

SECTION A – 65 MARKS

Question 1

In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.

- (i) Let L be a set of all straight lines in a plane. The relation R on L defined as 'perpendicular to' is: [1]

- (a) Symmetric and Transitive
- (b) Transitive
- (c) Symmetric
- (d) Equivalence

- (ii) The order and the degree of differential equation $1 + \left(\frac{dy}{dx}\right)^2 = \frac{d^2y}{dx^2}$ are: [1]

- (a) 2 and $\frac{3}{2}$
- (b) 2 and 3
- (c) 3 and 4
- (d) 2 and 1

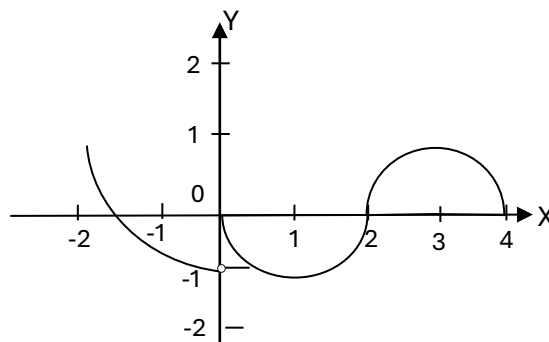
- (iii) Let A be a non-empty set. [1]

Statement 1: Identity relation on A is Reflexive.

Statement 2: Every Reflexive relation on A is an Identity relation.

- (a) Both the statements are true.
- (b) Both the statements are false.
- (c) Statement 1 is true and Statement 2 is false.
- (d) Statement 1 is false and Statement 2 is true.

- (iv) The graph of the function f is shown below. [1]

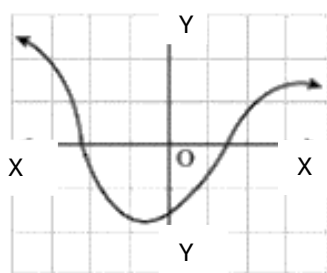


Of the following options, at what values of x is the function f **NOT** differentiable?

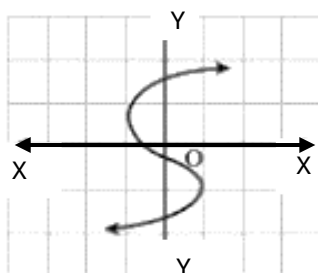
- (a) At $x = 0$ and $x = 2$
- (b) At $x = 1$ and $x = 3$

- (c) At $x = -1$ and $x = 1$
- (d) At $x = -1.5$ and $x = 1.5$
- (v) The value of $\operatorname{cosec}\left(\sin^{-1}\left(\frac{-1}{2}\right)\right) - \sec\left(\cos^{-1}\left(\frac{-1}{2}\right)\right)$ is equal to: [1]
- (a) -4
- (b) 0
- (c) -1
- (d) 4
- (vi) The value of $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ is: [1]
- (a) $\frac{\pi}{2}$
- (b) $\frac{2\pi}{3}$
- (c) $\frac{\pi}{6}$
- (d) $\frac{\pi}{12}$
- (vii) **Assertion:** Let the matrices $A = \begin{pmatrix} -3 & 2 \\ -5 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -2 \\ 5 & -3 \end{pmatrix}$ be such that [1]
 $A^{100}B = BA^{100}$
Reason: $AB = BA$ implies $A^nB = BA^n$ for all positive integers n .
- (a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.
- (b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.
- (c) Assertion is true and Reason is false.
- (d) Assertion is false and Reason is true.
- (viii) If $\int (\cot x - \operatorname{cosec}^2 x) e^x dx = e^x f(x) + c$ then $f(x)$ will be: [1]
- (a) $\cot x + \operatorname{cosec} x$
- (b) $\cot^2 x$
- (c) $\cot x$
- (d) $\operatorname{cosec} x$
- (ix) In which one of the following intervals is the function $f(x) = x^3 - 12x$ increasing? [1]
- (a) $(-2, 2)$
- (b) $(-\infty, -2) \cup (2, \infty)$
- (c) $(-2, \infty)$
- (d) $(-\infty, 2)$
- (x) If A and B are symmetric matrices of the same order, then $AB - BA$ is: [1]
- (a) Skew – symmetric matrix

- (b) Symmetric matrix
- (c) Diagonal matrix
- (d) Identity matrix
- (xi) Find the derivative of $y = \log x + \frac{1}{x}$ with respect to x . [1]
- (xii) Teena is practicing for an upcoming Rifle Shooting tournament. The probability of her shooting the target in the 1st, 2nd, 3rd and 4th shots are 0.4, 0.3, 0.2 and 0.1 respectively. Find the probability of at least one shot of Teena hitting the target. [1]
- (xiii) Which one of the following graphs is a function of x ? [1]



Graph A



Graph B

- (xiv) Evaluate: $\int_0^6 |x + 3| dx$ [1]
- (xv) Given that $\frac{1}{y} + \frac{1}{x} = \frac{1}{12}$ and y decreases at a rate of 1 cm s^{-1} , find the rate of change of x when $x = 5 \text{ cm}$ and $y = 1 \text{ cm}$ [1]

Comments of Examiners

- (i) Some candidates had confusion between symmetric and transitive relations.
- (ii) The question was found to be easy by most of the candidates and was answered through direct observation.
- (iii) Few candidates misread the options (c) and (d).
- (iv) Many candidates marked the incorrect option due to a lack of understanding of the concept of differentiability check of the function at a point graphically.
- (v) Very few candidates marked this question incorrectly.
- (vi) Most of the candidates answered this question correctly.
- (vii) A large number of candidates marked this question incorrectly.
- (viii) Many candidates answered this question correctly.
- (ix) The majority of candidates marked it correctly.
- (x) Most of the candidates were not able to answer this part correctly.
- (xi) Several candidates marked this question correctly.
- (xii) A large number of candidates failed to comprehend the implication of at least one shot.
- (xiii) Almost all candidates failed to answer correctly due to a lack of knowledge of the vertical test on graphs to identify a curve as a function.
- (xiv) The majority of candidates attempted this part correctly. However, a few candidates made simplification errors.
- (xv) Almost all the candidates responded to this question correctly.

Suggestions for teachers

- Revise all properties of a relation on a defined Set.
- Discuss the concept with the help of extensive examples.
- Revise all relations on a set and provide good practice of different types of questions.
- Discuss numerous examples how to check differentiability of a function at a point graphically.
- Understand and revise all the formulae of inverse trigonometric functions and how to apply it while solving problems.
- Revise all formulae of special integrals.
- Give sufficient practice of different types of questions involving matrix multiplication.
- Provide rigorous practice of solving different types of questions based on integration.
- Make sure to provide sufficient practice of finding intervals in which the function increases/decreases with the help of the sign of the derivative of the function.
- Discuss all properties of symmetric and skew symmetric matrices and their applications.
- Revise the standard functions differentiation.
- Emphasise more on drawing inferences from the vocabulary of probability and understanding the questions based on independent events.
- Revise the concepts and definition of a function and concept of the vertical line test.
- Give sufficient practice of problems involving modulus functions.
- Encourage students to solve different types of questions based on the rate of measure.

