COMEDK 2023 Morning Shift

Mathematics

Question 1

The value of $a^{\log_b c} - c^{\log_b a}$, where $a, b, c > 0$ but $a, b, c \neq 1$, is

Options:

A. a B. b C. c D. 0

Answer: D

Solution:

Let $y = C^{\log_b a}$

$\Rightarrow \log_C y = \log_b a$

$\Rightarrow \frac{\log y}{\log c} = \frac{\log a}{\log b}$ $[\because \log_a b = \frac{\log b}{\log a}]$

$\Rightarrow \frac{\log y}{\log a} = \frac{\log c}{\log b} \Rightarrow \log_a y = \log_b c$

$\Rightarrow y = a^{\log_b c}$ $[\because \log_a x = y \Rightarrow a^y = x]$

$\therefore a^{\log_b c} - c^{\log_b a} = a^{\log_b c} - a^{\log_b c}$

= 0

Question 2

The slope of the tangent to the curve, $y = x^2 - xy$ at $(1, \frac{1}{2})$ is

Options:

A. $\frac{4}{3}$

B. $\frac{2}{3}$

C. $\frac{3}{4}$

D. $\frac{3}{2}$

Answer: C

Solution:

Given curve, $y = x^2 - xy$.

On differentiating the equation, $y = x^2 - xy$ w.r.t. x, we get

$$\frac{dy}{dx} = 2x - \left(x\frac{dy}{dx} + y\right)$$

$$\Rightarrow (1+x)\frac{dy}{dx} = 2x - y \Rightarrow \frac{dy}{dx} = \frac{2x-y}{1+x}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1/2)} = \frac{2(1)-\left(\frac{1}{2}\right)}{1+(1)} = \frac{3/2}{2} = \frac{3}{4}$$

Question 3

The value of $\lim_{x\to 0}\frac{e^{ax}-e^{bx}}{2x}$ is equal to

Options:

A. $\frac{a+b}{2}$

B. $\frac{a-b}{2}$

C. $\frac{e^{ab}}{2}$

D. 0

Answer: B

Solution:

$$\lim_{x\to 0} \frac{e^{ax} - e^{bx}}{2x}$$

$$= \lim_{x\to 0} \frac{\left(1+ax+\frac{(ax)^2}{2!}+\frac{(ax)^3}{3!}+\ldots\right) - \left(1+bx+\frac{(bx)^2}{2!}+\frac{(bx)^3}{3!}\right)}{2x}$$

$$= \lim_{x\to 0} \frac{x\left[(a-b)+\left(\frac{a^2x}{2!}+\frac{a^3x^2}{3!}+\ldots\right)+\left(\frac{b^2x}{2!}+\frac{b^3x^2}{3!}+\ldots\right)\right]}{2x}$$

$$= \frac{1}{2}[(a-b)+(0+0+\ldots)+(0+0+\ldots)]$$

$$= \frac{a-b}{2}$$

Question 4

The points of intersection of circles $(x+1)^2 + y^2 = 4$ and $(x-1)^2+y^2=9$ are $(a,\pm b)$, then (a, b) equals to

Options:

A. $\left(1.25, \frac{3}{4}\sqrt{7}\right)$

B. $\left(-1.25, \frac{3}{4}\sqrt{7}\right)$

C. $(-1, 2)$

D. $(1, 3)$

Answer: B

Solution:

Given, $(x+1)^2+y^2=4$ ... (i)

and $(x-1)^2+y^2=9$ ... (ii)

On subtracting Eq. (ii) from Eq. (i), we get

$$(x+1)^2-(x-1)^2=4-9$$
$$\Rightarrow (x^2+2x+1)-(x^2-2x+1)=-5$$

$\Rightarrow 4x=-5$

$\Rightarrow x=-1.25$

On putting, $x=-1.25$ into Eq. (i), we get

$(-0.25)^2 + y^2 = 4$

$\Rightarrow y^2=3.9375 \Rightarrow y=\pm\sqrt{3.9375}$

$\Rightarrow y=\pm \frac{3}{4}\sqrt{7}$

$\therefore a=-1.25$ and $b=\frac{3}{4}\sqrt{7}$

$\therefore (a,b)=\left(-1.25, \frac{3}{4}\sqrt{7}\right)$

Question 5

The approximate value of $f(5.001)$, where $f(x)=x^3-7x^2+10$

Options:

A. -39.995

B. -38.995

C. -37.335

D. -40.995

Answer: A

Solution:

First, break the number 5.001 as $x=5$ and $\Delta x=0.001$ and use the relation

$f(x+\Delta x)\approx f(x)+\Delta x f'(x)$

consider $f(x)=x^3-7x^2+10 \Rightarrow f'(x)=3x^2-14x$

Therefore,

$f(x + \Delta x) \approx (x^3 - 7x^2 + 10) + \Delta x\,(3x^2 - 14x)$

$\Rightarrow f(5.001) \approx (5^3 - 7(5)^2 + 10) + (0.001)(3(5)^2 - 14(5))$

$= (125 - 175 + 10) + (0.001)(75 - 70)$

$= -40 + (0.001)(5) = -40 + 0.005 = -39.995$

Question 6

The circle $x^2 + y^2 + 3x - y + 2 = 0$ cuts an intercept on X-axis of length

Options:

A. 3 B. 4 C. 2

D. 1

Answer: D

Solution:

Given equation of circle is $x^2+y^2+3x-y+2=0$.

On comparing this equation with $x^2+y^2+2gx+2fy+c=0$, we get $g = \frac{3}{2}, f = -\frac{1}{2}$ and $c = 2$

Now, the length of intercept on X-axis $= 2\sqrt{g^2 - c}$

$= 2\sqrt{\left(\frac{3}{2}\right)^2 - 2} = 2\sqrt{\frac{9}{4} - 2} = 2\sqrt{\frac{1}{4}} = 2\left(\frac{1}{2}\right) = 1$

Question 7

Let $f(x) = a + (x-4)^{\frac{4}{9}}$, then minima of $f(x)$ is

Options:

A. 4

B. a

C. a − 4

D. None of these

Answer: B

Solution:

$\because f(x) = a + (x-4)^{4/9}$

$\therefore f'(x) = 0 + \frac{4}{9}(x-4)^{-5/9}$

Clearly, at $x = 4$, $f'(x)$ is not defined

Hence, $x = 4$ is the point of extremum.

$\because f(4) = a + (4-4)^{4/9} = a$

$\therefore$ The minimum value of $f(x)$ is a.

Question 8

If $f(x) = \begin{cases} 2\sin x & ; \quad -\pi \le x \le \frac{-\pi}{2} \\ a\sin x + b & ; \quad -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \cos x & ; \quad \frac{\pi}{2} \le x \le \pi \end{cases}$ and it is continuous on $[-\pi, \pi]$, then

Options:

A. $a = 1$ and $b = 1$

B. $a = -1$ and $b = -1$

C. $a = -1$ and $b = 1$

D. $a = 1$ and $b = -1$

Answer: D

Solution:

At, $x = \frac{\pi}{2}$

LHL $= \lim_{x\to\pi/2^-} (a\sin x + b) = a + b$

RHL $= \lim_{x\to\pi/2^+} (\cos x) = 0$

Since, $f(x)$ is continuous at $x = \pi/2$

$\therefore \quad a + b = 0$ .... (i)

At, $x = -\frac{\pi}{2}$

LHL $= \lim_{x\to-\pi/2^-} (2\sin x) = -2$

RHL $= \lim_{x\to-\pi/2^+} (a\sin x + b) = -a + b$

Since, $f(x)$ is continuous at $x = -\pi/2$

$\therefore \quad -a + b = -2$ ... (ii)

On solving Eqs. (i) and (ii), we get $a = 1$ and $b = -1$

Question 9

The value of $\lim_{x\to\infty} \left(\frac{x^2 - 2x + 1}{x^2 - 4x + 2}\right)^{2x}$ is

Options:

A. e^2

B. e^4

C. e

D. e^{16}

Answer: B

Solution:

Let $L=\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^{2x}$

$\Rightarrow \ln L = \lim_{x\to\infty}\left[2x\ln\left(\frac{x^2-2x+1}{x^2-4x+2}\right)\right]$

$\Rightarrow \ln L = \lim_{x\to\infty}\left[2x\ln\left(1+\frac{2x-1}{x^2-4x+2}\right)\right]$

$\Rightarrow \ln L = \lim_{x\to\infty}\left[2x\left(\frac{2x-1}{x^2-4x+2}-\frac{\left(\frac{2x-1}{x^2-4x+2}\right)^2}{2}+\frac{\left(\frac{2x-1}{x^2-4x+2}\right)^3}{3}\right)\ldots\right]$

$\Rightarrow \ln L = \left(\lim_{x\to\infty}\frac{2x(2x-1)}{x^2-4x+2}\right)-0+0\ldots$

$\Rightarrow \ln L = \lim_{x\to\infty}\frac{4x^2\left(1-\frac{1}{2x}\right)}{x^2\left(1-\frac{4}{x}+\frac{2}{x^2}\right)}$

$\ln L = \frac{4(1-0)}{(1-0+0)}$

$\Rightarrow \ln L = 4$

$\Rightarrow L = e^4$

Question 10

$S \equiv x^2+y^2-2x-4y-4=0$ and $S' \equiv x^2+y^2-4x-2y-16=0$ are two circles the point $(-2,-1)$ lies

Options:

A. inside S' only

B. inside S only

C. inside S and S'

D. outside S and S'

Answer: A

Solution:

$S_{(-2,-1)} = (-2)^2+(-1)^2-2(-2)-4(-1)-4$

$= 4+1+4+4-4 = 9 > 0$

$\therefore (-2,-1)$ lies outside of S

$S(-2,-1) = (-2)2 + (-1)2 - 4(-2) - 2(-1) - 16$

$=4+1+8+2-16=-1<0$

$\therefore (-2,-1)$ lies inside of S'

Thus, $(-2,-1)$ lies inside S' only.

