

# MATHEMATICS

Test Admission Ticket No.

Question Booklet Version Code
<b>A</b>
<i>(Write this Code on your OMR Answer sheet)</i>

Question Booklet Sr. No.
84681
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1. If  $\vec{u} = \vec{a} - \vec{b}$ ,  $\vec{v} = \vec{a} + \vec{b}$ , and  $|\vec{a}| = |\vec{b}| = 2$ , then  $|\vec{u} \times \vec{v}|$  is

a)  $2\sqrt{16 - (\vec{a} \cdot \vec{b})^2}$

b)  $2\sqrt{4 - (\vec{a} \cdot \vec{b})^2}$

c)  $\sqrt{16 - (\vec{a} \cdot \vec{b})^2}$

d)  $\sqrt{4 - (\vec{a} \cdot \vec{b})^2}$

$$\begin{aligned} |\vec{u} \times \vec{v}| &= |(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})| = 2\vec{a} \times \vec{b} = 2|\vec{a}||\vec{b}|\sin\theta \\ &= 2|\vec{a}||\vec{b}|\sqrt{1 - \cos^2\theta} \\ &= 2\sqrt{|\vec{a}|^2|\vec{b}|^2 - |\vec{a}||\vec{b}|\cos^2\theta} \\ &= 2\sqrt{4 \times 4 - (\vec{a} \cdot \vec{b})^2} \\ &= 2\sqrt{16 - (\vec{a} \cdot \vec{b})^2} \quad \text{(a)} \end{aligned}$$

2. The volume of the tetrahedron formed by the points (1, 1, 1), (2, 1, 3), (3, 2, 2) and (3, 3, 4) in cubic units is

a)  $\frac{5}{6}$

b)  $\frac{6}{5}$

c) 5

d)  $\frac{2}{3}$

$$\vec{AB} = \hat{i} + 2\hat{k}, \quad \vec{AC} = 2\hat{i} + \hat{j} + \hat{k}, \quad \vec{AD} = 2\hat{i} + \hat{j} + 3\hat{k}$$

Volume of the tetrahedron

$$= \frac{1}{6} [\vec{AB} \times \vec{AC} \cdot \vec{AD}] = \frac{5}{6} \quad \text{(a)}$$

3. Unit vector perpendicular to  $\hat{i} - 2\hat{j} + 2\hat{k}$  and lying in the plane containing  $\hat{i} + \hat{j} - 2\hat{k}$  and  $-\hat{i} + 2\hat{j} + \hat{k}$  is

a)  $8\hat{i} - 7\hat{j} + 11\hat{k}$

b)  $8\hat{i} + 7\hat{j} - 11\hat{k}$

c)  $8\hat{i} - 7\hat{j} - 11\hat{k}$

d)  $\frac{1}{\sqrt{234}}(8\hat{i} - 7\hat{j} - 11\hat{k})$

only  $\frac{1}{\sqrt{234}}(8\hat{i} - 7\hat{j} - 11\hat{k})$  is a unit vector  
and  $\perp$  to  $\hat{i} - 2\hat{j} + 2\hat{k}$  (d)

Space for calculation / rough work  
S.W.A.Y.



4. In the group  $(\mathbb{Q} - \{-1\}, \star)$  under the binary operation  $\star$  defined by  $a \star b = a + b + ab$  the inverse of 10 is

a)  $\frac{1}{10}$

b)  $\frac{11}{10}$

c)  $\frac{-11}{10}$

**d)**

$\frac{-10}{11}$

5. In the group  $\{1, 2, 3, 4, 5, 6\}$  under multiplication mod 7,  $2^{-1} \times 4 =$

a) 1

**c)**

b) 4

2

d) 3

6. The group  $(\mathbb{Z}, +)$  has

a) exactly one subgroup

b) only two subgroups

c) no subgroups

**d) infinitely many subgroups.**

d) infinitely many subgroups

Space for calculation / rough work

7. If  $3x \equiv 5 \pmod{7}$ , then

$$3x \equiv 5 \pmod{7}, \text{ then } x = 4$$

a)  $x \equiv 2 \pmod{7}$

Ans :-

$$x \equiv 4 \pmod{7}$$

b)  $x \equiv 3 \pmod{7}$

(C)

c)  $x \equiv 4 \pmod{7}$

d) none of these

8. The argument of the complex number  $\sin\left(\frac{6\pi}{5}\right) + i\left(1 + \cos\frac{6\pi}{5}\right)$  is

a)  $\frac{\pi}{10}$

(C)

b)  $\frac{5\pi}{6}$

c)  $\frac{-\pi}{10}$

d)  $\frac{2\pi}{5}$

9. The maximum value of  $n < 101$  such that  $1 + \sum_{k=1}^n i^k = 0$  is

a) 96

(C)

b) 97

c) 99

d) 100

Space for calculation / rough work

16. The value of  $(-1 + \sqrt{-3})^{62} + (-1 - \sqrt{-3})^{62}$  is

a)  $2^{62}$

b)  $2^{61}$

c)  $-2^{62}$

d) 0

$$2^{62} \left[ \left( \frac{-1 + \sqrt{-3}}{2} \right)^{62} + \left( \frac{-1 - \sqrt{-3}}{2} \right)^{62} \right]$$

$$2^{62} \{ \omega^{62} + \omega^{124} \} = 2^{62} \{ \omega^2 + \omega \} = 2^{62} (-1) = -2^{62}$$

