BOARD QUESTION PAPER : JULY 2024
MATHEMATICS AND STATISTICS

Time: 3 Hrs. **Max. Marks: 80**

General instructions:
*The question paper is divided into **FOUR** sections.*
- *(1) **Section A:** Q. 1 contains **Eight** multiple choice type of questions, each carrying **Two** marks. Q. 2 contains **Four** very short answer type questions, each carrying **One** mark.*
- *(2) **Section B:** Q. 3 to Q. 14 contain **Twelve** short answer type questions, each carrying **Two** marks. (Attempt any **Eight**)*
- *(3) **Section C:** Q. 15 to Q. 26 contain **Twelve** short answer type questions, each carrying **Three** marks. (Attempt any **Eight**)*
- *(4) **Section D:** Q. 27 to Q. 34 contain **Eight** long answer type questions, each carrying **Four** marks. (Attempt any **Five**)*
- *(5) Use of log table is allowed. Use of calculator is not allowed.*
- *(6) Figures to the right indicate full marks.*
- *(7) Use of graph paper is not necessary. Only rough sketch of graph is expected.*
- *(8) For each multiple choice type of question, only the first attempt will be considered for evaluation.*
- *(9) Start answer to each section on a new page.*

SECTION – A

Q.1. Select and write the correct answer for the following multiple choice type of questions: **[16]**

i. $\cos\left[\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{2}\right)\right]=$ _______.

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{\pi}{4}$ (2)

ii. If θ is the angle between two vectors $\bar{a}$ and $\bar{b}$ and $|\bar{a}\cdot\bar{b}|=|\bar{a}\times\bar{b}|$ then θ is equal to _______.

(A) 0 (B) $\frac{\pi}{4}$ or $\frac{3\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π or $\frac{\pi}{6}$ (2)

iii. The angle between the lines $\bar{r}=(\hat{i}+2\hat{j}-3\hat{k})+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$ and $\bar{r}=(5\hat{i}-2\hat{j}+7\hat{k})+\mu(\hat{i}+2\hat{j}+2\hat{k})$ is _______.

(A) $\cos^{-1}\left(\frac{17}{21}\right)$ (B) $\cos^{-1}\left(\frac{20}{21}\right)$ (C) $\cos^{-1}\left(\frac{18}{21}\right)$ (D) $\cos^{-1}\left(\frac{19}{21}\right)$ (2)

iv. The perpendicular distance of the plane $\bar{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})=5$, from the origin is _______.

(A) $\frac{5}{\sqrt{14}}$ units (B) $\frac{5}{14}$ units (C) 5 units (D) $\frac{\sqrt{14}}{5}$ units (2)

v. If $x=e^{\frac{x}{y}}$ then $\frac{dy}{dx}=$ _______.

(A) $1-\frac{y}{x}$ (B) $1+\frac{y}{x}$ (C) $\frac{x-y}{x\log x}$ (D) $\frac{x+y}{x\log x}$ (2)

vi. $y=c^2+\frac{c}{x}$ is solution of _______.

(A) $x^4\left(\frac{dy}{dx}\right)^2-x\frac{dy}{dx}-y=0$ (B) $x^2\left(\frac{dy}{dx}\right)^2+y=0$

(C) $x^3\left(\frac{d^2y}{dx^2}\right)-x\frac{dy}{dx}+y=0$ (D) $x\frac{d^2y}{dx^2}=4y$ (2)

vii. Given that $X \sim B(n, p)$. If $n = 10$ and $p = 0.4$ then $E(X)$ and $Var(X)$ respectively are ________.
(A) 4, 0.24 (B) 0.4, 0.24 (C) 4, 2.4 (D) 3, 0.24 (2)

viii. The approximate value of $\tan(44°30')$, given that $1° = 0.0175^c$, is ________.
(A) 0.8952 (B) 0.9528 (C) 0.9285 (D) 0.9825 (2)

Q.2. Answer the following questions: **[4]**

i. Find the combined equation of the pair of lines $2x + y = 0$ and $3x - y = 0$ (1)

ii. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ (1)

iii. Evaluate: $\int \frac{5^x}{3^x} dx$ (1)

iv. Write the integrating factor (I.F.) of the differential equation $\frac{dy}{dx} + y = e^{-x}$. (1)

SECTION – B

Attempt any EIGHT of the following questions: **[16]**

Q.3. If the statements p, q are true statements and r, s are false statements, then determine the truth value of the statement pattern:
$(q \wedge r) \vee (\sim p \wedge s)$ (2)

Q.4. Find the inverse of matrix A by elementary row transformations, where $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ (2)

Q.5. Find the polar co-ordinates of the point whose Cartesian co-ordinates are $(1, -\sqrt{3})$. (2)

Q.6. Find the acute angle between the lines represented by $xy + y^2 = 0$ (2)

Q.7. Using the truth table, show that the statement pattern $p \to (q \to p)$ is a tautology. (2)

Q.8. If $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$ then find the value of x, where $0 < 3x < 1$. (2)