Question 11

A number n is chosen at random from $s=\{1,2,3,\ldots,50\}$. Let $A=\{n\in s: n$ is a square $\}$, $B=\{n\in s:n$ is a prime$\}$ and $C=\{n\in s: n$ is a square$\}$. Then, correct order of their probabilities is

Options:

A. $p(A)<p(B)<p(C)$

B. $p(A)>p(B)>p(C)$

C. $p(B)<p(A)<p(C)$

D. $p(A)>p(c)>p(B)$

Answer: B

Solution:

Given, $S=\{1,2,3...,50\}$

$A + \{n \in S : n + \frac{50}{n} > 27\}$

$= \{n \in S : n2 - 27n + 50 > 0\}$

$= \{n \in S : (n-25)(n-2) > 0\}$

$= \{n \in S : n < 2 \text{ or } n > 25\}$

$= \{1, 26, 27, 28, \ldots, 50\}$

$\Rightarrow n(A) = 26$

$B = \{n \in S : n \text{ is prime }\}$

$= \{2, 3, 5, 7, 11, 13, 17, 19, 23,$

$29, 31, 37, 41, 43, 47\}$

$\Rightarrow n(B) = 15$

$C = \{n \in S : n \text{ is a square }\} = \{1, 4, 9, 16, 25, 36, 49\}$

$\Rightarrow n(C) = 7$

$\therefore \quad p(A) = \frac{n(A)}{n(S)} = \frac{26}{50},$

$\Rightarrow \quad p(B) = \frac{n(B)}{n(S)} = \frac{15}{50},$

$p(C) = \frac{n(C)}{n(S)} = \frac{7}{50}$

$\therefore p(A) > p(B) > p(C)$

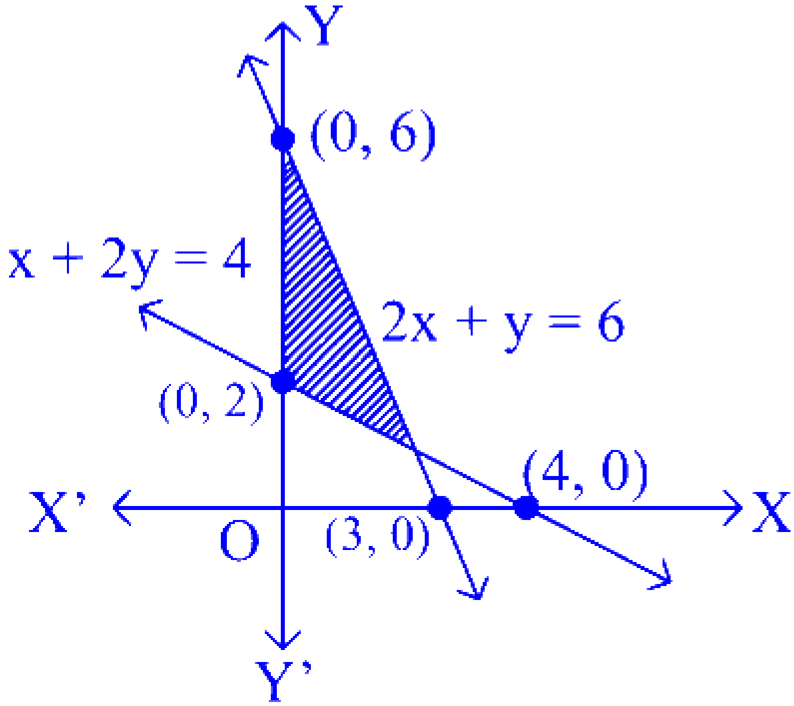
Question 12

The feasible region for the inequations

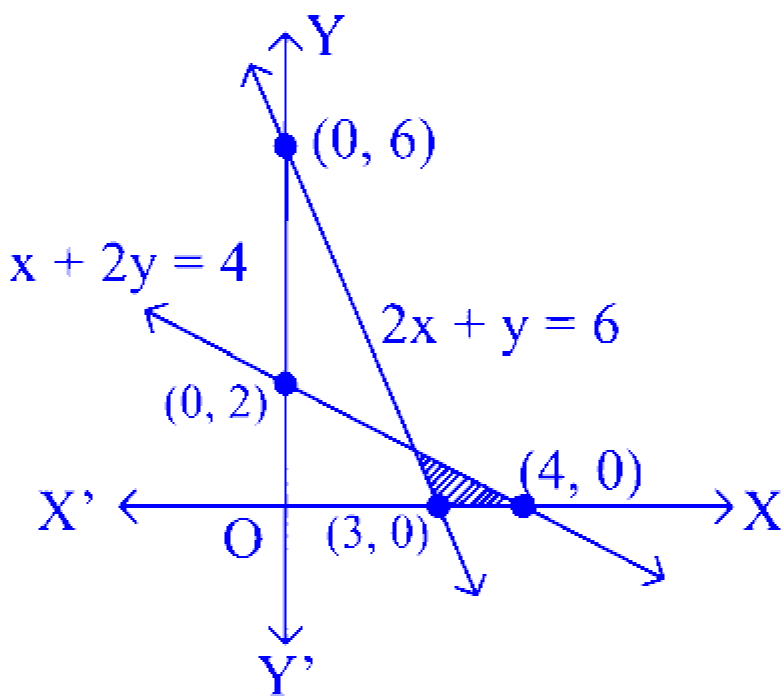
$x + 2y \geq 4, 2x + y \leq 6, x, y \geq 0$ is

Options:

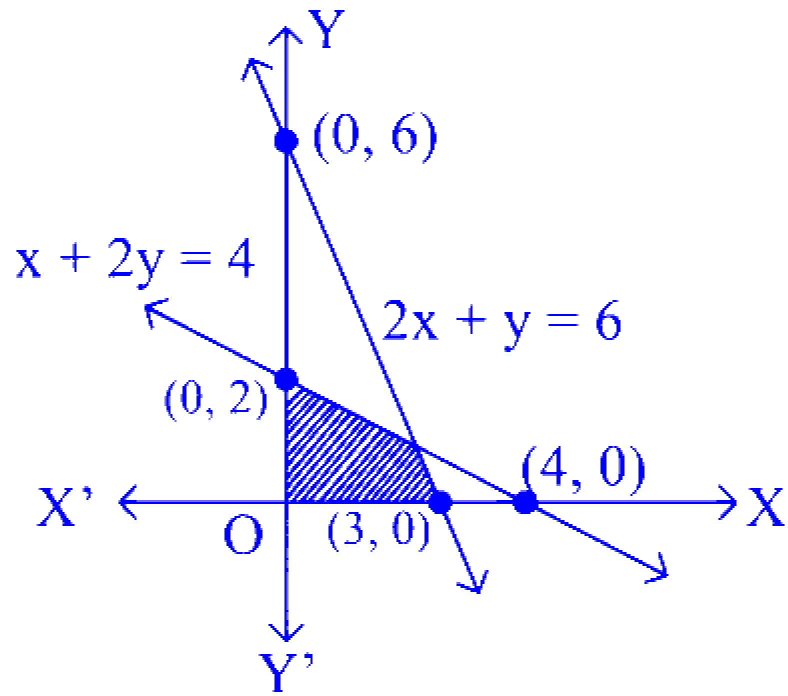
A.