17. All complex numbers  $z$  which satisfy the equation  $\left| \frac{z-6i}{z+6i} \right| = 1$  lie on the

a) imaginary axis

b) real axis

c) neither of the axes

d) none of these

$$\left| \frac{(x+iy)-6i}{x+iy+6i} \right| = 1 \text{ or, } \frac{(x+iy)-6i}{x+iy+6i} \times \frac{x-iy-6i}{x-iy-6i} = 1$$

$$= \frac{(y^2+x^2+12y-36) + i12x}{x^2+(y+6)^2}$$

Solve it

12. The value of  $\sin \left[ \cot^{-1} \left\{ \cos \left( \tan^{-1} x \right) \right\} \right]$  is

a)  $\left( \frac{1-x^2}{\sqrt{2-x^2}} \right)$

b)  $\left( \frac{2+x^2}{\sqrt{1+x^2}} \right)$

c)  $\left( \frac{\sqrt{x^2-2}}{\sqrt{x^2-1}} \right)$

d)  $\left( \frac{x^2-1}{\sqrt{x^2-2}} \right)$

$$= \sin \left[ \cot^{-1} \left\{ \cos \left( \cos^{-1} \frac{1}{\sqrt{1+x^2}} \right) \right\} \right]$$

$$= \sin \left[ \cot^{-1} \left\{ \frac{1}{\sqrt{1+x^2}} \right\} \right]$$

$$= \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$$

Space for calculation / rough work

13. The value of  $\alpha (\neq 0)$  for which the function  $f(x) = 1 + \alpha x$  is the inverse of itself is

- a) -2
- b) 2
- c) -1
- d) 1

(C)

Let  $y = f(x)$ ,  $y = 1 + \alpha x \Rightarrow x = \frac{y-1}{\alpha}$

$f(x)$  is the inverse of it self

$$\frac{x-1}{\alpha} = (1 + \alpha x)$$

$$\text{or, } (\alpha^2 - 1)x + (\alpha + 1) = 0$$

$$(\alpha + 1) \{ \alpha x - x + 1 \} = 0$$

14. If  $x^r$  occurs in the expansion of  $(x + \frac{1}{x})^n$ , then its coefficient is  $\alpha = -1$

- a)  $\frac{n!}{(r!)^2}$
- b)  $\frac{n!}{(r+1)!(r-1)!}$
- c)  $\frac{n!}{\left(\frac{n+r}{2}\right)! \left(\frac{n-r}{2}\right)!}$
- d)  $\frac{n!}{\left[\left(\frac{r}{2}\right)!\right]^2}$

(C)

$k$ th term =  ${}^n C_k x^k \left(\frac{1}{x}\right)^{n-k}$ ; coefficient of  $x^{2k-n}$

Power of  $x$ ;  $x^{2k-n}$   ${}^n C_k$

Let  $x^{2k-n} = x^r$

$$r = 2k - n \Rightarrow k = \frac{n+r}{2}$$

Now  ${}^n C_k = {}^n C_{\frac{n+r}{2}} = \frac{n!}{\frac{n+r}{2}! \cdot \frac{n-r}{2}!}$

15. If  $\tan A - \tan B = x$  and  $\cot B - \cot A = y$  then  $\cot(A-B) =$

- a)  $\frac{1}{y} - \frac{1}{x}$
- b)  $\frac{1}{x} - \frac{1}{y}$
- c)  $\frac{1}{x} + \frac{1}{y}$
- d) none of these

(C)

$$\cot(A-B) = \frac{1 + \tan A \cdot \tan B}{\tan A - \tan B} \quad \text{--- (1)}$$

$$\frac{1}{\tan B} - \frac{1}{\tan A} = \frac{\tan A - \tan B}{\tan A \cdot \tan B} = y \quad \text{--- (2)}$$

Given:  $\tan A - \tan B = x$  --- (3)

Eq (2) + 3  $\div$   $\tan A \cdot \tan B = x/y$ , put this value in eq (1)

$$\cot(A-B) = \frac{1 + x/y}{x} = \frac{1}{x} + \frac{1}{y}$$

Space for calculation / rough work

✓  $\cos^2 \frac{\pi}{12} \cos^2 \frac{\pi}{4} \cos^2 \frac{5\pi}{12} =$

✓  $\frac{3}{2}$

b)  $\frac{3-\sqrt{3}}{2}$

c)  $\frac{2}{3}$

d)  $\frac{2}{3+\sqrt{3}}$

(a)