MARKING SCHEME

Question 1

(i)	(c) or Symmetric
(ii)	(d) or 2 and 1
(iii)	(c) or Statement 1 is true and Statement 2 is false.
(iv)	(a) or At $x = 0$ and $x = 2$
(v)	(b) or 0
(vi)	(d) or $\frac{\pi}{12}$
(vii)	(a) or Both Assertion and Reason are true and Reason is the correct explanation for Assertion.
(viii)	(c) or $\cot x$
(ix)	(b) or $(-\infty, -2) \cup (2, \infty)$
(x)	(a) or Skew – symmetric matrix
(xi)	$\frac{1}{x} - \frac{1}{x^2}$
(xii)	$0.6976 : \frac{436}{625} = 0.697 = 0.7$
(xiii)	Graph A
(xiv)	36
(xv)	25 cms^{-1}

Question 2

[2]

- (i) Let $f: R - \left\{\frac{-1}{3}\right\} \rightarrow R - \{0\}$ be defined as $f(x) = \frac{5}{3x+1}$ is invertible. Find $f^{-1}(x)$

OR

- (ii) If $f: R \rightarrow R$ is defined by $f(x) = \frac{2x-7}{4}$, show that $f(x)$ is one - one and onto.

Comments of Examiners

- (i) Almost all candidate calculated $f^{-1}(y)$ instead of $f^{-1}(x)$.
 (ii) The majority of candidates, after writing $x = (4y + 7)/2$ and claiming it to be onto function, did so without justification.

Suggestions for teachers

- Provide rigorous practice that after finding $f^{-1}(y)$ to find $f^{-1}(x)$, y need to be replaced by x .
- Emphasise identifying whether the function is onto, with reasoning and justification.

MARKING SCHEME

Question 2

(i)

$$y = \frac{5}{3x+1}$$

$$x = \frac{5-y}{3y}$$

$$f^{-1}(x) = \frac{5-x}{3x}; x \neq 0$$

OR

(ii)

$$f(x_1) = f(x_2) \Rightarrow \frac{2x_1 - 7}{4} = \frac{2x_2 - 7}{4}$$

$$\Rightarrow x_1 = x_2$$

f is 1 - 1

$$x = \frac{4y + 7}{2}$$

f is onto function ($\forall y \in R$ there exist a pre-image i.e $x \in R$)

Question 3

[2]

Find the value of the determinant given below, without expanding it at any stage.

$$\begin{vmatrix} \beta\gamma & 1 & \alpha(\beta + \gamma) \\ \gamma\alpha & 1 & \beta(\gamma + \alpha) \\ \alpha\beta & 1 & \gamma(\alpha + \beta) \end{vmatrix}$$

Comments of Examiners

Some candidates committed errors while applying properties determinants and ignored the condition “without expanding at any stage.”

Suggestions for teachers

- Teach all properties of determinants with the help of suitable examples and their applications.
- Emphasise the application of relevant properties, not any property.

MARKING SCHEME

Question 3

$$C_3 \rightarrow C_3 + C_1$$

$$\begin{vmatrix} \beta\gamma & 1 & \alpha\beta + \gamma\alpha + \beta\gamma \\ \gamma\alpha & 1 & \beta\gamma + \beta\alpha + \gamma\alpha \\ \alpha\beta & 1 & \gamma\alpha + \gamma\beta + \alpha\beta \end{vmatrix}$$

Take common $\alpha\beta + \gamma\alpha + \beta\gamma$ from C_3

$$\alpha\beta + \gamma\alpha + \beta\gamma \begin{vmatrix} \beta\gamma & 1 & 1 \\ \gamma\alpha & 1 & 1 \\ \alpha\beta & 1 & 1 \end{vmatrix} = 0 \text{ (as } C_2 \text{ and } C_3 \text{ are identical)}$$

Question 4

[2]

- (i) Determine the value of k for which the following function is continuous at $x = 3$.

$$f(x) = \begin{cases} \frac{(x+3)^2 - 36}{x-3} & ; x \neq 3 \\ k & ; x = 3 \end{cases}$$

OR

- (ii) Find a point on the curve $y = (x - 2)^2$ at which the tangent is parallel to the line joining the chord through the points (2, 0) and (4, 4).

Comments of Examiners

- (i) Majority of the candidates answered this part correctly, and only a few of them were unable to evaluate the limit of the function.

OR

- (ii) Some candidates were unable to equate slope of tangent (dy/dx), with the slope of the chord.

Suggestions for teachers

- Provide practice on questions based on continuity and differentiability.
- Explain thoroughly the geometrical significance of dy/dx .
- Give lots of practice on questions based on the applications of the geometrical significance of dy/dx .

MARKING SCHEME

Question 4

- (i) As $f(x)$ is continuous at $x = 3$
 $\therefore \lim_{x \rightarrow 3} f(x) = f(3)$
 $\lim_{x \rightarrow 3} \frac{(x-3)(x+9)}{(x-3)} = k$
 $\lim_{x \rightarrow 3} (x+9) = k$
 $k = 12$

OR

- (ii) Given curve $y = (x - 2)^2$
 On difference between we have

$$\frac{dy}{dx} = 2(x - 2)$$

 Slope of the chord joining points (2,0) and (4,4).

$$m = \frac{4-0}{4-2} = \frac{4}{2} = 2$$

 According to tangent parallel to the cord joining the points

$$\frac{dy}{dx} = \text{Slope of the chord}$$

$$\Rightarrow 2(x - 2) = 2$$

$$\Rightarrow x - 2 = 1$$

$$\Rightarrow x = 3$$

$$y = (3 - 2)^2 = 1$$

Question 5

[2]

Evaluate: $\int_0^{2\pi} \frac{1}{1+e^{\sin x}} dx$

Comments of Examiners

A number of candidates were able to answer this question correctly. However, many candidates failed to recognise that it is a question based on the properties of definite integration.

Suggestions for teachers

- Revise all properties of definite integrals.
- Provide ample practice in solving questions on each property.