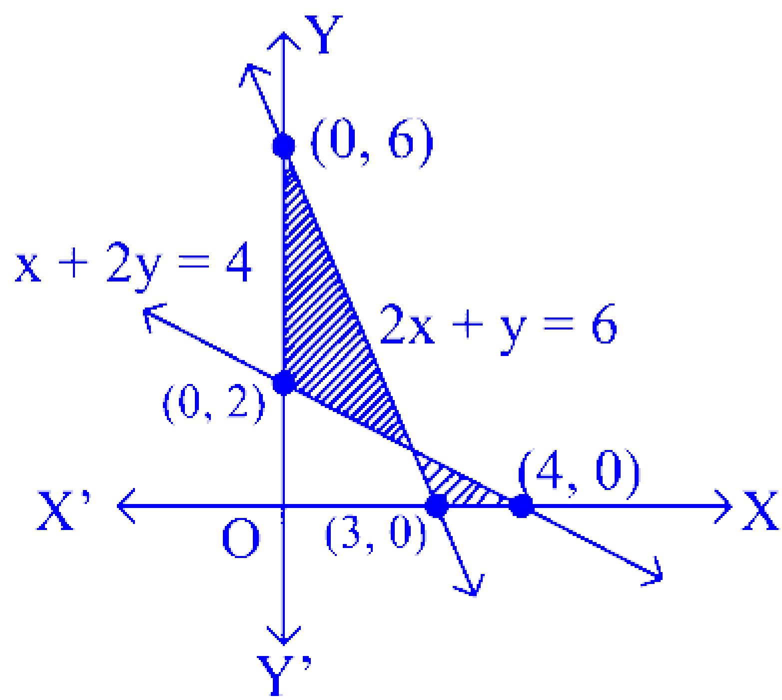
B.



C.



D.



Answer: A

Solution:

The given inequations are $x+2y \geq 4, 2x+y \leq 6, x, y \leq 0$.

According to the inequtions $x, y \geq 0$, the feasible region be the first quadrant (including positive X and positive Y-axis). According to the inequation $x+2y \geq 4$, the feasible region be the region above or on the line

$x+2y=4$.

According to the inequation $2x+y\le 6$, the feasible region be the region below or on the line $2x+y=6$. Now, the common feasible region will be the required feasible region.

Thus, option (a) is correct.

Question 13

The maximum value of $Z = 10x + 16y$, subject to constraints $x \ge 0,\ y \ge 0,\ x + y \le 12, 2x+y\le 20$ is

Options:

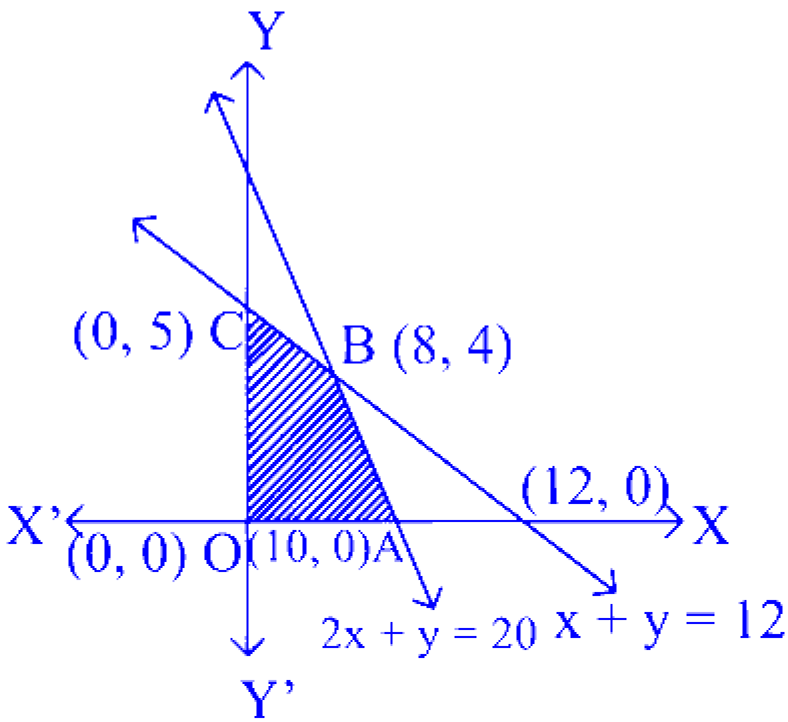
A. 144 B. 192 C. 120 D. 240

Answer: B

Solution:

Given, constraints are $x \ge 0, y \ge 0,\ x + y\le 12, 2x+y\le 20$

The feasible region is $OABC\ O.$



$\because Z = 10x + 16y$

At, $O(0, 0), Z = 10(0) + 16(0) = 0$

At, $A(10, 0), Z = 10(10) + 16(0) = 100$

At, $B(8, 4) Z = 10(8) + 16(4) = 144$

At, $C(0, 12), Z = 10(0) + 16(12) = 192$

Hence, the maximum value of Z is 192.

Question 14

If $A = \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix}$, then A^{-1} equals to

Options:

A. $\begin{bmatrix} 2 & 1 \\ -3/2 & -1 \end{bmatrix}$

B. $\begin{bmatrix} 2 & -1 \\ -3/2 & 1 \end{bmatrix}$

C. $\begin{bmatrix} -2 & 1 \\ 3/2 & -1 \end{bmatrix}$

D. $\begin{bmatrix} -2 & -1 \\ 3/2 & 1 \end{bmatrix}$

Answer: B

Solution:

Given, $A = \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} = 2\times4-3\times2=8-6=2$

Now, $A_{11}=4, A_{12}=-3, A_{21}=-2$ and $A_{22}=2$

$\therefore adj A = \begin{bmatrix} 4 & -3 \\ -2 & 2 \end{bmatrix}^T = \begin{bmatrix} 4 & -2 \\ -3 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} adj A = \frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2 & -1 \\ -3/2 & 1 \end{bmatrix}$

Question 15

If A is a matrix of order 4 such that $A(\text{adj} A) = 10\,I$, then $|\text{adj} A|$ is equal to

Options:

A. 10

B. 100

C. 1000

D. 10000

Answer: C

Solution:

Given, $A(\text{adj}A)=10I$

We know that $A(\text{adj}A)=|A|I$

$\therefore \quad 10I=|A|I$
$\Rightarrow |A|=10$
We know that $|\text{adj}A|=|A|^{n-1}$, where n is order of A

$\therefore |\text{adj } A| = |A|^{4-1} = 10^3 = 1000$

Question 16

If $A = \begin{bmatrix} k+1 & 2 \\ 4 & k-1 \end{bmatrix}$ is a singular matrix, then possible values of k are

Options:

A. ± 1

B. ± 2

C. ± 3

D. ± 4

Answer: C

Solution:

Given $A=\begin{bmatrix} k+1 & 2 \\ 4 & k-1 \end{bmatrix}$ is a singular matrix.

$\therefore |A|=0$
$\Rightarrow \begin{vmatrix} k+1 & 2 \\ 4 & k-1 \end{vmatrix} = 0$
$\Rightarrow (k+1)(k-1)-4\times 2=0$
$\Rightarrow k^2-1-8=0$
$\Rightarrow k^2-9=0$
$\Rightarrow k^2=9$
$\Rightarrow k=\pm 3$

Question 17

The angle between the vectors $\mathbf{a}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $\mathbf{b}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ is

Options:

A. $\sin^{-1}(1/9)$

B. $\sin^{-1}(8/9)$

C. $\cos^{-1}(8/9)$

D. $\cos^{-1}(1/9)$

Answer: D

Solution:

We have, $\mathbf{a}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $\mathbf{b}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}-2\hat{\mathbf{k}}$

Clearly, $|\mathbf{a}|=\sqrt{1+4+4}=\sqrt{9}=3$

and $|\mathbf{b}|=\sqrt{1+4+4}=\sqrt{9}=3$

$\because$ $\mathbf{a}\cdot\mathbf{b}=|a||b|\cos\vartheta$

$\Rightarrow$ $\cos\vartheta=\frac{1\cdot1+2\cdot2+2\cdot(-2)}{3\times3}$

$\Rightarrow$ $\cos\vartheta=\frac{1+4-4}{9}\Rightarrow\cos\vartheta=\frac{1}{9}$

$\therefore$ $\theta=\cos^{-1}\left(\frac{1}{9}\right)$

Question 18

If the vectors $\mathbf{a}=2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\hat{\mathbf{k}};\mathbf{b}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $\mathbf{c}=m\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ are coplanar, then the value of m is

Options:

A. $\frac{5}{8}$

B. $\frac{8}{5}$

C. $\frac{-7}{4}$

D. $\frac{2}{3}$

Answer: B

Solution:

Since, vectors a, b and c are coplanar

$\therefore [\mathbf{a}\ \mathbf{b}\ \mathbf{c}]=0 \Rightarrow a\cdot(b\times c)=0$

Now, $\mathbf{b}\times\mathbf{c} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 2 & -1 \\ m & -1 & 2 \end{vmatrix}$

$=\hat{\mathbf{i}}(4-1)-\hat{\mathbf{j}}(2+m)+\hat{\mathbf{k}}(-1-2m)$

$=3\hat{\mathbf{i}}-(2+m)\hat{\mathbf{j}}-(1+2m)\hat{\mathbf{k}}$

and

$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})=(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\hat{\mathbf{k}})\cdot(3\hat{\mathbf{i}}-(2+m)\hat{\mathbf{j}}-(1+2m)\hat{\mathbf{k}})$