$\cos^2 15^\circ + \frac{1}{2} + \cos^2 75^\circ$

$= \cos^2 15^\circ \cdot 1 - \sin^2 75^\circ + \frac{1}{2}$

$= \cos^2 15^\circ - \sin^2 75^\circ + \frac{3}{2}$

$= \cos^2 15^\circ - \cos^2 (90^\circ - 15^\circ) + \frac{3}{2}$

$= \frac{3}{2} + \cos^2 15^\circ - \cos^2 15^\circ$

$= \frac{3}{2}$

19. If  
a)  
b)  
c)  
d)  
20. If  
a)  
b)  
c)  
d)  
21. If

17. If  $\sin \theta$ ,  $\cos \theta$ , and  $\tan \theta$  are in GP then  $\cot^2 \theta - \cot^4 \theta$  is

✓ a) 1

b)  $\frac{1}{2}$

c) 2

d) 3

(a)

$\cos^2 \theta = \sin \theta \cdot \tan \theta \Rightarrow \cos^3 \theta = \sin^2 \theta$

or,  $\cos^3 \theta = 1 - \cos^2 \theta$  or,  $\cos^3 \theta + \cos^2 \theta - 1 = 0$

Solve it for  $\theta$  and replace in  $\cot^2 \theta - \cot^4 \theta = \underline{1}$

a)  
b)  
c)  
d)

18. If  $\frac{3x^2 - 2x + 4}{(x-1)^2} = \frac{A_1}{x+1} + \frac{A_2}{(x+1)^2} + \frac{A_3}{(x+1)^3} + \frac{A_4}{(x+1)^4} + \frac{A_5}{(x+1)^5} + \frac{A_6}{(x+1)^6}$ , then

$(-A_1 + A_2 + A_3, A_2 - A_4 - A_6) =$

a) (0, 0)

b) (-8, -12)

c) (8, -12)

✓ d) (-8, 12)

(d)

Put  $x=0$ ,

then  $A_1 + A_2 + A_3 + A_4 + A_5 + A_6 = 4$

only "d" (-8, 12) satisfies

the solution

22. The  
a)  
b)  
c)  
d)

Space for calculation / rough work

19. If  $\log_2(2^{x-1}+6) + \log_2(4^{x-1}) = 5$ , then  $x =$  ;  $\log_2(2^{x-1}+6)(2^{2x-2}) = 5$   
 or,  $(2^{x-1}+6)(2^{2x-2}) = 2^5$ ; Let  $y = 2^{x-1}$   
 Possible  $(y+6)y^2 = 32$  or  $(y-2)(y^2+8y+16) = 0$   
 soln  $y = 2^{x-1} = 2^1 \therefore x-1 = 1$  or,  $x = 2$

- a) 4
- b) 1
- c) 3
- d) 2

20. If  $a, b, c, d$  are the roots of the equation  $x^4 + 2x^3 + 3x^2 + 4x + 5 = 0$ , then  $1+a^2+b^2+c^2+d^2$  is equal to  
 $1+(a^2+b^2+c^2+d^2) = 1+(a+b+c+d)^2 - 2(ab+ac+ad+bc+bd+cd)$   
 $= 1 + (\text{Sum of roots})^2 - 2(\text{Sum of multiplication of roots})$   
 $= 1 + 2^2 - 2 \times 3 = 5 - 6 = -1$  Ans

- a) -2
- b) -1
- c) 2
- d) 1

21. If  $C_0, C_1, C_2, \dots, C_n$  are binomial coefficients of order  $n$ , then the value of  $\frac{C_1}{2} + \frac{C_2}{4} + \frac{C_3}{6} + \dots =$

$(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$   
 Integrate both side from 0 to 1:  
 $\frac{2^n - 1}{n+1} = C_0 + C_1/2 + C_2/3 + \dots + C_n/n+1$  — (1)  
 again  $(1-x)^n = C_0 - C_1x + C_2x^2 + \dots + (-1)^n C_nx^n$   
 Integrate both side 0 to 1 ( $\div$ )  
 $\frac{1}{n+1} = C_0 - C_1/2 + C_2/3 - \dots$  — (2) Perform (1) - (2)  
 result  $\Rightarrow \frac{2^n - 1}{n+1} = C_1/2 + \frac{C_2}{4} + \frac{C_3}{6} + \dots$

- a)  $\frac{2^n + 1}{n+1}$
- b)  $\frac{2^n - 1}{n+1}$
- c)  $\frac{2^n + 1}{n-1}$
- d)  $\frac{2^n}{n+1}$

