.



Now,

$PS = at^2 + 3 = 3t^2 + 3$

$QS = 3 + \frac{a}{t^2} = 3 + \frac{3}{t^2}$

$PS.QS = \frac{147}{4} \Rightarrow 3(t^2 + 1)\left(1 + \frac{1}{t^2}\right) = \frac{49}{4}$

$12t^4 - 25t^2 + 12 = 0$

$t^2 = \frac{4}{3}$ or $t^2 = \frac{3}{4}$

If $t = \frac{2}{\sqrt{3}} \Rightarrow -\frac{1}{t} = -\frac{\sqrt{3}}{2}$

If $t = -\frac{2}{\sqrt{3}} \Rightarrow -\frac{1}{t} = \frac{\sqrt{3}}{2}$

If $t = \frac{\sqrt{3}}{2} \Rightarrow -\frac{1}{t} = \frac{-2}{\sqrt{3}}$

If $t = -\frac{\sqrt{3}}{2} \Rightarrow -\frac{1}{t} = \frac{2}{\sqrt{3}}$

Thus, if $\left(t, -\frac{1}{t}\right) = \left(\frac{2}{\sqrt{3}}, -\frac{\sqrt{3}}{2}\right)$

Equation of circle having PQ as diameter.

$(x - at^2)\left(x - \frac{a}{t^2}\right) + (y - 2at)\left(y + \frac{2a}{t}\right) = 0$

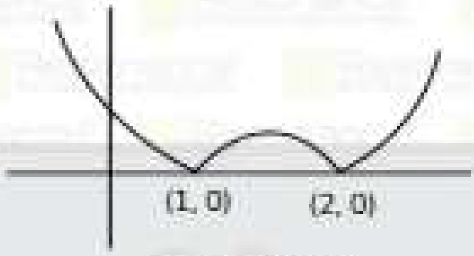
$(x - 4)\left(x - \frac{9}{4}\right) + (y - 4\sqrt{3})(y + 3\sqrt{3})$

$4x^2 + 4y^2 - 25x - 4\sqrt{3}y - 108 = 0$

16. Let $f(x) = (x^2 + 1)\,|x^2 - ax + 2| + \cos|x|$. If $f(x)$ is non-differentiable at $x = \alpha = 2$ and $x = \beta$. Find the distance of the point (α, β) from the line $12x + 5y + 10 = 0$

Ans. (3)

Sol. $(x^2 + 1)$ and $\cos|x|$ are always differentiable, $g(x) = x^2 - ax + 2$ will be non-differentiable at points where $g(x) = 0$



$y = |x^2 - ax + 2|$

$\Rightarrow g(\alpha = 2) = 0$

$\Rightarrow 4 - 2a + 2 = 0 \Rightarrow a = 3$

$g(x) = x^2 - 3x + 2 = (x - 2)(x - 1)$

$\alpha = 2$ and $\beta = 1$

$(\alpha, \beta) = (2, 1)$

Distance $= \left|\dfrac{12 \times 2 + 5 \times 1 + 10}{\sqrt{12^2 + 5^2}}\right| = \left|\dfrac{39}{13}\right| = 3$