$=2(3)+3(2+m)-4(1+2m)$

$=6+6+3m-4-8m=8-5m$

$\therefore \mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})=0$

$\therefore 8-5m=0 \Rightarrow m=\frac{8}{5}$

Question 19

The maximum value of $Z = 12x + 13y$, subject to constraints $x \geq 0, y \geq 0, x+y \leq 5$ and $3x+y \leq 9$ is

Options:

A. 63

B. 65

C. 60

D. 117

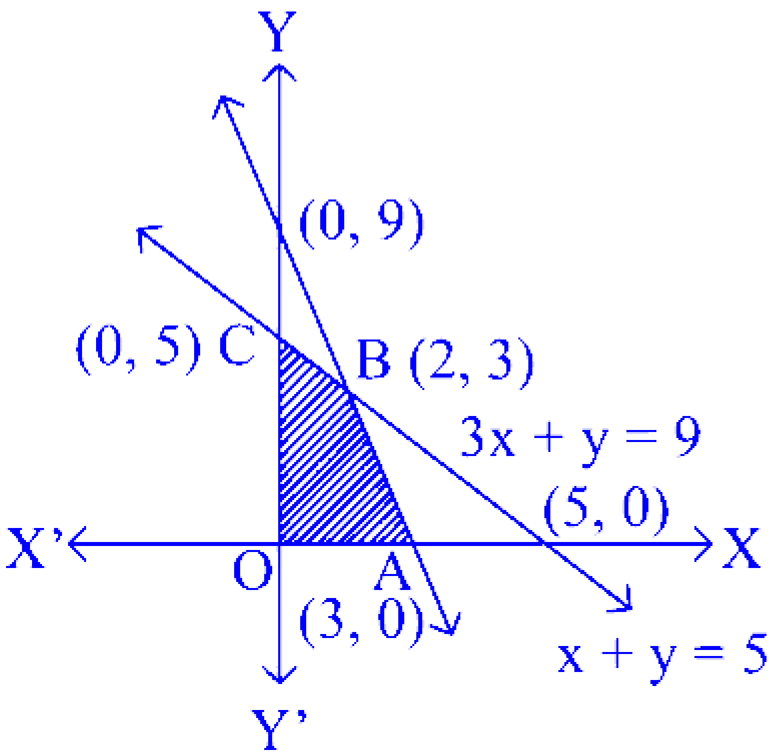
Answer: B

Solution:

Given constraints are

$x \geq 0, y \geq 0,\ x + y \leq 5$ and $3x + y \leq 9$ and $z = 12x + 13y$

The feasible region is $OABC\ O$.



$\because Z = 12x + 13y$

At, $O(0, 0), Z = 12(0) + 13(0) = 0$

At, $A(3, 0), Z = 12(3) + 13(0) = 36$

At, $B(2, 3), Z = 12(2) + 13(3) = 63$

At, $C(0, 5), Z = 12(0) + 13(5) = 65$

Here, maximum value of Z is 65.

Question 20

$\mathbf{a} = 2\hat{i} + \hat{j} - \hat{k}, \mathbf{b} = \hat{i} - \hat{j}$ and $\mathbf{c} = 5\hat{i} - \hat{j} + \hat{k}$, then unit vector parallel to $\mathbf{a} + \mathbf{b} - \mathbf{c}$ but in opposite direction is

Options:

A. $\frac{1}{3}(2\hat{i}-\hat{j}+2\hat{k})$

B. $\frac{1}{2}(2\hat{i}-\hat{j}+2\hat{k})$

C. $\frac{1}{3}(2\hat{i}-\hat{j}-2\hat{k})$

D. None of these

Answer: A

Solution:

Given, $\mathbf{a}=2\hat{i}+\hat{j}-\hat{k}, \mathbf{b}=\hat{i}-\hat{j}$ and $\mathbf{c}=5\hat{i}-\hat{j}+\hat{k}$

$\therefore \mathbf{a}+\mathbf{b}-\mathbf{c}=(2+1-5)\hat{i}+(1-1+1)\hat{j}+(-1+0-1)\hat{k}$

$=-2\hat{i}+\hat{j}-2\hat{k}=-(2\hat{i}-\hat{j}+2\hat{k})$

Now, the unit vector in the direction of $\mathbf{a}+\mathbf{b}-\mathbf{c}$ be

$\frac{-(2\hat{i}-\hat{j}+2\hat{k})}{\sqrt{2^2+(-1)^2+2^2}}=-\frac{1}{3}(2\hat{i}-\hat{j}+2\hat{k})$

$\therefore$ The required unit vector be $\frac{1}{3}(2\hat{i}-\hat{j}+2\hat{k})$

Question 21

The place $x-2y+z=0$ is parallel to the line

Options:

A. $\frac{x-3}{4}=\frac{y-4}{5}=\frac{z-3}{6}$

B. $\frac{x-2}{1}=\frac{y-2}{-2}=\frac{z-3}{1}$

C. $\frac{x-2}{2}=\frac{y-3}{3}=\frac{z-4}{4}$

D. $\frac{x-4}{3}=\frac{y-5}{4}=\frac{z-6}{3}$

Answer: A

Solution:

Consider the equation of line given in option (a). The DR's of this line or (4,5,6).

We know that if the line $\frac{x-x_0}{a_1}=\frac{y-y_0}{b_1}=\frac{z-z_0}{c_1}$ is parallel to the plane $a_2x+b_2y+c_2z+d=0$, then $a_1a_2+b_1b_2+c_1c_2=0$, that is the normal to the plane is perpendicular to the line.

Here, the vector $\hat{i}-2\hat{j}+\hat{k}$ is normal to the plane $x-2y+z=0$ and $4(1)+5(-2)+6(1)=4-10+6=10-10=0$

So, option (a) is correct.

Question 22

$\int \frac{x\,dx}{2(1+x)^{3/2}}$ is equal to

Options:

A. $\frac{2+x}{\sqrt{1+x}}+C$

B. $\frac{2+x}{x\sqrt{1+x}}+C$

C. $\frac{x}{\sqrt{1+x}}+C$

D. $-\frac{x}{\sqrt{1+x}}+C$

Answer: A

Solution:

Let $I=\int \frac{x\,dx}{2(1+x)^{3/2}}$

On putting, $1+x=t$, we get $dx=dt$

$\therefore \quad I = \int \frac{(t-1)dt}{2t^{3/2}} = \frac{1}{2}[\int t^{-1/2} dt - \int t^{-3/2} dt]$

$= \frac{1}{2}[\frac{t^{1/2}}{1/2} - \frac{t^{-1/2}}{-1/2}] + C$

$= \frac{1}{2} \times 2 [\sqrt{t} + \frac{1}{\sqrt{t}}] + C$

$= \frac{t+1}{\sqrt{t}} + C = \frac{x+2}{\sqrt{1+x}} + C$

Question 23

$\int \frac{4^x}{\sqrt{1-16^x}} dx$ is equal to

Options:

A. $(\log 4) \sin^{-1} 4^x + C$

B. $\frac{1}{4} \sin^{-1} (4^x) + C$

C. $\frac{1}{\log 4} \sin^{-1} 4^x + C$

D. $4 \log 4 \sin^{-1} 4 + C$

Answer: C

Solution:

Let $I = \int \frac{4^x}{\sqrt{1-16^x}} dx$

$= \int \frac{4^x}{\sqrt{1-(4^x)^2}} dx$

On putting, $4^x = t$, we get $4^x \log 4 dx = dt$

$\Rightarrow \quad 4^x dx = \frac{dt}{\log 4}$

$\therefore \quad I = \frac{1}{\log 4} \int \frac{dt}{\sqrt{1 - t^2}}$

$= \frac{1}{\log 4} \sin^{-1} t + C$

$= \frac{1}{\log 4} \sin^{-1} 4^x + C$

Question 24

$\int_{-\pi/2}^{\pi/2} \sin^2 x \, dx$ is equal to

Options:

A. 0

B. π

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

Answer: C

Solution:

Let $f(x) = \sin^2 x$

Now, $f(-x) = \sin^2(-x) = (\sin(-x))^2$

$= (-\sin x)^2 = \sin^2 x = f(x)$

So, f is an even function

$$\therefore \int_{-\pi/2}^{\pi/2} \sin^2 x = 2\int_0^{\pi/2} \sin^2 x\,dx$$

$$\left[\because \int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx, \text{ if } f \text{ is even}\right]$$

$$2\int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2} dx = \left[x - \frac{\sin 2x}{2}\right]_0^{\pi/2}$$

$$= \left(\frac{\pi}{2} - \frac{\sin 2\times\pi/2}{2}\right) - \left(0 - \frac{\sin(2\times 0)}{2}\right)$$

$$= \left(\frac{\pi}{2} - 0\right) - (0-0) = \frac{\pi}{2}$$

Question 25

The lines $\frac{x-1}{2} = \frac{y-4}{4} = \frac{z-2}{3}$ and $\frac{1-x}{1} = \frac{y-2}{5} = \frac{3-z}{a}$ are perpendicular to each other, then a equals to

Options:

A. -6

B. 6

C. $\frac{22}{3}$

D. $-\frac{22}{3}$

Answer: B

Solution:

Let $L_1: \frac{x-1}{2} = \frac{y-4}{4} = \frac{z-2}{3}$

and $L_2: \frac{1-x}{1} = \frac{y-2}{5} = \frac{3-z}{a}$

the line L_2 can be written as $\frac{x-1}{-1} = \frac{y-2}{5} = \frac{z-3}{-a}$

Now, the DR's of lines L_1 and L_2 are (2,4,3) and $(-1, 5, -a)$ respectively.