MARKING SCHEME

Question 5

$$\begin{aligned}
 I &= \int_0^{2\pi} \frac{1}{1+e^{\sin x}} dx \\
 &= \int_0^{2\pi} \frac{1}{1+e^{\sin(2\pi-x)}} dx \\
 &= \int_0^{2\pi} \frac{1}{1+e^{-\sin x}} dx = \int_0^{2\pi} \frac{e^{\sin x}}{1+e^{\sin x}} dx \\
 2I &= \int_0^{2\pi} \frac{1}{1+e^{\sin x}} dx + \int_0^{2\pi} \frac{e^{\sin x}}{1+e^{\sin x}} dx \\
 &= \int_0^{2\pi} \frac{1+e^{\sin x}}{1+e^{\sin x}} dx = \int_0^{2\pi} dx = (2\pi - 0) \\
 I &= \pi
 \end{aligned}$$

Question 6

[2]

Evaluate: $P(A \cup B)$ if $2P(A) = P(B) = \frac{5}{13}$ and $P(A|B) = \frac{2}{5}$

Comments of Examiners

Most of the candidates attempted this question correctly. A few candidates made errors due to lack of understanding of the concept of the conditional probability.

Suggestions for teachers

- Revise all concepts of probability.
- Provide practice of solving a good number of problems.

MARKING SCHEME

Question 6

$$P(A) = \frac{5}{26}, P(B) = \frac{5}{13}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = \frac{2}{5} \times \frac{5}{13} = \frac{2}{13} = 0.15$$

$$P(A \cup B) = \frac{5}{26} + \frac{5}{13} - \frac{2}{13} = \frac{11}{26}$$

Question 7

[4]

If $y = 3 \cos(\log x) + 4 \sin(\log x)$, show that $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$

Comments of Examiners

Many candidates made errors in applying the chain rule correctly, especially in the second order differentiation. Also, the candidates made simplification errors.

Suggestions for teachers

- Explain successive differentiation and their applications by solving a good number of problems.
- Provide practice to students on a variety of problems.

MARKING SCHEME

Question 7

$$y = 3 \cos(\log x) + 4 \sin(\log x)$$

$$\frac{dy}{dx} = -3 \sin(\log x) \cdot \frac{1}{x} + 4 \cos(\log x) \cdot \frac{1}{x}$$

$$x \frac{dy}{dx} = -3 \sin(\log x) + 4 \cos(\log x)$$

$$xy_2 + y_1 = -3 \cos(\log x) \cdot \frac{1}{x} - 4 \sin(\log x) \cdot \frac{1}{x} = -y \frac{1}{x}$$

$$x^2 y_2 + x y_1 + y = 0$$

Question 8

[4]

(i) Solve for x : $\sin^{-1}\left(\frac{x}{2}\right) + \cos^{-1} x = \frac{\pi}{6}$

OR

(ii) If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, show that $x^2 - y^2 - z^2 + 2yz\sqrt{1-x^2} = 0$

Comments of Examiners

- (i) The majority of the candidates knew the identities of inverse trigonometric functions but made mistakes in algebraic simplifications. However, most of the candidates attempted the problem using lengthy calculations but were unable to solve it further.

OR

- (ii) While answering this part, some candidates had taken incorrect combination of terms, hence failed to prove the result.

Suggestions for teachers

- Revise all properties and formulae of inverse trigonometric functions.
- Provide practice of solving variety of problems.
- Ensure students understand on applying the most appropriate property.
- Emphasise more on sum/difference of two or more terms and adequate practice of solving questions.

MARKING SCHEME

Question 8

(i) $\sin^{-1} \frac{x}{2} = \frac{\pi}{6} - \cos^{-1} x$
 $\frac{x}{2} = \sin\left(\frac{\pi}{6} - \cos^{-1} x\right)$
 $\frac{x}{2} = \frac{x}{2} - \frac{\sqrt{3}}{2} \sin[(\sin^{-1} \sqrt{1-x^2})]$ or $\cos^{-1} x = \sin^{-1} \sqrt{1-x^2}$
 $\frac{x}{2} = \frac{x}{2} - \frac{\sqrt{3}}{2} \sqrt{1-x^2}$
 $x = \pm 1$
 $x = 1$ is the only solution.

OR

(ii) $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$
 $\sin^{-1} x + \sin^{-1} y = \pi - \sin^{-1} z$
 $\sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}) = \pi - \sin^{-1} z$
 $x\sqrt{1-y^2} + y\sqrt{1-x^2} = \sin(\pi - \sin^{-1} z) = \sin(\sin^{-1} z) = z$
 $x\sqrt{1-y^2} = z - y\sqrt{1-x^2}$
 Squaring on both sides,
 $x^2(1-y^2) = z^2 - 2yz\sqrt{1-x^2} + y^2(1-x^2)$
 $x^2 - y^2 - z^2 + 2yz\sqrt{1-x^2} = 0$

Question 9

[4]

- (i) Evaluate: $\int x^2 \cos x \, dx$

OR

- (ii) Evaluate: $\int \frac{x+7}{x^2+4x+7} \, dx$

Comments of Examiners

- (i) Many candidates successfully attempted this part. However, some candidates made incorrect selections of the first and second functions while applying integration by parts, and thus failed in solving the question correctly.

OR

- (ii) Most of the candidates attempted it correctly. Some candidates failed to split the integral into two parts correctly, while others were unable to express the second part in the correct form of special integral.

Suggestions for teachers

- Provide rigorous practice of solving questions based on the concept of integration by parts and learn to identify the first and second functions and importance of the same while applying integration by parts.
- Teach all standard methods of integration including all types of special integrals.
- Encourage students to learn multiple techniques to solve indefinite integrals of different types.

MARKING SCHEME

Question 9

- | | |
|-----|---|
| (i) | <p>Applying Integration by parts</p> $I = x^2 \sin x - \int 2x \sin x \, dx$ <p>Applying integration by parts again</p> $I = x^2 \sin x + 2x \cos x - 2 \sin x + c$ |
|-----|---|

OR

- | | |
|------|---|
| (ii) | $\int \frac{x+7}{x^2+4x+7} \, dx = \int \frac{x+2+5}{x^2+4x+7} \, dx$ $= \int \frac{x+2}{x^2+4x+7} \, dx + \int \frac{5}{x^2+4x+7} \, dx$ <p>$I = I_1 + I_2$</p> <p>$I_1 = \int \frac{x+2}{x^2+4x+7} \, dx$ let $x^2+4x+7 = t$, $2(x+2) = \frac{dt}{dx}$</p> $(x+2)dx = \frac{1}{2} dt$ $I_1 = \int \frac{1}{t} \cdot \frac{1}{2} dt = \frac{1}{2} \int \frac{1}{t} dt = \frac{1}{2} \log t = \frac{1}{2} \log x^2+4x+7 $ $I_2 = \int \frac{5}{x^2+4x+7} \, dx = 5 \int \frac{1}{(x+2)^2+3} \, dx$ |
|------|---|

$$= 5 \int \frac{1}{(x+2)^2 + (\sqrt{3})^2} dx = \frac{5}{\sqrt{3}} \tan^{-1} \left(\frac{x+2}{\sqrt{3}} \right)$$

$$I = \frac{1}{2} \log|x^2 + 4x + 7| + \frac{5}{\sqrt{3}} \tan^{-1} \left(\frac{x+2}{\sqrt{3}} \right) + C$$

[4]

Question 10

A jewellery seller has precious gems in white and red colour which he has put in three boxes. The distribution of these gems is shown in the table given below:

Box	Number of Gems	
	White	Red
I	1	2
II	2	3
III	3	1

He wants to gift two gems to his mother. So, he asks her to select one box at random and pick out any two gems one after the other without replacement from the selected box. The mother selects one white and one red gem.