22. The value of  $(0.2)^{\log_{\sqrt{5}}(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots + \infty)}$  is

$(0.2)^{\log_{\sqrt{5}}(\frac{1}{4} + \frac{1}{4} \cdot \frac{1}{2} + \frac{1}{4}(\frac{1}{2})^2 + \dots + \infty)}$   
 $= (0.2)^{\log_{\sqrt{5}}(\frac{\sqrt{4}}{1} / 1 - 1/2)}$   
 $= (0.2)^{\log_{\sqrt{5}}(1/2)}$   
 $= (\frac{1}{5})^{-\log_{\sqrt{5}} 2} = \frac{1}{\sqrt{5}^{\log_5 2}} = 5^{\log_5 2}$   
 $= 5 \log_5 4 = 4$

- a) 4
- b)  $\frac{1}{4}$
- c) 2
- d)  $\frac{1}{2}$

Space for calculation / rough work



27. If  $n(A) = n(B) = m$ , then the number of possible bijections from  $A$  to  $B$  is

27.  
toug

- a)  $m$
- b)  $m^2$
- c)  $m!$
- d)  $2m$

28.  $\sin^{-1} [x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}] =$

28.

- a)  $\sin^{-1} x - \sin^{-1} \sqrt{1-x}$
- b)  $\sin^{-1} x + \sin^{-1} \sqrt{1-x}$
- c)  $\sin^{-1} x - \sin^{-1} \sqrt{x}$
- d)  $\sin^{-1} x + \sin^{-1} \sqrt{x}$

$$\sin^{-1} [x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}]$$

$$= \sin^{-1} [x\sqrt{1-(\sqrt{x})^2} - \sqrt{x}\sqrt{1-x^2}] \quad \text{--- (1)}$$
 W.K.T.  $\sin^{-1}(x+y) = \sin^{-1} [x\sqrt{1-y^2} - y\sqrt{1-x^2}] \quad \text{--- (2)}$ 
 Compare (1) & (2) :  

$$\boxed{\sin^{-1} x - \sin^{-1} \sqrt{x}}$$

29. If  $\tan \theta + \tan 4\theta + \tan 7\theta = \tan \theta \tan 4\theta \tan 7\theta$ , then the general solution is

30.

- a)  $\theta = \frac{n\pi}{4}$
- b)  $\theta = \frac{n\pi}{12}$
- c)  $\theta = \frac{n\pi}{6}$
- d) none of these

31. If a circle with the point  $(-1, 1)$  as its center touches the straight line  $x+2y+9=0$  then the coordinates of the points of contact is

31.

- a)  $(-3, 3)$
- b)  $(-3, -3)$
- c)  $(0, 0)$
- d)  $(\frac{7}{3}, -\frac{17}{3})$

Let eqn of line  $x+2y+9=0$  is  $y = -\frac{1}{2}x - \frac{9}{2}$  --- (1)
   
 Eqn of OA  $\Rightarrow 1 = -2 + c$ 
  
 $c = -3$ 
  
 $y = -\frac{1}{2}x - 3$  --- (2)
   
 Solve eqn (1) & (2) :  
 Space for calculation/rough work.

$x = -3$   
 $y = -3$   
 Ans we r - (b)

27. If the circles  $x^2 + y^2 + 2gx + 2fy = 0$ , and  $x^2 + y^2 + 2g'x + 2f'y = 0$  touch each other, then

For given condition:-

$$\frac{2g}{2g'} = \frac{2f}{2f'} \Rightarrow \boxed{f'g = g'f}$$

- a)  $fg = f'g'$  (b)
- b)  $f'g = fg'$
- c)  $ff' = gg'$
- d) none of these

28. The number of common tangents to the circles  $x^2 + y^2 = 4$  and  $x^2 + y^2 - 4x + 2y - 4 = 0$  is

- a) 1
  - b) 2
  - c) 3
  - d) 4
- Eqns are:  $x^2 + y^2 = (2)^2$  ; (1) Cut each other at two places:-  
 (b)  $(x-2)^2 + (y+1)^2 = 3^2$  ; (2) So, no. common tangents = 2

29. The length of the tangent drawn from any point on the circle  $x^2 + y^2 - 4x + 6y - 4 = 0$  to the circle  $x^2 + y^2 - 4x + 6y = 0$  is

- a) 8
  - b) 4
  - c) 2
  - d) none of these
- (c) Length of tangent from  $x^2 + y^2 + 2gx + 2fy + c_1$  to circle  $x^2 + y^2 + 2g'x + 2f'y + c_2 = 0$  is  $\sqrt{c_2 - c_1}$   
 here  $c_2 = 4, c_1 = 0$   
 So, length =  $\sqrt{4-0} = 2$

30. If the foci of the ellipse  $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$  and the hyperbola  $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$  coincide, then the value of  $b^2$  is