Since, L_1 and L_2 are perpendicular to each other.

$\therefore 2(-1) + 4)(5) + 3(-a) = 0$
$\Rightarrow -2+20-3a=0$
$\Rightarrow -3a=-18 \Rightarrow a=6$

Question 26

If two lines $L1 : \frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $L_2 : \frac{x-3}{1} = \frac{y-k}{2} = z$ intersect at a point, then $2k$ is equal to

Options:

A. 9 B. $\frac{1}{2}$

C. $\frac{9}{2}$

D. 1

Answer: A

Solution:

Let $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = \lambda$

Now, any point P that lies on the lines L 1 has the form $(1+2\lambda, -1+3, 1+4)$ λ .

Now, on putting $x = 1 + 2\lambda, y = -1+3$ λ and$ $z=1+4\lambda$ into the equation of lines L 2, we get

$$\frac{1+2\lambda-3}{1} = \frac{-1+3\lambda-k}{2} = 1+4\lambda$$

$$\Rightarrow \frac{1+2\lambda-3}{1} = 1+4\lambda$$

$$\Rightarrow -2\lambda = 3 \Rightarrow \lambda = \frac{-3}{2}$$

and $\frac{-1+3\lambda-k}{2} = 1+4\lambda$

$\Rightarrow -1+3\lambda-k = 2+8\lambda$

$\Rightarrow -5\lambda = 3+k$

$\Rightarrow -5\left(-\frac{3}{2}\right) = 3+k$ $[\because \lambda = -3/2]$

$\Rightarrow k = \frac{15}{2} - 3$

$\Rightarrow k = \frac{9}{2}$

$\Rightarrow 2k = 9$

Question 27

A five-digits number is formed by using the digits 1,2,3,4,5 with no repetition. The probability that the numbers 1 and 5 are always together, is

Options:

A. $\frac{2}{5}$

B. $\frac{1}{5}$

C. $\frac{3}{5}$

D. $\frac{1}{4}$

Answer: A

Solution:

The total number of possible five-digit numbers $= 5!$

The total number of possible five-digit numbers in which 1 and 5 are always together $= 2 \times 4!$

$\therefore$ Required probability $= \frac{2 \times 4!}{5!} = \frac{2 \times 4!}{5 \times 4!} = \frac{2}{5}$

Question 28

If a number n is chosen at random from the set {11,12,13,......,30}. Then, the probability that n is neither divisible by 3 nor divisible by 5, is

Options:

A. $\frac{7}{20}$

B. $\frac{9}{20}$

C. $\frac{11}{20}$

D. $\frac{13}{20}$

Answer: C

Solution:

Here, number which are divisible by either 3 or 5 are 12, 15, 18, 20, 21, 24, 27, 30.

∴ Total numbers =9

P(number either divisible by 3 or 5) $= \frac{9}{20}$

P(number neither divisible by 3 nor 5)

$= 1 - P$(number either divisible by 3 or 5)

$= 1 - \frac{9}{20} = \frac{11}{20}$

Question 29

Three vertices are chosen randomly from the nine vertices of a regular 9-sided polygon. The probability that they form the vertices of an isosceles triangle, is

Options:

A. $\frac{4}{7}$

B. $\frac{3}{7}$

C. $\frac{2}{7}$

D. $\frac{5}{7}$

Answer: B

Solution:

Number of triangles formed

$$={}^{9}C_{3}$$

Number of isosceles triangles $= 9 \times \left(\frac{9-1}{2}\right)$

$=9\times4=36$

So, required probability

$$= \frac{36}{{}^{9}C_{3}} = \frac{36}{\frac{9!}{3!(9-3)!}} = \frac{36\times3\times2\times6!}{9\times8\times7\times6!} = \frac{3}{7}$$

Question 30

If A,B and C are mutually exclusive and exhaustive events of a random experiment such that $P(B)=\frac{3}{2}P(A)$ and $P(C) = \frac{1}{2}P(B)$, then $P(A\cup C)$ equals to

Options:

A. $\frac{10}{13}$

B. $\frac{3}{13}$

C. $\frac{6}{13}$

D. $\frac{7}{13}$

Answer: D

Solution:

Given, $P(B) = \frac{3}{2}P(A)$ and $P(C) = \frac{1}{2}P(B)$

Since, A, B and C are mutually exclusive and exhaustive events.

$\therefore \quad P(A) + P(B) + P(C) = 1$

$\Rightarrow \quad P(A) + \frac{3}{2}P(A) + \frac{1}{2} \times \frac{3}{2}P(A) = 1$

$\Rightarrow \quad P(A)\left(1 + \frac{3}{2} + \frac{3}{4}\right) = 1$

$\Rightarrow \quad P(A)\left(\frac{13}{4}\right) = 1 \Rightarrow P(A) = \frac{4}{13}$

$\therefore \quad P(C) = \frac{1}{2} \times \frac{3}{2}P(A) = \frac{3}{4} \times \frac{4}{13} = \frac{3}{13}$

Also, A, B and C are mutually exclusive.

$\therefore P(A \cap B) = P(B \cap C) = P(C \cap A) = 0$

Now, $P(A \cup C) = P(A) + P(C) - P(A \cap C)$

$= \frac{4}{13} + \frac{3}{13} - 0 = \frac{7}{13}$

Question 31

Using mathematical induction, the numbers a_n are defined by $a_0 = 1, a_{n+1} = 3n^2 + n + a_n, (n \geq 0)$. Then, a_n is equal to

Options:

A. $n^3 + n^2 + 1$

B. $n^3 - n^2 + 1$

C. $n^3 - n^2$

D. $n^3 + n^2$

Answer: B

Solution:

Given, $a_0 = 1, a_{n+1} = 3n^2 + n + a_n$

$\Rightarrow \quad a_1 = 3(0)^2 + (0) + a_0 = 0 + 0 + 1 = 1$

$\Rightarrow \quad a_2 = 3(1)^2 + (1) + a_1 = 3 + 1 + 1 = 5$

From option (b),

Let $P(n) = n^3 - n^2 + 1$

$P(0)=(0)^3-(0)^2+1=1=a_0$

$P(1)=(1)^3-(1)^2+1=1-1+1=a_1$

$P(2)=(2)^3-(2)^2+1=8-4+1=5=a_2$

Thus, $a_n = n^3 - n^2 + 1$

Question 32

If 49^n+16^n+k is divisible by 64 for $n \in N$, then the least negative integral value of k is

Options:

A. -1

B. -2

C. -3

D. -4

Answer: A

Solution:

Let $P(n)=49^n+16^n+k$

For $n=1$, we get

$P(1) = 49(1) + 16(1) + k = 65 + k$

As $P(1)$ is divisible by 64 , we take

$k=-1$

$\therefore P(1)=65-1=64$, which is divisible by 64 .

Thus, the least negative integral value of k be -1 .

Question 33

$2^{3n} - 7n - 1$ is divisible by

Options:

A. 64 B. 36 C. 49 D.

25

Answer: C

Solution:

Let $P(n) = 23n - 7n - 1$

$\Rightarrow P(1) = 23(1) - 7(1) - 1 = 8 - 8 = 0$
$\Rightarrow P(2) = 23(2) - 7(2) - 1 = 64 - 15 = 49$

$P(1)$ and $P(2)$ are divisible by 49 .

Let $P(k) = 2^{3k} - 7k - 1 = 49t$, where t is an integer

Now,

$P(k+1) = 23(k+1) - 7(k+1) - 1 = 23k \cdot 23 - 7k - 7 - 1$
$= 8(23k - 7k - 1) + 49k$
$= 8(49t) + 49k$
$= 49(8t+k)$, where $8t+k$ is an integer

Thus, $23n - 7n - 1$ is divisible by 49.