Calculate the probability that the gems drawn are from Box II.

Comments of Examiners

Many candidates were unable to recognise that the two gems were drawn one after the other without replacement from the selected box, hence calculated probabilities incorrectly. However, some candidates applied total probability theorem but did not apply Baye's theorem thereafter.

Suggestions for teachers

- Explain Baye's theorem with proper details along with illustrations.
- Teach extensively with examples the concept of required probability of a specific known event to the sum of the conditional probabilities of all events under consideration.
- Explain and provide good practice of examples on the concept of drawing one after the other without replacement and with replacement.

MARKING SCHEME

Question 10

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

$$P\left(\frac{A}{E_1}\right) = \frac{2}{3} ; P\left(\frac{A}{E_2}\right) = \frac{3}{5} ; P\left(\frac{A}{E_3}\right) = \frac{1}{2}$$

$$P\left(\frac{E_2}{A}\right) = \frac{\frac{1}{3} \times \frac{3}{5}}{\frac{1}{3} \times 2 + \frac{1}{3} \times \frac{3}{5} + \frac{1}{3} \times \frac{1}{2}}$$

$$= \frac{18}{53} = 0.34$$

Question 11

[6]

A furniture factory uses three types of wood namely, teakwood, rosewood and satinwood for manufacturing three types of furniture, that are, table, chair and cot.

The wood requirements (in tonnes) for each type of furniture are given below:

	Table	Chair	Cot
Teakwood	2	3	4
Rosewood	1	1	2
Satinwood	3	2	1

It is found that 29 tonnes of teakwood, 13 tonnes of rosewood and 16 tonnes of satinwood are available to make all three types of furniture.

Using the above information, answer the following questions:

- Express the data given in the table above in the form of a set of simultaneous equations.
- Solve the set of simultaneous equations formed in subpart (i) by matrix method.
- Hence, find the number of table(s), chair(s) and cot(s) produced.

Comments of Examiners

Most of the candidates attempted and solved it correctly. However, few candidates were unable to form the system of equations in three variables. In addition to this, a very few candidates made computational errors in the process of calculating inverse matrix.

Suggestions for teachers

- Give good number of questions where students have to form the system of equations in three variables in linear form.
- Build good understanding and practice of identifying the minors, cofactors and adjoint matrix.
- Provide illustrations while explaining the concept of inverse matrix and its properties.

MARKING SCHEME

Question 11

(i)

Let x be the number of tables made. y be the number of chairs made. z be the number of cots made.

According to this, the set of Equations are:

$$2x + 3y + 4z = 29$$

$$x + y + 2z = 13$$

$$3x + 2y + z = 16$$

(ii)

By Matrix rule, we have

$$A \cdot X = B$$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix} ; X = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} ; B = \begin{bmatrix} 29 \\ 13 \\ 16 \end{bmatrix}$$

$$X = A^{-1} B$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$|A| = 2(1 - 4) - 3(1 - 6) + 4(2 - 3)$$

$$= 2(-3) - 3(-5) + 4(-1)$$

$$= -5 + 15 - 4 = -10 + 15 = 5 \neq 0$$

$$\therefore A^{-1} \text{ exists}$$

Let A_{ij} be the cofactor matrix

$$A_{ij} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$A_{11} = -3 ; A_{12} = +5 ; A_{13} = -1$$

$$A_{21} = +5 ; A_{22} = -10 ; A_{23} = +5$$

$$A_{31} = 2 ; A_{32} = 0 ; A_{33} = -1$$

$$\text{adj } A = [A_{ij}]' = \begin{bmatrix} -3 & 5 & -1 \\ 5 & -10 & 5 \\ 2 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -3 & 5 & 2 \\ 5 & -10 & 0 \\ -1 & 5 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

Any six cofactors

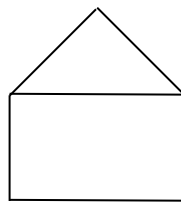
$$= \frac{1}{5} \begin{bmatrix} -3 & 5 & 2 \\ 5 & -10 & 0 \\ -1 & 5 & -1 \end{bmatrix}$$

	Now, $X = A^{-1} B$ $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{5} \begin{bmatrix} -3 & 5 & 2 \\ 5 & -10 & 0 \\ -1 & 5 & -1 \end{bmatrix} \begin{bmatrix} 29 \\ 13 \\ 16 \end{bmatrix}$ $= \begin{bmatrix} \frac{87}{5} + 13 + \frac{32}{5} \\ 29 - 26 + 0 \\ -\frac{29}{5} + 13 - \frac{16}{5} \end{bmatrix} = \begin{bmatrix} -11 + 13 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ $\therefore x = 2 ; y = 3 ; z = 4$
(iii)	$\therefore$ Number of Table = 2 Number of Chair = 3 Number of Cot = 4

Question 12

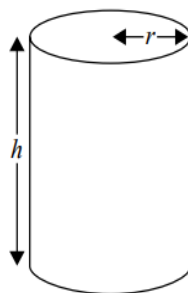
[6]

- (i) Mrs. Roy designs a window in her son's study room so that the room gets maximum sunlight. She designs the window in the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the window that will admit maximum sunlight into the room.



OR

- (ii) Sumit has bought a closed cylindrical dustbin. The radius of the dustbin is ' r ' cm and height is ' h ' cm. It has a volume of $20\pi \text{ cm}^3$.



- Express h in terms of ' r ', using the given volume.
- Prove that the total surface area of the dustbin is $2\pi r^2 + \frac{40\pi}{r}$
- Sumit wants to paint the dustbin. The cost of painting the base and top of the dustbin is ₹ 2 per cm^2 and the cost of painting the curved side is ₹ 25 per cm^2 . Find the total cost in terms of ' r ', for painting the outer surface of the dustbin including the base and top.

- (d) Calculate the minimum cost for painting the dustbin.

Comments of Examiners

- (i) A large number of candidates were unable to express the area of the window in terms of one variable. A few candidates used incorrect formula of area of equilateral triangle. Few more candidates failed to simplify the function obtained in terms of one variable. Some candidates were unable to complete the second order derivative test.

OR

- (ii) Some of the candidates failed to form the required function. However, some more candidates were using incorrect formula of volume of cylinder/total surface area of cylinder. Many candidates failed to simplify the cost function obtained in variable. In most cases, the second order differentiation test was ignored.