- a) 25
  - b) 9
  - c) 16
  - d) 4
- For hyperbola =  $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ ; eccentricity  $e = \frac{225}{144} = \frac{15}{12}$   
 Since foci of ellipse coincide: foci  $(\pm \frac{12}{5} \times \frac{15}{12}, 0) = (\pm 3, 0)$   
 $\Rightarrow 5e = 3$  or,  $e = 3/5$   
 since  $b^2 = a^2(1 - e^2)$  or  $b^2 = 25(1 - 9/25) = 16$  ✓

31. The latus rectum of the ellipse is half the minor axis. Then its eccentricity is

- a)  $\frac{1}{\sqrt{2}}$
  - b)  $\frac{1}{\sqrt{3}}$
  - c)  $\frac{\sqrt{3}}{2}$
  - d) none of these
- latus Rectum =  $2b^2/a$ , L minor axis =  $2b$   
 Given  $\frac{2b^2}{2} = 2b^2/a \Rightarrow a = 2b$   
 W.K.T.  $b^2 = a^2(1 - e^2)$   
 $1 - e^2 = 1/4$  or,  $e^2 = 3/4$   
 $\boxed{e = \sqrt{3}/2}$

Space for calculation / rough work

32. The ends of the latus rectum of the parabola  $x^2 + 10x - 16y + 25 = 0$  are

- a) (3,4), (-13,4)
- b) (5,-8), (-5,8)
- c) (3,-4), (13,4)
- d) (-3,-4), (13,-4)

(a)

$$(x+5)^2 = 4(4)y$$

$$x^2 = 4ay$$

vertex = (-5,0), focus = (-5,4)

eqn of axis  $\Rightarrow x = 4$

only (3,4) eqn of a line  $\perp$  to axis and passing through focus is  $y = 4$ ; (3,4), (-13,4)

Substituting the parabola and their y-coordinates is 4

33. Which of the following functions is differentiable at  $x=0$ ?

- a)  $\cos(|x|) + |x|$
- b)  $\cos(|x|) - |x|$
- c)  $\sin(|x|) + |x|$
- d)  $\sin(|x|) - |x|$

(d)

34. If  $x = a \left( \cos t + \log \tan \frac{t}{2} \right)$ ,  $y = a \sin t$ , then  $\frac{dy}{dx} =$

- a)  $\tan t$
- b)  $\cot t$
- c)  $-\cot t$
- d)  $-\tan t$

(a)

Find  $\frac{dy}{dt}$  and  $\frac{dx}{dt}$

$$\frac{(dy/dt)}{(dx/dt)} = \frac{\cos t \sin t}{\cos^2 t} = \tan t$$

35. If  $\begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan \theta \\ -\tan \theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ , then

- a)  $a=1, b=1$
- b)  $a=\cos 2\theta, b=\sin 2\theta$
- c)  $a=\sin 2\theta, b=\cos 2\theta$
- d)  $a=\cos \theta, b=\sin \theta$

(b)

$$\begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} \begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix}^{-1} = \frac{1}{\sec^2 \theta}$$

$$= \begin{bmatrix} 1 - \tan^2 \theta & -2 \tan \theta \\ 2 \tan \theta & 1 - \tan^2 \theta \end{bmatrix} \frac{1}{\sec^2 \theta}$$

$$= \begin{bmatrix} \cos^2 \theta - \sin^2 \theta & -2 \sin \theta \cos \theta \\ 2 \sin \theta \cos \theta & \cos^2 \theta - \sin^2 \theta \end{bmatrix}$$

Space for calculation / rough work

Answers:  
 $\begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix}^{-1} = \frac{1}{\sec^2 \theta} \begin{bmatrix} 1 & \tan \theta \\ -\tan \theta & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan \theta \\ -\tan \theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$\left. \begin{aligned} a &= \cos 2\theta \\ b &= \sin 2\theta \end{aligned} \right\}$$

Slu...

36. If  $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ , then  $A^n$  is

a)  $\begin{bmatrix} 1 & 2^n - 2 \\ 0 & 1 \end{bmatrix}$

b)  $\begin{bmatrix} 1 & n^2 \\ 0 & 1 \end{bmatrix}$

c)  $\begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$

d)  $\begin{bmatrix} 1 & n^2 \\ 1 & 1 \end{bmatrix}$

(c)  $\begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$

37. If  $\alpha, \beta, \gamma$  are the roots of the equation  $x^3 + px + q = 0$  then the value of the determinant  $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$  is

a)  $q$

b)  $0$

c)  $p$

d)  $p^2 - 2q$

38. The number of distinct real roots of  $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix}$  in the interval  $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$  is