Q.9. Find the points on the curve given by $y = x^3 - 6x^2 + x + 3$ where the tangents are parallel to the line $y = x + 5$. (2)

Q.10. Evaluate: $\int \frac{e^x(1+x)dx}{\sin^2(xe^x)}$. (2)

Q.11. The displacement of a particle at a time t is given by $s = 2t^3 - 5t^2 + 4t - 3$. Find the time when acceleration is 14 ft/sec^2. (2)

Q.12. Evaluate: $\int_0^{\pi/4} \sqrt{1 + \sin 2x}\, dx$ (2)

Q.13. The probability distribution of X is as follows:

$X = x$	0	1	2	3	4
$P(X = x)$	0.1	k	2k	2k	k

Find (a) k
(b) $P(X < 2)$ (2)

Q.14. Find the particular solution of:
$r\frac{dr}{d\theta} + \cos\theta = 5$ at $r = \sqrt{2}$ and $\theta = 0$ (2)

SECTION – C

Attempt any EIGHT of the following questions: **[24]**

Q.15. In ΔABC, if a cos A = b cos B then prove that the triangle is either a right angled or an isosceles triangle. (3)

Q.16. Are the four points A(1, –1, 1), B(–1, 1, 1), C(1, 1, 1) and D(2, –3, 4) co-planar? Justify your answer. (3)

Q.17. Find the difference between the slopes of the lines given by $(\tan^2\theta + \cos^2\theta)x^2 - 2xy\tan\theta + (\sin^2\theta)y^2 = 0$ (3)

Q.18. Find the vector equation of the line passing through the point $\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ and perpendicular to the vectors $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}-\hat{j}+\hat{k}$. (3)

Q.19. Let $\bar{a}$ and $\bar{b}$ be non-collinear vectors. If vector $\bar{r}$ is co-planar with $\bar{a}$ and $\bar{b}$ then prove that there exists unique scalars t_1 and t_2 such that $\bar{r} = t_1\bar{a} + t_2\bar{b}$. Hence find t_1 and t_2 for $\bar{r} = \hat{i}+\hat{j}, \bar{a} = 2\hat{i}-\hat{j}, \bar{b} = \hat{i}-2\hat{j}$. (3)

Q.20. Find the equation of the plane passing through the intersection of the planes $x + 2y + 3z + 4 = 0$ and $4x + 3y + 2z + 1 = 0$ and the origin. (3)

Q.21. Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\sqrt{\frac{3-x}{3+x}}\right)$ (3)

Q.22. Find the approximate value of $f(x) = x^3 + 5x^2 - 2x + 3$ at $x = 1.98$. (3)

Q.23. Evaluate: $\int \frac{\sin(x+a)}{\cos(x-b)}\,dx$ (3)

Q.24. Solve the differential equation $dr + (2r\cot\theta + \sin 2\theta)\,d\theta = 0$ (3)

Q.25. Let X ~ B(10, 0.2). Find

(a) $P(X = 1)$

(b) $P(X \geq 1)$ (3)

Q.26. Find the expected value, variance and standard deviation of r.v. X whose p.m.f. is given as:

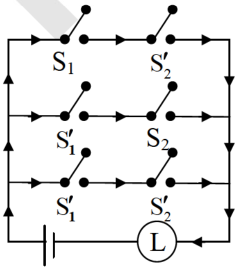
$X = x$	1	2	3
P(X)	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

(3)

SECTION – D

Attempt any FIVE of the following questions: **[20]**

Q.27. Give an alternative arrangement for the following circuit, so that the new circuit has minimum switches:



(4)

Q.28. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ then find A^{-1} by Adjoint method. (4)

Q.29. In ΔABC, D and E are points on BC and AC respectively such that BD = 2DC and AE = 3EC. Let P be the point of intersection of AD and BE. Find ratio $\frac{BP}{PE}$ using the vector method. (4)

Q.30. A firm manufactures two products A and B on which profit earned per unit are ₹3 and ₹4 respectively. Each product is processed on two machines M_1 and M_2. The product A requires one minute of processing time on M_1 and two minutes on M_2, while product B requires one minute on M_1 and one minute on M_2.
Machine M_1 is available for use not more than 450 minutes, while M_2 is available for 600 minutes during any working day.
Find the number of units of products A and B to be manufactured to get maximum profit. (4)

Q.31. If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is differentiable function of x such that the composite function $y = f[g(x)]$ is a differentiable function of x then prove that: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.

Hence find $\frac{d}{dx}\left(\frac{1}{\sqrt{\sin x}}\right)$ (4)

Q.32. Evaluate: $\int x^2 \sin 3x dx$ (4)

Q.33. Prove that:

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx, \quad \text{if f is an even function}$$

$$= 0, \quad \text{if f is an odd function}$$

Hence find the value of $\int_{-1}^{1} \tan^{-1}x \, dx$ (4)

Q.34. Find the area of the ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Hence write area of $\frac{x^2}{25} + \frac{y^2}{16} = 1$ (4)