Suggestions for teachers

- Ensure that students are familiar with mensuration formulae.
- Emphasise on writing the function in terms of one variable with the help of given information.
- Provide rigorous practice to identify the given information and function to be maximised/minimised.
- Encourage students to learn the second order differentiation test and its importance in applications.
- Familiarise students with different types of shapes and interpretations.
- Provide rigorous practice of solving problems (application type).
- Focus students' attention to learn the second order differentiation test and its importance in applications.

MARKING SCHEME

Question 12

- (i) $P = 12\text{m}$ (Perimeter of window)
- $$2x + 3y = 12$$
- $$2x = 12 - 3y$$
- $$x = 6 - \frac{3}{2}y$$
- $\therefore$ $A = \text{Area of rectangle} + \text{Area of equilateral Triangle}$
- $$= xy + \frac{\sqrt{3}}{4}y^2$$
- $$= (6 - \frac{3}{2}y)y - \frac{\sqrt{3}}{4}y^2$$
- $$A = 6y - \frac{3}{2}y^2 + \frac{\sqrt{3}}{4}y^2$$
- $$\frac{dA}{dy} = 6 - 3y + \frac{\sqrt{3}}{2}y$$
- $$\frac{dA}{dy} = 0$$
- $$y\left(\frac{\sqrt{3}}{2} - 3\right) = -6$$
- $$y = -6 \left[\frac{2}{\sqrt{3}-6} \right] = \frac{12}{6-\sqrt{3}}$$

		$\frac{d^2A}{dy^2} = -3 + \frac{\sqrt{3}}{2} < 0$ $\therefore \text{Area is maximum}$ $\therefore x = 6 - \frac{3}{2} \times \frac{12}{6-\sqrt{3}}$ $= \frac{36-6\sqrt{3}-18}{6-\sqrt{3}}$ $= \frac{18-6\sqrt{3}}{6-\sqrt{3}} = 1.78$ $\therefore x = \frac{18-6\sqrt{3}}{6-\sqrt{3}}$ $Y = \frac{12}{6-\sqrt{3}} = 2.81$
OR		
(ii)	(a)	$\pi r^2 h = 20\pi$ $h = \frac{20}{r^2}$
	(b)	$\text{TSA} = 2\pi r^2 + 2\pi r h$ $= 2\pi r^2 + \frac{40\pi}{r}$
	(c)	$C = 2(2\pi r^2) + 25(2\pi r h)$ $= 4\pi r^2 + \frac{1000\pi}{r}$
	(d)	$\frac{dC}{dr} = 8\pi r - \frac{1000\pi}{r^2}$ $\frac{dC}{dr} = 0 \Rightarrow r = 5$ $\frac{d^2C}{dr^2} \Big _{r=5} > 0$ $C_{\min} = \text{Rs. } 300\pi$

Question 13

[6]

- (i) Solve the following differential equation:

$$2ye^{\frac{x}{y}} dx + \left(y - 2xe^{\frac{x}{y}}\right) dy = 0, \text{ given } x = 0 \text{ and } y = 1$$

OR

- (ii) Solve the following differential equation:

$$x(x^2 - 1) \frac{dy}{dx} = 1, y = 0, \text{ given } x = 2$$

Comments of Examiners

- (i) Very few candidates attempted this part correctly. Most of the candidates were unable to identify the homogenous differential equation. Some of the candidates, after converting into standard expression, applied incorrect methods of integration and failed to get required result. Many candidates did not obtain a particular solution.
- (ii) Majority of the candidates attempted it correctly. Few candidates failed to integrate correctly RHS of the equation. Many candidates did not obtain a particular solution.

Suggestions for teachers

- Illustrate all types of differential equations and give adequate practice.
- Emphasise on methods of identifying the types of the differential equations.
- Provide more practice in the areas like integration by substitution and other standard formats of integrals.
- Illustrate all types of differential equations and give adequate practice.
- Emphasise on methods of identifying the types of the differential equations.
- Explain the significance of Constant C in the solution of differential equation.

MARKING SCHEME

Question 13

(i)

Solution:

$$\begin{aligned}
 2ye^{\frac{x}{y}} dx &= \left(2xe^{\frac{x}{y}} - y \right) dy \\
 \Rightarrow \frac{dx}{dy} &= \frac{2xe^{\frac{x}{y}} - y}{2ye^{\frac{x}{y}}} \\
 x &= vy \Rightarrow \frac{x}{y} = v \\
 \frac{dx}{dy} &= v + y \frac{dv}{dy} \\
 v + y \frac{dv}{dy} &= \frac{2ve^v - 1}{2e^v} \\
 \Rightarrow y \frac{dv}{dy} &= \frac{2ve^v - 1 - 2ve^v}{2e^v} \\
 \int 2e^v dv &= \int -\frac{1}{y} dy \\
 \Rightarrow 2e^v &= -\log|y| + C \\
 2e^{\frac{x}{y}} + \log|y| &= C \\
 \text{at } x = 0 \text{ and } y = 1, \text{ the value of } C &= 2 \\
 \text{So, the solution is: } 2e^{\frac{x}{y}} + \log|y| &= 2
 \end{aligned}$$

OR

(ii)

Given Differential Equation is

$$x(x^2 - 1) \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{x(x-1)(x+1)}$$

$$\int dy = \int \frac{dx}{x(x-1)(x+1)}$$

By Integrating factor;

$$\frac{1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$$1 = A(x-1)(x+1) + B(x)(x+1) + C(x)(x-1)$$

When; $x = 1$;

$$2B = 1$$

$$B = \frac{1}{2}$$

$$1 = C(-1)((-2))$$

$$C = \frac{1}{2}$$

On comparing the coefficient of x^2 termWe have $A + B + C = 0$

$$A + \frac{1}{2} + \frac{1}{2} = 0$$

$$A + 1 = 0$$

$$A = -1$$

Now, we have

$$\int \frac{dx}{x(x-1)(x+1)}$$

$$y = \int \left[\frac{-1}{x} + \frac{\frac{1}{2}}{x-1} + \frac{\frac{1}{2}}{x+1} \right] dx$$

$$y = -\int \frac{1}{x} dx + \frac{1}{2} \int \frac{dx}{x-1} + \frac{1}{2} \int \frac{dx}{x+1}$$

$$y = -\log|x| + \frac{1}{2}\log|x-1| + \frac{1}{2}\log|x+1| + C$$

According to When $x = 2$; $y = 0$

$$C = -\log|2| + \frac{1}{2}\log|1| + \frac{1}{2}\log|3| + C$$

$$C = \log 2 - \frac{1}{2}\log 1 - \frac{1}{2}\log 3$$

$$C = \log \frac{2}{\sqrt{3}}$$

$$y = -\log|x| + \frac{1}{2}\log|x-1| + \frac{1}{2}\log|x+1| + \log \frac{2}{\sqrt{3}}$$

OR

$$\int dy = \int \frac{x dx}{x^2(x^2-1)}$$

$$x^2 = t$$

$$x dx = \frac{1}{2} dt$$

$$y = \frac{1}{2} \int \frac{dt}{t(t-1)} \quad \frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1}$$

$$A = -1, B = 1$$

$$y = \frac{1}{2} \int \left[\frac{-1}{t} + \frac{1}{t-1} \right] dt$$

$$y = \frac{1}{2} [\log(t-1) - \log t] + c$$

$$y = \frac{1}{2} [\log(x^2-1) - 2 \log x] + c$$

$$y = \log \frac{(x^2-1)^{\frac{1}{2}}}{x} + c$$

$$x = 2, y = 0$$

$$c = \frac{2}{\sqrt{3}}$$

$$y = \log \frac{(x^2-1)^{\frac{1}{2}}}{x} \times \frac{2}{\sqrt{3}}$$

Question 14

[6]

A primary school teacher wants to teach the concept of 'larger number' to the students of Class II.