a)  $0$

b)  $1$

c)  $2$

d)  $3$

Space for calculation / rough work

Suraj

Suraj

39. The sum of non-prime positive divisors of 450 is

- a) 1209
- b) 1299
- c) 1199
- d) 1099

40. The last digit of  $\sum_{\substack{1 < p < 100 \\ p \text{ - prime}}} p! - \sum_{n=1}^{50} (2n)!$  is

- a) 2
- b) 4
- c) 6
- d) 8

41. The interval I such that  $\int_0^1 \frac{dx}{\sqrt{1+x^4}} \in I$  is given by

- a)  $(0, \frac{1}{\sqrt{2}})$
- b)  $[\frac{1}{\sqrt{2}}, 1]$
- c)  $[\sqrt{2}, 2]$
- d)  $[\sqrt{2}, \frac{7}{4}]$

$$(1+x^4) < (1+x^2)^2 \Rightarrow \sqrt{1+x^4} < 1+x^2$$

$$\text{or, } \frac{1}{\sqrt{1+x^4}} > \frac{1}{1+x^2} \quad \text{or, } \frac{1}{1+x^2} < \frac{1}{\sqrt{1+x^4}}$$

$$\frac{1}{\sqrt{1+x^4}} < 1 \text{ always.}$$

$$\text{So, } \int_0^1 \frac{1}{1+x^2} dx < \int_0^1 \frac{1}{\sqrt{1+x^4}} dx < \int_0^1 1 dx$$

42.  $\int_0^{\frac{\pi}{2}} \log(\tan x) dx =$

- a)  $\frac{\pi}{2}$
- b) 0
- c) 1
- d)  $\frac{\pi}{4}$

$$\int_0^{\frac{\pi}{2}} \log(\tan x) dx = \int_0^{\frac{\pi}{2}} \log(\sin x) dx - \int_0^{\frac{\pi}{2}} \log(\cos x) dx$$

$$= 0$$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \log(\sin x) dx = \int_0^{\frac{\pi}{2}} \log(\cos x) dx$$

Space for calculation / rough work

43. The value of  $\int_{-2}^2 (ax^3 + bx + c) dx$  depends on the

- a) value of b
- b) value of c
- c) value of a
- d) values of a and b

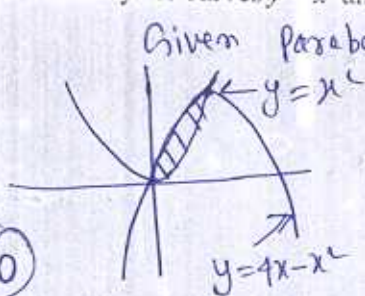
(b)

Since  $ax^3 + bx$  is an odd function  $\int_{-2}^2 (ax^3 + bx) dx = 0$   
 Hence  $\int_{-2}^2 (ax^3 + bx + c) dx = \int_{-2}^2 c dx$ ;  $\therefore$  integral depends upon the value of 'c'

44. The area of the region bound by the curves  $y = x^2$  and  $y = 4x - x^2$  is

- a)  $\frac{16}{3}$  sq. units
- b)  $\frac{8}{3}$  sq. units
- c)  $\frac{4}{3}$  sq. units
- d)  $\frac{2}{3}$  sq. units

(b)



Given parabolas are  $y = x^2$ ,  $(y-4) = -(x-2)^2$

x-coordinates on intersect pt. = 0 or 2

$$\text{area} = \int_0^2 (4x - x^2 - x^2) dx = \int_0^2 (4x - 2x^2) dx$$

$$\left[ 2x^2 - \frac{2}{3}x^3 \right]_0^2 = \underline{\underline{8/3}}$$

45. The particular solution of  $\frac{y}{x} \frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ , when  $x=1, y=2$  is

- a)  $5(1+y^2) = 2(1+x^2)$
- b)  $2(1+y^2) = 5(1+x^2)$
- c)  $5(1+y^2) = (1+x^2)$
- d)  $(1+y^2) = 2(1+x^2)$

(b)

$$\frac{1}{2} \int \frac{2y dy}{1+y^2} = \frac{1}{2} \int \frac{2x dx}{1+x^2}; \text{ or, } \frac{1}{2} \log(1+y^2)$$

$$= \frac{1}{2} \log(1+x^2) + c$$

or,  $\log\left(\frac{1+y^2}{1+x^2}\right) = c$   
 Put  $x=1, y=2$  Put value of c in eqn  
 $c = \log 5/5$   $\therefore 2(1+y^2) = 5(1+x^2)$

46. The solution of the differential equation  $\frac{dy}{dx} = (x+y)^2$  is

- a)  $\frac{1}{x+y} = c$
- b)  $\sin^{-1}(x+y) = x + c$
- c)  $\tan^{-1}(x+y) = c$
- d)  $\tan^{-1}(x+y) = x + c$

(d)

Put  $x+y = z \Rightarrow \frac{dy}{dx} + 1 = \frac{dz}{dx}$

Now given eqn

$$\frac{dz}{dx} - 1 = z^2 \text{ or, } \frac{dz}{dx} = 1 + z^2$$

$$\int dx = \int \frac{dz}{1+z^2}$$

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$c + x = \tan^{-1}(x+y)$

47. The maximum value of  $\left(\frac{1}{x}\right)^{2x^2}$  is

- 1.  $\int e^x$
- a)
- b)
- c)
- d)