Question 34

The sum of n terms of the series, $\frac{4}{3} + \frac{10}{9} + \frac{28}{27} + \ldots$ is

Options:

A. $\frac{3n(2n+1)+1}{2(3n)}$

B. $\frac{3n(2n+1)-1}{2(3n)}$

C. $\frac{3^n n-1}{2(3^n)}$

D. $\frac{3n-1}{2}$

Answer: B

Solution:

Given series is

$\frac{4}{3}+\frac{10}{9}+\frac{28}{27}+\dots$

The sum of the given series upto n-terms

$\frac{4}{3}+\frac{10}{9}+\frac{28}{27}+\dots$ upto n-terms

$=\left(1+\frac{1}{3}\right)+\left(1+\frac{1}{9}\right)+\left(1+\frac{1}{}\right)+\dots$ upto n -terms

$=(1+1+1+\dots$ upto n-terms$)$

$+\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\dots\right.$ upto n - terms $\left.\right)$

$=n+\frac{1}{3}\left(\frac{1-\frac{1}{3^n}}{1-\frac{1}{3}}\right)=n+\frac{(3^n-1)}{}=\frac{3^n(2n+1)-1}{2(3^n)}$

Question 35

The value of $\frac{1}{2!}+\frac{2}{3!}+\dots+\frac{99}{100!}$ is equal to

Options:

A. $\frac{100!-1}{100!}$

B. $\frac{100!+1}{100!}$

C. $\frac{999!-1}{999!}$

D. $\frac{999!+1}{999!}$

Answer: A

Solution:

Given, $\frac{1}{2!}+\frac{2}{3!}+\ldots+\frac{99}{100!}$

$= \frac{2-1}{2!}+\frac{3-1}{3!}+\frac{4-1}{4!}+\ldots+\frac{100-1}{100!}$

$= \left(\frac{1}{1!}-\frac{1}{2!}\right)+\left(\frac{1}{2!}-\frac{1}{3!}\right)+\left(\frac{1}{3!}-\frac{1}{4!}\right)+\ldots+\left(\frac{1}{99!}-\frac{1}{100!}\right)$

$= 1-\frac{1}{100!}=\frac{100!-1}{100!}$

Question 36

If the sum of 12th and 22nd terms of an AP is 100, then the sum of the first 33 terms of an AP is

Options:

A. 1700 B. 1650 C. 3300 D. 3500

Answer: B

Solution:

Here, $T_{12}=a+11d$ and $T_{22}=a+21d$

Since, $100=T_{12}+T_{22}$

$\therefore 100=a+11d+a+21d$

$\Rightarrow a+16d=50 \ldots$ (i)

Now,

$S_{33}=\frac{33}{2}[2a+(33-1)d]$

$= 33(a+16d)=33\times 50$ [From Eq. (i)]

$= 1650$

Thus, required sum be 1650 .

Question 37

The differential equation of all non-vertical lines in a plane is

Options:

A. $\frac{d^2y}{dx^2} = 0$

B. $\frac{d^2x}{dy^2} = 0$

C. $\frac{dy}{dx} = 0$

D. $\frac{dx}{dy} = 0$

Answer: A

Solution:

The general equation of all non-vertical lines in a plane is $ax + by = 1$, where $b \neq 0$.

On differentiating both sides w.r.t. x, we get

$$a + b\frac{dy}{dx} = 0$$

Again, differentiating w.r.t x, we get

$$b\frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = 0 \quad [\because b \neq 0].$$

Question 38

The general solution of $\left(\frac{dy}{dx}\right)^2 = 1 - x^2 - y^2 + x^2y^2$ is

Options:

A. $2\sin^{-1} y = x\sqrt{1 - x^2} + \sin^{-1} x + C$

B. $\cos^{-1} y = \cos^{-1} x$

C. $\sin^{-1} y = \frac{1}{2}\sin^{-1} x + C$

D. $2\sin^{-1} y = x\sqrt{1-y^2} + C$

Answer: A

Solution:

Given, $\left(\frac{dy}{dx}\right)^2 = 1 - x^2 - y^2 + x^2y^2$

$\Rightarrow \left(\frac{dy}{dx}\right)^2 = (1-y^2) - x^2(1-y^2) = (1-x^2)(1-y^2)$

$\therefore \frac{dy}{dx} = \sqrt{(1-x^2)(1-y^2)}$

$\Rightarrow \frac{dy}{\sqrt{1-y^2}} = \sqrt{1-x^2}\,dx$

On integrating both sides, we get

$$\int \frac{dy}{\sqrt{1-y^2}} = \int \sqrt{1-x^2}\,dx$$

$\Rightarrow \sin^{-1} y = \frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1} x + \frac{C}{2}$

$\Rightarrow 2\sin^{-1} y = x\sqrt{1-x^2} + \sin^{-1} x + C$

Question 39

The solution of the differential equation $\left(\frac{dy}{dx}\right)\tan y = \sin(x+y) + \sin(x-y)$ is

Options:

A. $\sec x = -2\sec y + C$

B. $\sec y = 2\cos x + C$

C. $\sec y = -2\cos x + C$

D. $\sec x = -2\cos y + C$

Answer: C

Solution:

Given, differential equation is

$$\left(\frac{dy}{dx}\right)\tan y = \sin(x+y)+\sin(x-y)$$

$$\Rightarrow \left(\frac{dy}{dx}\right)\tan y=2\sin\left(\frac{x+y+x-y}{2}\right)$$

$$\Rightarrow \left(\frac{dy}{dx}\right)\frac{\sin y}{\cos y}=2\sin x\cos y$$

$$\Rightarrow \frac{\sin y}{\cos^2 y}dy=2\sin x dx$$

On integration both sides, we get

$$\int \frac{\sin y}{\cos^2 y} dy=\int 2\sin x dx$$

$$\Rightarrow -\frac{(\cos y)^{-2+1}}{(-2+1)}=-2\cos x+C$$

$$\Rightarrow \frac{1}{\cos y}=-2\cos x+C \Rightarrow \sec y=-2\cos x + C$$

Question 40

Find ${}^nC_{21}$, if ${}^nC_{10} = {}^nC_{12}$

Options:

A. 1 B. 21

C. 22 D. 2

Answer: C

Solution:

We know that if ${}^nC_x = {}^nC_y$, then either $x = y$

or $x+y=n$

Since, ${}^nC_{10} = {}^nC_{12}$

$\therefore 10+12=n$

$\Rightarrow n=22$

Now, ${}^nC_{21} = {}^{22}C_{21}$

$$= \frac{22!}{(22-21)!21!} = \frac{22!}{1!21!} = \frac{22\times21!}{1\times21!} = 22$$

Question 41

In a trial, the probability of success is twice the probability of failure. In six trials, the probability of at most two failure will be

Options:

A. $\frac{60}{0}$ 72

B. $\frac{900}{729}$

C. $\frac{400}{729}$

D. $\frac{496}{729}$

Answer: D

Solution:

Let the probability of failure and success be p and q respectively.

Let X represents the number of failure

According to the question, $q = 2p$

$\because\ p+q=1$ and $q=2p$

$\therefore\ p=\frac{1}{3}$ and $q=\frac{2}{3}$

Now, required probability $= P(X \le 2)$

$= P(X=0) + P(X=1) + P(X=2)$

$= {}^6C_0\, p^0 q^6 + {}^6C_1\, p^1 q^5 + {}^6C_2\, p^2 q^4$

$= \left(\frac{2}{3}\right)^6 + 6\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^5 + 15\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^4$

$= \frac{1}{729}(64+192+240) = \frac{496}{729}$

Question 42

If $\cos A = m\cos B$ and $\cot\left(\frac{A+B}{2}\right) = \lambda \tan\left(\frac{B-A}{2}\right)$, then λ is equal to

Options:

A. $\frac{m}{m-1}$

B. $\frac{m+1}{m}$

C. $\frac{m+1}{m-1}$

D. None of these

Answer: C

Solution:

Given, $\cos A = m\cos B \Rightarrow \frac{\cos A}{\cos B} = m$

On applying componendo and dividendo rule, we get

$$\frac{\cos A+\cos B}{\cos A-\cos B} = \frac{m+1}{m-1}$$

$$\Rightarrow \frac{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)} = \frac{m+1}{m-1}$$

$$\Rightarrow \frac{\cot\left(\frac{A+B}{2}\right)}{\tan\left(\frac{B-A}{2}\right)} = \frac{m+1}{m-1}$$

$$\Rightarrow \cot\left(\frac{A+B}{2}\right) = \left(\frac{m+1}{m-1}\right)\tan\left(\frac{B-A}{2}\right)$$

$$\therefore \lambda = \frac{m+1}{m-1}$$

Question 43

The expression $\frac{2\tan A}{1-\cot A} + \frac{2\cot A}{1-\tan A}$ can be written as

Options:

A. $\sin 2A + \cos 2A$

B. $2 \sec A \operatorname{cosec} A + 2$

C. $\tan 2A + \cot 2A$

D. $\sec 2A + \operatorname{cosec} 2A$

Answer: B

Solution:

Given, $\frac{2\tan A}{1-\cot A} + \frac{2\cot A}{1-\tan A}$

$= \frac{\frac{2\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}} + \frac{\frac{2\cos A}{\sin A}}{1-\frac{\sin A}{\cos A}}$

$= (\sin A)\left(\frac{2\sin A}{\cos A}\right) + \left(\frac{\cos A}{}\right)\left(\frac{2\cos A}{}\right)A$

$= \frac{2}{\sin A-\cos A}\left[\frac{\sin A^2}{\cos A} - \frac{\cos A^2}{}\right]$