To teach this concept, he conducts an activity in his class. He asks the children to select two numbers from a set of numbers given as 2, 3, 4, 5 one after the other without replacement.

All the outcomes of this activity are tabulated in the form of ordered pairs given below:

	2	3	4	5
2	(2,2)	(2,3)	(2,4)	
3	(3,2)	(3,3)		(3,5)
4	(4,2)		(4,4)	(4,5)
5		(5,3)	(5,4)	(5,5)

- Complete the table given above.
- Find the total number of ordered pairs having one larger number.
- Let the random variable X denotes the larger of two numbers in the ordered pair. Now complete the probability distribution table for X given below:

X	3	4	5
$P(X = x)$			

- Find the value of $P(X < 5)$
- Calculate the expected value of the probability distribution.

Comments of Examiners

Many candidates attempted part (i) and (ii) of the question correctly.

Suggestions for teachers

Provide rigorous practice of solving activity-based probability distribution function problems.

MARKING SCHEME

Question 14

(i)	(2,5), (3,4), (4,3), (5,2)				
(ii)	12				
(iii)	X	3	4	5	
	P(X= x)	$\frac{2}{12} = \left(\frac{1}{6}\right)$	$\frac{4}{12} = \left(\frac{1}{3}\right)$	$\frac{6}{12} = \left(\frac{1}{2}\right)$	
(iv)	$P(X < 5) = \frac{6}{12} = \frac{1}{2}$				
(v)	$E(x) = 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{3}\right) + 5\left(\frac{1}{2}\right)$ $= \frac{1}{2} + \frac{4}{3} + \frac{5}{2} = \frac{3+8+15}{6} = \frac{26}{6} = \frac{13}{3}$				

SECTION B – 15 MARKS

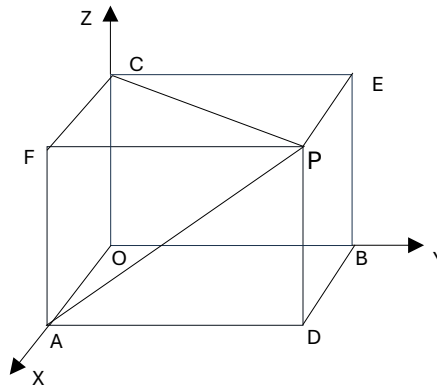
Question 15

[5]

In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

- (i) If $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} - 3\hat{k}$ then the value of $|\vec{a} - 2\vec{b}|$ will be:
- (a) $\sqrt{85}$
- (b) $\sqrt{86}$
- (c) $\sqrt{87}$
- (d) $\sqrt{88}$
- (ii) If a line makes an angle α, β and γ with positive direction of the coordinate axes, then the value of $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$ will be:
- (a) 1
- (b) 3
- (c) -2
- (d) 2

- (iii) In the figure given below, if the coordinates of the point P are (a, b, c), then what are the perpendicular distances of P from XY, YZ and ZX planes respectively?



- (iv) If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 5\hat{i} - 3\hat{j} + \hat{k}$, find the projection of $\vec{b}$ on $\vec{a}$
- (v) Find a vector of magnitude 20 units parallel to the vector $2\hat{i} + 5\hat{j} + 4\hat{k}$.

Comments of Examiners

- (i) Most of the candidates attempted this part correctly.
- (ii) A large number of candidates answered this part correctly. However, a few candidates were unable to identify due to lack of knowledge of direction angles and direction cosines.
- (iii) Most of the candidates attempted this question incorrectly.
- (iv) Some candidates were unable to apply correct formula to find the projection of $\vec{b}$ on $\vec{a}$.
- (v) Majority of the candidates attempted it correctly.

Suggestions for teachers

- Provide rigorous practice of solving problems based on the vector concepts including magnitude of the vector.
- Emphasise on the basic concepts of direction angles/ direction cosines and also direction ratios.
- Develop strong understanding on the geometrical significance of X-coordinate, Y-coordinate and Z-coordinate of a point in the space.
- Emphasise on learning concepts of vector dot product and cross product and geometrical significance of each.
- Provide rigorous practice in understanding concepts like parallel vectors and vectors along a given direction.

MARKING SCHEME

Question 15

(i)	(b) or $\sqrt{86}$
(ii)	(d) or 2
(iii)	(c, a, b)
(iv)	Projection $\vec{b}$ on $\vec{a} = 3$
(v)	$\frac{20}{3\sqrt{5}}(2i + 5j + 4k)$

Question 16

[2]

- (i) If $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are non-zero vectors, then prove that either $\vec{b} = \vec{c}$ or $\vec{a}$ and $(\vec{b} - \vec{c})$ are parallel.

OR

- (ii) If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$, find the angle between $\vec{a}$ and $\vec{b}$.

Comments of Examiners

- (i) Some candidates failed to establish distributive law from the given equation of vectors.

OR

- (ii) Majority of the candidates attempted this part correctly, but some made calculation errors.

Suggestions for teachers

- Emphasize the significance of when the cross product of two vectors is equal to zero.
- Provide rigorous practice in solving problems based on the concepts of the dot product and cross product.