- a)  $x^{1/2}$
- b)  $\sqrt{e}$
- c) 1
- d)  $e^2$

48. Let  $x$  be a number which exceeds its square by the greatest possible quantity, then  $x =$

- a)
- b)
- c)
- d)

- a)  $1/2$
- b)  $1/4$
- c)  $3/4$
- d)  $1/3$

Go by option: For  $x = \frac{1}{2}$ ,  $\frac{1}{2} - \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

49. The subtangent at  $x = \pi/2$  on the curve  $y = x \sin x$  is

- 2.  $\int \frac{x}{x}$

- a) 0
- b) 1
- c)  $\pi/2$
- d) none of these

Slope  $\frac{dy}{dx} \Big|_{x=\pi/2} = 1$ ;  $y = x + c$ ;  $P(\pi/2, 0)$  lies on  $y = x \sin x$ ;  $\therefore c = \pi/2$   
 eqn of line  $\Rightarrow y = x + \pi/2$  (distance b/w origin and axis intersect Pt =  $\pi/2$ )

50. The value of  $\int \frac{10^{x/2}}{\sqrt{10^{-x} - 10^x}} dx$  is

- b)

- a)  $\frac{1}{\log_e 10} \sin^{-1}(10^x) + c$
- b)  $2\sqrt{10^{-x} + 10^x} + c$
- c)  $\frac{1}{\log_e 10} \sinh^{-1}(10^x) + c$
- d)  $\frac{-1}{\log_e 10} \sinh^{-1}(10^x) + c$

$$\int \frac{10^{x/2} dx}{\sqrt{10^{-x} - 10^x}} = \int \frac{10^{x/2} 10^{x/2} dx}{\sqrt{1 - (10^x)^2}}$$

$$= \int \frac{10^x dx}{\sqrt{1 - (10^x)^2}}; y = 10^x = e^{x \log_e 10}$$

$$\frac{dy}{dx} = (\log_e 10) e^{x \log_e 10}$$

$$= \int \frac{\log_e 10 (e^{x \log_e 10}) dx}{\log_e 10 \sqrt{1 - (10^x)^2}}$$

$$= \frac{1}{\log_e 10} \int \frac{dy}{\sqrt{1 - y^2}}; y = 10^x = e^{x \log_e 10}$$

$$= \frac{1}{\log_e 10} \sin^{-1}(10^x) + c$$

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11.  $\int e^x \left\{ \frac{1 + \sin x \cos x}{\cos^2 x} \right\} dx =$

$\int e^x \left\{ \frac{1}{\cos^2 x} + \frac{\sin x \cos x}{\cos^2} \right\} dx$

a)  $e^x \cos x + c$

b)  $e^x \sec x \tan x + c$

c)  $e^x \tan x + c$

d)  $e^x \cos^2 x - 1 + c$

(c)  $\int e^x (\sec^2 x + \tan x) dx \equiv \int e^x (f'(x) + f(x)) dx$   
 $= e^x \tan x + c$

12.  $\int \frac{x^2 + 1}{x^4 + 1} dx$

(d)

a)  $\frac{1}{\sqrt{2}} \log_e(x^2 + 1) + c$

b)  $\frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{x^2 + 1}{x\sqrt{2}} \right) + c$

c)  $\frac{1}{\sqrt{2}} \tan^{-1}(x^2 - 1) + c$

d)  $\frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{x^2 - 1}{x\sqrt{2}} \right) + c$

13. The locus of the mid point of the intercept of the line  $x \cos \alpha + y \sin \alpha = p$  between the coordinate axes is

a)  $x^2 + y^2 = 4p^2$

b)  $x^2 + y^2 = p^2$

c)  $x^2 + y^2 = 4p^{-2}$

d)  $x^2 + y^2 = p^2$

(a)

when  $x = 0, y = p \operatorname{cosec} \alpha$

$y = 0, x = p \operatorname{sec} \alpha$

mid point  $\equiv \left( \frac{p \operatorname{sec} \alpha}{2}, \frac{p \operatorname{cosec} \alpha}{2} \right)$

So,  $x = \frac{p \operatorname{sec} \alpha}{2}; y = \frac{p \operatorname{cosec} \alpha}{2}$

$\therefore \cos \alpha = \frac{p}{2x}; \sin \alpha = \frac{p}{2y}$

W.K.T

$\cos^2 \alpha + \sin^2 \alpha = 1$

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$\frac{p^2}{4x^2} + \frac{p^2}{4y^2} = 1$