$= \frac{2}{\sin A-\cos A}\left[\frac{\sin A^3-\cos A^3}{\sin A\cos A}\right]$

$= \frac{2(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cos A)}{(\sin A - \cos A)(\sin A \cos A)}$

$= 2\left(\frac{1}{\sin A\cos A} + 1\right)$

$= 2(\sec A \operatorname{cosec} A + 1) = 2 \sec A \operatorname{cosec} A + 2$

Question 44

The general solution of $2 \cos 4x + \sin^2 2x = 0$ is

Options:

A. $x = \frac{n\pi}{2} \pm \sin^{-1}\left(\frac{1}{5}\right)$

B. $x = \frac{n\pi}{4} + \frac{(-1)^n}{4} \sin^{-1}\left(\pm \frac{2\sqrt{2}}{3}\right)$

C. $x = \frac{n\pi}{2} \pm \cos^{-1}\left(\frac{1}{5}\right)$

D. $x = \frac{n\pi}{4} + \frac{(-1)^n}{4} \cos^{-1}\left(\frac{1}{5}\right)$

Answer: B

Solution:

Given, $2\cos 4x + \sin^2 2x = 0$

$\Rightarrow 2\cos 4x + \left(\frac{1-\cos 4x}{2}\right) = 0$

$\Rightarrow 3\cos 4x + 1 = 0$

$\Rightarrow \cos 4x = -\frac{1}{3}$

$\Rightarrow \sin 4x = \pm \frac{2\sqrt{2}}{3}$

$\Rightarrow 4x = n\pi + (-1)^n \sin^{-1}\left(\pm \frac{2\sqrt{2}}{3}\right)$

$\Rightarrow x = \frac{n\pi}{4} + \frac{(-1)^n}{4} \sin^{-1}\left(\pm \frac{2\sqrt{2}}{3}\right)$

Question 45

If $2f(x^2) + 3f\left(\frac{1}{x^2}\right) = x^2 - 1, \forall x \in R - \{0\}$, then $f(x^8)$ is equal to

Options:

A. $\frac{(1-x^8)(2x^8+3)}{5x^8}$

B. $\frac{(1+x^8)(2x^8-3)}{5x^8}$

C. $\frac{(1-x^8)(2x^8-3)}{5x^8}$

D. None of these

Answer: A

Solution:

Given, $2f(x^2)+3f\left(\frac{1}{x^2}\right)=x^2-1$ .... (i)

Replacing x by $\frac{1}{x}$, we get

$2f\left(\frac{1}{x^2}\right)+3f(x^2)=\frac{1}{x^2}-1$ ..... (ii)

On multiplying Eq. (i) by 2, Eq. (ii) by 3 and then subtracting Eq. (i) from Eq. (ii), we get

$$5f(x^2)=3\left(\frac{1}{x^2}-1\right)-2(x^2-1)$$

$$\Rightarrow 5f(x^2)=\frac{3}{x^2}-2x^2-1$$

$$\Rightarrow f(x^2)=\frac{1}{5}\left(\frac{3}{x^2}-2x^2-1\right)$$

$$\Rightarrow f(x^2)=\frac{1}{5x^2}(3-2x^4-x^2)$$

$$\Rightarrow f(x^2)=\frac{(2x^2+3)(1-x^2)}{5x^2}$$

$$\therefore f(x^8)=\frac{(1-x^8)(2x^8+3)}{5x^8}$$

Question 46

If $A=\{a,b,c\}$, $B=\{b,c,d\}$ and $C=\{a,d,c\}$, then $(A-B)\times(B\cap C)$ is equal to

Options:

A. $\{(a,c),(a,d)\}$

B. $\{(a,b),(c,d)\}$

C. $\{(c,a),(d,a)\}$

D. $\{(a,c),(a,d),(b,d)\}$

Answer: A

Solution:

Given, $A=\{a,b,c\}$, $B=\{b,c,d\}$ and $C=\{a,d,c\}$

Now, $A-B=\{a,b,c\}-\{b,c,d\}=\{a\}$

and $B\cap C=\{b,c,d\}\cap\{a,d,c\}=\{c,d\}$

$\therefore (A-B)\times(B\cap C)=\{a\}\times\{c,d\}=\{(a,c),(a,d)\}$

Question 47

If $n(A)=p$ and $n(B)=q$, then the numbers of relations from the set A to the set B is

Options:

A. $2p+q$

B. $2pq$

C. $p+q$

D. pq

Answer: B

Solution:

Given; $n(A)=p$ and $n(B)=q$

$\therefore n(A\times B)=pq$

The number of relations from a set A to a set B is same as the total number of subset of the set $A\times B$.

We know that if $n(A)=k$, then $n(P(A))=2k$

Now, the total number of subset of $A\times B$ be $2pq$

$\therefore$ Then number of relations from the set A to the set B is $2pq$.

Question 48

If $z=\sqrt{3}+i$, then the argument of z^2e^{z-i} is equal to

Options:

A. $e^{\pi/3}$

B. $\frac{\pi}{3}$

C. π

D. $e^{\pi/6}$

Answer: B

Solution:

Given, $Z = \sqrt{3} + i$

$\therefore \arg\left(z^2 e^{z-i}\right) = \arg\left[(\sqrt{3}+i)^2 e^{\sqrt{3}+i-i}\right]$

$= \arg\left[(2+2\sqrt{3}i)e^{\sqrt{3}}\right]$

$= \arg\left[2e^{\sqrt{3}}(1+\sqrt{3}i)\right]$

$= \arg[(1 + \sqrt{3}i)]$

$= \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \tan^{-1}\left(\tan\frac{\pi}{3}\right) = \frac{\pi}{3}$

Question 49

If $i = \sqrt{-1}$ and n is a positive integer, then $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is equal to

Options:

A. 1

B. i

C. i^n

D. 0

Answer: D

Solution:

Given, $i^n + i^{n+1} + i^{n+2} + i^{n+3} = i^n(1 + i + i^2 + i^3)$

$= i^n(1 + i + (-1) + (i^2)i)$

$= i^n(1 + i + (-1) + (-1)i)$

$= i^n[(1 + i) - (1 + i)] = i^n(0) = 0$

Question 50

If $\left(\frac{3}{2} + \frac{\sqrt{3}i}{2}\right)^{50} = 3^{25}(x + iy)$, where x and y are real, then the ordered pair $(2x, 2y)$ is

Options:

A. $(-6, 0)$

B. $(0, 6)$

C. $(0, -6)$

D. $(1, \sqrt{3})$

Answer: D

Solution:

We have, $\left(\frac{3}{2}+i\frac{\sqrt{3}}{2}\right)^{50}=3^{25}(x+iy)$

$\Rightarrow (\sqrt{3})^{50}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^{50}=3^{25}(x+iy)$

$\Rightarrow 3^{25}\left[-i\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\right]^{50}=3^{25}(x+iy)$

$\Rightarrow (-i)^{50}\omega^{50}=x+iy$

$\Rightarrow (i^{4})^{12}\cdot i^{2}\cdot(\omega^{3})^{16}\cdot\omega^{2}=x+iy$

$\Rightarrow (1)^{12}\cdot(-1)\cdot(1)^{16}\cdot\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)=x+iy$

$\Rightarrow \frac{1}{2}+i\frac{\sqrt{3}}{2}=x+iy$

$\Rightarrow 1+i\sqrt{3}=2x+i(2y)$

$\therefore (2x,2y)=(1,\sqrt{3})$

Question 51

There are 10 points in a plane out of which 4 points are collinear. How many straight lines can be drawn by joining any two of them?

Options:

A. 39 B. 40 C. 45 D. 21

Answer: B

Solution:

From 10 given points, ${}^{10}C_2$ straight lines can be drawn.

But 4 points are collinear, using 4 points, ${}^{4}C_2$ straight lines can be drawn.

From 4 col linear points, 1 straight line can be drawn. So, total number of straight lines $= {}^{10}C_2 - {}^{4}C_2 + 1$

$= \frac{10!}{8!2!} - \frac{4!}{2!2!} + 1$

$= 45 - 6 + 1 = 40$

Question 52

The total number of numbers greater than 1000 but less than 4000 that can be formed using 0, 2, 3, 4 (using repetition allowed) are

Options:

A. 125 B. 105 C. 128 D. 625

Answer: C

Solution:

Since, numbers should be greater than 1000 but less than 4000.

∴ The first digit: must be either 2 or 3.

It is clear that required numbers must be 4 digit numbers.

Now, there are four choices $(0, 2, 3, 4)$ for each unit, ten and hundred place digit.