MARKING SCHEME

Question 16

(i)	$(\vec{a} \times \vec{b}) - (\vec{a} \times \vec{c}) = 0$ $\vec{a} \times (\vec{b} - \vec{c}) = 0$ $\vec{a} \parallel (\vec{b} - \vec{c})$ OR $(\vec{b} - \vec{c}) = 0$ $\vec{b} = \vec{c}$
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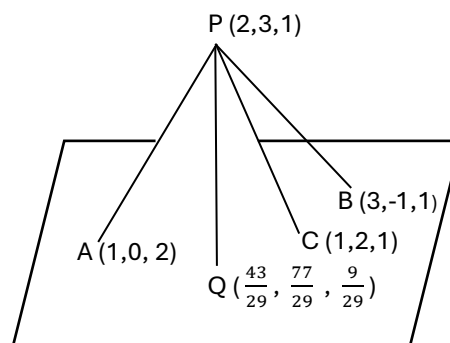
OR

- (ii) Let θ be the angle between $\vec{a}$ and $\vec{b}$
 $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$
 $|\vec{a}||\vec{b}|\sin\theta = |\vec{a}||\vec{b}|\cos\theta$
 $\sin\theta = \cos\theta$
 $\tan\theta = 1 \Rightarrow \theta = 45^\circ$

Question 17

[4]

A mobile tower is situated at the top of a hill. Consider the surface on which the tower stands as a plane having points A(1, 0, 2), B(3, -1, 1) and C(1, 2, 1) on it. The mobile tower is tied with three cables from the points A, B and C such that it stands vertically on the ground. The top of the tower is at point P(2, 3, 1) as shown in the figure below. The foot of the perpendicular from the point P on the plane is at the point Q ($\frac{43}{29}$, $\frac{77}{29}$, $\frac{9}{29}$)



Answer the following questions.

- Find the equation of the plane containing the points A, B and C.
- Find the equation of the line PQ.
- Calculate the height of the tower.

Comments of Examiners

The majority of the candidates attempted the question correctly using 3D geometry, while very few attempted it using the vector method. Some candidates made errors in solving the equations and finding values in terms of A, B, and C using the cross-multiplication rule.

Suggestions for teachers

- Provide practice of finding solutions (equation of a plane) for the different type of questions.
- Discuss all possible conditions that may occur in the questions to find the equation of a plane.

MARKING SCHEME

Question 17

(i)	Equation to the plane is, $\begin{bmatrix} x-1 & y-0 & z-2 \\ 0 & 2 & -1 \\ 2 & -1 & -1 \end{bmatrix} = 0$ $3x + 2y + 4z = 11$
(ii)	Equation to PQ, $\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-1}{4}$
(iii)	Height of the tower = Perpendicular distance of the point (2, 3, 1) to the plane $3x + 2y + 4z - 11 = 0.$ $h = \frac{ 3.2+2.3+4.1-11 }{\sqrt{3^2+2^2+4^2}}$ $= \frac{5}{\sqrt{29}} \text{ unit.}$

Question 18

[4]

- (i) Using integration, find the area bounded by the curve $y^2 = 4ax$ and the line $x = a$

OR

- (ii) Using integration, find the area of the region bounded by the curve $y^2 = 4x$ and $x^2 = 4y$

Comments of Examiners

- (i) A large number of candidates solved this question correctly. However, some candidates did not understand how to find the limits or the region bounded by the curve and the straight line. Many candidates also drew incorrect sketches of the graphs of the curves.

OR

- (ii) Some candidates were unfamiliar with finding the limits or the region bounded by the two curves. Others failed to express the resultant region as the difference of two functions.

Suggestions for teachers

- Provide rigorous practice in drawing the graphs of curves, interpreting and analyzing the curves, identifying the limits of integrals, and shading the region, etc.
- Learn the method of identifying the limits of the bounded region.

MARKING SCHEME

Question 18

(i)

$$y^2 = 4ax$$

$$y = 2\sqrt{ax}$$

$$\text{Area} = 2 \int_0^a 2\sqrt{ax} \, dx$$

$$= \frac{8}{3} a^{1/2} \left[\frac{x^{3/2}}{1} \right]_0^a$$

$$= \frac{8a^2}{3} \text{ Sq unit}$$

OR

(ii)

Given equation of the curve

$$y^2 = 4x \Rightarrow \frac{x^4}{16} = 4x \Rightarrow x^3 = 64 \Rightarrow x^3 = 64 \Rightarrow x=4 \text{ or } y=4$$

$$x^2 = 4y \Rightarrow y = \frac{x^2}{4}$$

$\therefore$ Required area

$$\int_0^4 2\sqrt{x} \, dx - \int_0^4 \frac{x^2}{4} \, dx$$

$$= 2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \int_0^4 - \frac{1}{4} \frac{x^3}{3} \int_0^4 = \frac{4}{3} \cdot 8 - \frac{1}{4} \cdot \frac{64}{3}$$

$$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3} \text{ sq. units.}$$

SECTION C - 15 MARKS

Question 19

[5]

In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

- (i) A company sells hand towels at ₹100 per unit. The fixed cost for the company to manufacture hand towels is ₹35,000 and variable cost is estimated to be 30% of total revenue. What will be the total cost function for manufacturing hand towels?

- (a) $35000 + 3x$
- (b) $35000 + 30x$
- (c) $35000 + 100x$
- (d) $35000 + 10x$

- (ii) If the correlation coefficient of two sets of variables (X, Y) is $\frac{-3}{4}$, which one of the following statements is true for the same set of variables?

- (a) Only one of the two regression lines has a negative coefficient.
- (b) Both regression coefficients are positive.
- (c) Both regression coefficients are negative.
- (d) One of the lines of regression is parallel to the x -axis.
- (iii) If the total cost function is given by $C = x + 2x^3 - \frac{7}{2}x^2$, find the Marginal Average Cost function (MAC).
- (iv) The equations of two lines of regression are $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$. Find the mean value of x and y .
- (v) The manufacturer of a pen fixes its selling price at ₹45, and the cost function is $C(x) = 30x + 240$. The manufacturer will begin to earn profit if he sells more than 16 pens. Why? Give one reason.

Comments of Examiners

- (i) Most of the candidates marked this part correctly. However, few candidates made errors in the process of simplification.
- (ii) Some of the candidates marked the incorrect option.
- (iii) Many candidates attempted this part correctly.
- (iv) Majority of the candidates responded to this question successfully. Very few computational errors made in the process of solving simultaneous equations.
- (v) Many candidates were unable to justify the situation.

Suggestions for teachers

- Emphasize learning the terms and formulae related to cost, revenue, and functions, along with a complete understanding of the concepts.
- Focus on regression coefficients and the relationship between regression coefficients and the correlation coefficient.
- Provide practice with all relevant terms and formulae related to cost functions, ensuring complete understanding.
- Elucidate the concept of finding the mean of variables x & y from the regression equations.
- Explain with examples the terms cost function, revenue function, profit function, breakeven point, and so on.
- Help students to understand the concept and significance of the **breakeven point**.

MARKING SCHEME

Question 19

(i)	(b) or $35000 + 30x$
(ii)	(c) or Both regression coefficients are negative.
(iii)	$C = x + 2x^3 - 3.5x^2$ $AC = 1 + 2x^2 - 3.5x$ $MAC = \frac{dAC}{dx} = 4x - 3.5$

(iv)	$\bar{x} = \frac{-4}{7}, \bar{y} = \frac{-11}{7}$
(v)	At $x = 16$, $R(x) = C(x)$ OR At $x = 16$, no profit and no loss $R(x) > C(x)$ if $x > 16$

Question 20

[2]

- (i) The Average Cost function associated with producing and marketing x units of an item is given by $AC = x + 5 + \frac{36}{x}$
- (a) Find the Total Cost function.
- (b) Find the range of values of x for which Average Cost is increasing.