$x^2 + y^2 = 4p^2$



54. If the line through  $A = (4, -5)$  is inclined at an angle  $45^\circ$  with the positive direction of the x-axis, then the coordinates of the two points on opposite sides of A at a distance of  $3\sqrt{2}$  units are

a)  $(7, 2), (1, 8)$

b)  $(7, 2), (1, -8)$

c)  $(7, -2), (1, -8)$

d)  $(7, 2), (-1, 8)$

Slope =  $\tan 45^\circ = 1$ ; eqn<sup>n</sup>  $y = x + C$   
 $P(4, 5)$  lies on line so,  $C = -9$   
 Now, eqn<sup>n</sup>  $\Rightarrow y = x - 9$   
 only  $(7, -2)$  and  $(1, -8)$  lies on above st. line  
 eqn<sup>n</sup>, do no need for further calculation

57. lim

a)

b)

c)

d)

55. If the line  $px + qy = 0$  coincides with one of the lines given by  $ax^2 + 2hxy + by^2 = 0$  then

a)  $ap^2 + 2hpq + bq^2 = 0$

b)  $aq^2 + 2hpq + bp^2 = 0$

c)  $aq^2 - 2hpq + bp^2 = 0$

d) none of these

$y = \frac{-p}{q}x$ ; put in given pair of lines  
 or,  $ax^2 + 2hx(\frac{-px}{q}) + b\frac{p^2}{q^2}x^2 = 0$   
 or,  $(aq^2 - 2hpq + bp^2)x^2 = 0$   
 Soln: either  $x = 0$  or  $aq^2 - 2hpq + bp^2 = 0$

58. The n

a) 3

b) 2

c) 1

d) 6

56. The function  $f(x) = \left( \frac{\log_e(1+ax) - \log_e(1-bx)}{x} \right)$  is undefined at  $x = 0$ . The value which should be assigned to  $f$  at  $x = 0$  so that it is continuous at  $x = 0$  is

a)  $\frac{a+b}{2}$

b)  $a+b$

c)  $\log_e(ab)$

d)  $a-b$

(b)

59. The a

a) 17

b) 14

c) 13

d) 12



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67.  $\lim_{n \rightarrow \infty} \frac{(1^2 + 2^2 + \dots + n^2) \sqrt{n}}{(n+1)(n+10)(n+100)} =$

a) 3

b)  $\frac{1}{3}$

c)  $\frac{2}{3}$

d)  $\infty$

(b)

$$\left( \lim_{n \rightarrow \infty} \frac{n(n+1)(2n+1)}{6(n+1)(n+10)(n+100)} \right) \left( \lim_{n \rightarrow \infty} \sqrt{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2n^2 + n}{6(n+10)(n+100)} (1) = \lim_{n \rightarrow \infty} \frac{2 + \frac{1}{n}}{6(1 + \frac{10}{n})(1 + \frac{100}{n})}$$

$$= \frac{2 + 0}{6(1+0)(1+0)} = \frac{2}{6} = \frac{1}{3}$$

58. The number of triangles in a complete graph with 10 non-collinear vertices is

a) 360

b) 240

c) 120

d) 60

(c)

$$\text{no. of triangle} = {}^{10}C_3 = \frac{10 \times 9 \times 8 \times 7}{2 \times 3} = 120$$

59. The angle between hands of a clock when the time is 4.25 AM is

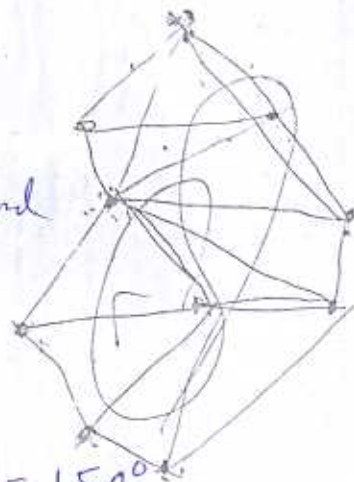
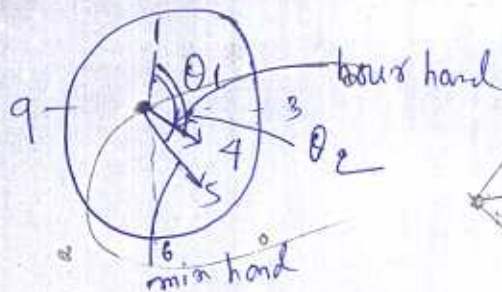
a)  $17 \frac{1}{2}^\circ$

b)  $14 \frac{1}{2}^\circ$

c)  $13 \frac{1}{2}^\circ$

d)  $12 \frac{1}{2}^\circ$

(a)



$$\theta_1 \text{ min hand} = \frac{360^\circ}{12} \times 5 = 150^\circ$$

$$\theta_2 \text{ hour hand} = \frac{360^\circ}{12} \times 4 + \frac{30^\circ}{60 \text{ min}} \times 25 \text{ min}$$

$$\theta_1 - \theta_2 = 150 - 132.5 = 17.5^\circ = 17 \frac{1}{2}^\circ$$

Space for calculation / rough work