Thus, total number $= {}^2C_1 \times 4 \times 4 \times 4$

$= 2 \times 4 \times 4 \times 4 = 128$

Question 53

A polygon of n sides has 105 diagonals, then n is equal to

Options:

A. 20

B. 21 C. 15

D. −14

Answer: C

Solution:

∵ The total number of lines joining any two points of the polygon is given by $^nC_2$

So, $^nC_2 = 105$

$\Rightarrow \frac{n(n-1)}{2} = 105$

$\Rightarrow n^2 - n = 210$

$\Rightarrow n^2 - n - 210 = 0$

$\Rightarrow n^2 - 15n + 14n - 210 = 0$

$\Rightarrow n(n-15) + 14(n-15) = 0$

$\Rightarrow (n-15)(n+14) = 0$

either $n - 15 = 0$ or $n + 14 = 0$

$\Rightarrow n = 15$ or -14

∵ Number of sides cannot be negative

$\therefore n = 15$

Question 54

Let the equation of pair of lines $y=m_1x$ and $y=m_2x$ can be written as $(y-m_1x)(y-m_2x)=0$. Then, the equation of the pair of the angle bisector of the line $3y^2-5xy-2x^2=0$ is

Options:

A. $x^2+5xy-y^2=0$

B. $x^2-5xy+y^2=0$

C. $x^2-xy+y^2=0$

D. $x^2+xy-y^2=0$

Answer: D

Solution:

$\because$ Equation of angles of bisector of pair of straight line, $ax^2 + 2hxy + by^2$ is $\frac{x^2-y^2}{a-b} = \frac{xy}{h}$

$\therefore$ For, $3y^2 - 5xy - 2x^2 = 0$

$a = 3, b = -2, h = -5$

So, equation of angle bisector is

$$\frac{x^2-y^2}{3-(-2)} = \frac{xy}{-5}$$

$$\Rightarrow \frac{x^2-y^2}{5} = \frac{xy}{-5} \Rightarrow x^2-y^2+xy = 0$$

Question 55

The distance of the point (3,4) from the line $3x+2y+7=0$ measured along the line parallel to $y-2x+7=0$ is equal to

Options:

A. $\frac{24\sqrt{5}}{7}$

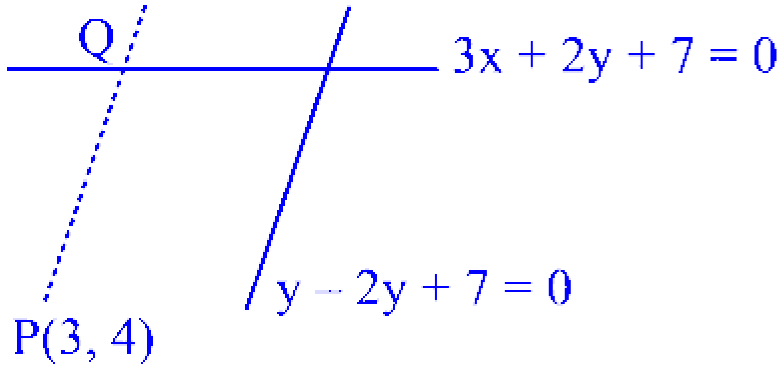
B. $3\sqrt{5}$

C. $\frac{23\sqrt{5}}{7}$

D. $4\sqrt{5}$

Answer: A

Solution:



The slope of the line, $-2y\ x+7=0$

$\Rightarrow y=2x-7$

Slope $(m)=2$

$\therefore$ Slope of $PQ=m_{PQ}=2$

Equation of PQ,

$(y-4)=2(x-3)$

$\Rightarrow y=2x-2$ ... (i)

and $3x+2y+7=0$ ... (ii)

On putting, the value of y in Eq. (ii), we get

$$3x+2(2x-2)+7=0$$

$\Rightarrow 7x=-3 \Rightarrow x=-\frac{3}{7}$

Then, $y=2\times\left(-\frac{3}{7}\right)-2=-\frac{20}{7}$

So, coordinates of $Q=\left(-\frac{3}{7},-\frac{20}{7}\right)$

Thus, distance

$$PQ=\sqrt{\left(3-\left(-\frac{3}{7}\right)\right)^2+\left(4-\left(-\frac{20}{7}\right)\right)^2}=\frac{24\sqrt{5}}{7}$$

Question 56

The slope of lines which makes an angle 60° with the line $y-3x-18=0$

Options:

A. $\frac{3\sqrt{3}-3}{1+3\sqrt{3}}, \frac{3\sqrt{3}-3}{1+3\sqrt{3}}$

B. $\frac{3-\sqrt{3}}{1+3\sqrt{3}}, \frac{3+\sqrt{3}}{1-3\sqrt{3}}$

C. $\frac{3}{1+\sqrt{3}}, \frac{3}{1-\sqrt{3}}$

D. $\sqrt{3}-1, \sqrt{3}+13$

Answer: B

Solution:

Slope of the line,

$y-3x+18=0 \Rightarrow y=3x-18 \Rightarrow$ Slope $(m_1)=3$

and angle $(\theta)=60^\circ$

so, $\tan 60^\circ = \frac{m_1-m_2}{1+m_1m_2}$

$\Rightarrow \sqrt{3} = \frac{3-m_2}{1+3m_2}$

$\Rightarrow \frac{3-m_2}{1+3m_2} = \pm\sqrt{3}$

Either, $\frac{3-m_2}{1+3m_2}=\sqrt{3}$ or $\frac{3-m_2}{1+3m_2}=-\sqrt{3}$

$\Rightarrow \quad 3-m_2=\sqrt{3}+3\sqrt{3}m_2$

or $\quad 3-m_2=-\sqrt{3}-3\sqrt{3}m_2$

$\Rightarrow \quad m_2(1+3\sqrt{3})=3-\sqrt{3}$

or $\quad m_2(1-3\sqrt{3})=3+\sqrt{3}$

$\Rightarrow m_2=\frac{3-\sqrt{3}}{1+3\sqrt{3}}$ or $m_2=\frac{3+\sqrt{3}}{1-3\sqrt{3}}$

$\therefore \quad m_2=\frac{3-\sqrt{3}}{1+3\sqrt{3}}, \frac{3+\sqrt{3}}{1-3\sqrt{3}}$

Question 57

3 and 5 are intercepts of a line $L=0$, then the distance of $L=0$ from $(3,7)$ is

Options:

A. $\sqrt{31}$

B. $\sqrt{34}$

C. $\frac{21}{\sqrt{34}}$

D. $\frac{\sqrt{34}}{31}$

Answer: C

Solution:

If 3 and 5 are intercepts of a line

$L = 0$, then

x-intercept $= a = 3$

y-intercept $= b = 5$

Equation of line is

$\frac{x}{3} + \frac{y}{5} = 1 \Rightarrow 5x + 3y - 15 = 0$

$\therefore$ Required distance

$= \frac{5(3)+3(7)-15}{\sqrt{52+32}} = \frac{21}{\sqrt{34}}$

Question 58

The total number of terms in the expansion of $(x+y)^{60} + (x-y)^{60}$ is

Options:

A. 60 B. 61

C. 30 D. 31

Answer: D

Solution:

$(x+y)^{60} = {}^{60}C_0 x^{60} + {}^{60}C_1 x^{59} y + \dots + {}^{60}C_{60} y^{60}$ .... (i)

$(x-y)^{60} = {}^{60}C_0 x^{60} - {}^{60}C_1 x^{59} y + \dots + {}^{60}C_{60} y^{60}$ .... (ii)

By adding Eq. (i) and Eq. (ii)

$(x+y)^{60} + (x-y)^{60}$

$= 2\,({}^{60}C_0 x^{60} + {}^{60}C_2 x^{58} y^2 + \dots + {}^{60}C_{60} y^{60})$ — 31 terms

Hence, the expansion of $(x+y)^{60}+(x-y)^{60}$ has 31 terms.

Question 59

The coefficient of x^{29} in the expansion of $(1-3x+3x^2-x^3)^{15}$ is

Options:

A. ${}^{45}C_{29}$

B. ${}^{45}C_{28}$

C. $-{}^{45}C_{16}$

D. ${}^{45}C_{30}$

Answer: C

Solution:

$(1-3x+3x^2-x^3)^{15} = [(1-x)^3]^{15}$
$= (1-x)^{45}$

So, $T_{r+1} = {}^{45}C_r (1)^{45-r}(-x)^r$

For coefficient of x^{29}, put $r=29$

Then, $T_{30} = {}^{45}C_{29}(1)^{45-29}(-x)^{29} = -{}^{45}C_{29} x^{29}$

Hence, coefficient of $x^{29} = -{}^{45}C_{29}$ and $-{}^{45}C_{29} = -{}^{45}C_{16}$

Question 60

In the expansion of $(1+3x+3x^2+x^3)^{2n}$, the term which has greatest binomial coefficient, is

Options:

A. $(3n)$ th term

B. $(3n+1)$ th term

C. $(3n-1)$ th term

D. $(3n+2)$ th term

Answer: B

Solution:

$\because$ Middle term has greatest binomial coefficient. In the expansion of $(1+3x+3x^2+x^3)^{2n}$

$= ((1+x)^3)^{2n} = (1+x)^{6n}$

$\because 6n$ is even

So, middle term of $(1+x)^{6n} = T_{\left(\frac{6n}{2}+1\right)}$

$= T_{(3n+1)} = (3n+1)$ th term.