OR

- (ii) A monopolist's demand function is $x = 60 - \frac{p}{5}$. At what level of output will marginal revenue be zero?

Comments of Examiners

- (i) Many candidates were unable to find the value of x , when average cost is increasing.
- OR**
- (ii) Most of those who attempted this question answered it correctly. Very few candidates made computational errors.

Suggestions for teachers

- Give adequate practice on the conditions for decreasing/increasing functions and the simplification of problems based on these concepts.
- Provide rigorous practice on all formula related to $R(x)$, $AR(x)$, $MR(x)$ etc. and problems based on these concepts.

MARKING SCHEME

Question 20

(i)	(a)	$C(x) = x^2 + 5x + 36$
	(b)	$\frac{d}{dx} \left(x + 5 + \frac{36}{x} \right) > 0 \Rightarrow 1 - \frac{36}{x^2} > 0$ $x^2 - 36 > 0 \Rightarrow (x - 6)(x + 6) > 0$  $x < -6 \text{ or } x > 6$

$\Rightarrow x > 6$ as x is non negative.

OR

- (ii) Demand $x = 60 - \frac{p}{5}$
 $P = 300 - 5x$
 Total revenue $R = px = 300x - 5x^2$
 Marginal Revenue (MR) $= \frac{d}{dx}(300x - 5x^2) = 300 - 10x$
 $MR = 0 \Rightarrow 300 - 10x = 0 \Rightarrow x = 30$

[4]

Question 21

- (i) XYZ company plans to advertise some vacancies. The Manager is asked to suggest the monthly salary for these vacancies based on the years of experience. To do so, the Manager studies the years of service and the monthly salary drawn by the existing employees in the company.

Following is the data that the Manager refers to:

Years of service (X)	11	7	9	5	8	6	10
Monthly salary (in ₹1000) (Y)	10	8	6	5	9	7	11

- (a) Find the regression equation of monthly salary on the years of service.
 (b) If a person with 13 years of experience applies for a job in this company, what monthly salary will be suggested by the Manager?

OR

- (ii) The line of regression of marks in Statistics (X) and marks in Accountancy (Y) for a class of 50 students is $3y - 5x + 180 = 0$. The average score in Accountancy is 44 and the variance of marks in Statistics is $\left(\frac{9}{16}\right)^{th}$ of variance of marks in Accountancy.

- (a) Find the average score in Statistics.
 (b) Find the coefficient of correlation between marks in Statistics and marks in Accountancy.

Comments of Examiners

- (i) Some candidates were confused between the regression coefficients b_{yx} & b_{xy} . Some of the candidates used the incorrect regression coefficient b_{xy} in the regression equation y on x . However, very few candidates lacked knowledge of the properties and the relationship between regression coefficients and correlation coefficients.

OR

- (ii) Most of the candidates attempted it correctly. A few candidates made errors in identifying the regression coefficient b_{xy} from the regression equation x on y . Some candidates did not know the difference between variance and standard deviation.

Suggestions for teachers

- Help students learn the concepts and properties of regression coefficients and correlation coefficients.
- Give practice of using correct regression coefficients in finding regression equation y on x / x on y .
- Encourage rigorous practice of solving different types of problems using regression coefficients.
- Ensure accuracy to be at the highest level.
- Focus students to learn formulae of regression coefficients b_{yx} and b_{xy} correctly using in terms r , σ_x and σ_y .

MARKING SCHEME

Question 21

(i)	(a)		x	y	xy	x^2	y^2
			11	10	110	121	100
			7	8	56	49	64
			9	6	54	81	36
			5	5	25	25	25
			8	9	72	64	81
			6	7	42	36	49
			10	11	110	100	121
			$\Sigma x = 56$	$\Sigma y = 56$	$\Sigma xy = 469$	$\Sigma x^2 = 476$	$\Sigma y^2 = 521$
		$\bar{x} = \frac{56}{7} = 8, \bar{y} = \frac{56}{7} = 8$ $b_{yx} = \frac{7 \times 469 - 56 \times 56}{7 \times 476 - (56)^2} = \frac{147}{196} = \frac{3}{4}$ $y - \bar{y} = b_{yx}(x - \bar{x})$ $\Rightarrow y - 8 = \frac{3}{4}(x - 8)$ $\Rightarrow 4y - 3x = 8$					

	(b)	For $x = 13$, $y = 11.75 = \frac{47}{4}$ Starting pay will be ₹ 11,750
OR		
(ii)	(a)	$3y - 5x + 180 = 0$ $x = \frac{3y}{5} + 36$ $b_{xy} = \frac{3}{5} = r \frac{\sigma_x}{\sigma_y}$ Also, $\frac{\sigma_x}{\sigma_y} = \frac{3}{4}$ (given) from (i) $\frac{3}{5} = r \left(\frac{3}{4} \right)$ $r = 0.8$
	(b)	Since, b_{xy} is (+)ve so r is (+)ve and $\bar{x}, \bar{y}$ lie on the regression line. $\bar{x} = \frac{3}{5} \cdot 44 + 36$ $= 62.4$

Question 22

[4]

Aman has ₹1500 to purchase rice and wheat for his grocery shop. Each sack of rice and wheat costs ₹180 and ₹120 respectively. He can store a maximum number of 10 bags in his shop. He will earn a profit of ₹11 per bag of rice and ₹9 per bag of wheat.

- Formulate a Linear Programming Problem to maximise Aman's profit.
- Calculate the maximum profit.

Comments of Examiners

Majority of the candidates attempted this question correctly. Some candidates were unable to identify the feasible region on the graph due to a lack of knowledge in solving linear inequalities graphically. Other candidates identified incorrect corner points when finding the maximum of the objective function.

Suggestions for teachers

- Support students in solving LPPs that involve knowledge of line sketching and linear inequalities.
- Help in identifying the feasible region with the help of corner points.
- Emphasise rigorous practice in solving different types LPPs.

MARKING SCHEME

Question 22

- (i) Rice: x bags
 Wheat: y bags
 Max $z = 11x + 9y$
 Constraints $x + y \leq 10, 3x + 2y \leq 25, x \geq 0, y \geq 0$

(ii)	Corner points	Profit= $40x + 50y$
	(0,0)	0
	(0,10)	90
	(5,5)	100
	$\left(\frac{25}{3}, 0\right)$	91.67

Max profit = ₹ 100 when 5 bags of rice and 5 bags of wheat are purchased.

Note: For questions having more than one correct answer/solution, alternate correct answers/solutions, apart from those given in the marking scheme, have also been accepted.