

Mathematics:

1. Calculus:

- "If $y=\sin(x^2)$, find $\frac{dy}{dx}$."
 - Solution: Using the chain rule, $\frac{dy}{dx}=\cos(x^2) \cdot 2x=2x\cos(x^2)$.
- "Evaluate the integral: $\int_0^{\pi/2} \cos(x) dx$."
 - Solution: $\int \cos(x) dx = \sin(x)$. Therefore, $\int_0^{\pi/2} \cos(x) dx = \sin(\pi/2) - \sin(0) = 1 - 0 = 1$.

2. Algebra:

- "Find the determinant of the matrix: $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$."
 - Solution: The determinant is $(2 \times 4) - (1 \times 3) = 8 - 3 = 5$.
- "What is the number of ways of selecting 3 balls from 5 different balls?"
 - Solution: This is a combination problem. We need to calculate 5C_3 . ${}^5C_3 = \frac{5!}{3!(5-3)!} = \frac{5!}{3!2!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 2 \times 1} = 2 \times 1 \times 1 = 2$.

3. Coordinate Geometry:

- "Find the equation of a line passing through the points (1, 2) and (3, 4)."
 - Solution: The slope is $\frac{4-2}{3-1} = \frac{2}{2} = 1$. Using the point-slope form, $y-2=1(x-1)$, which simplifies to $y=x+1$.
- "Find the equation of a circle with center (0,0) and radius of 5."
 - Solution: The equation of a circle with center (h,k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$. Therefore, $x^2 + y^2 = 25$.

4. Trigonometry:

- "Solve for x: $\sin(2x) = \frac{1}{2}$ "
 - Solution: $2x = \frac{\pi}{6} + 2n\pi$ or $2x = \frac{5\pi}{6} + 2n\pi$, where n is an integer. Thus, $x = \frac{\pi}{12} + n\pi$ or $x = \frac{5\pi}{12} + n\pi$.

5. Find the equation of the tangent to the curve $y=x^2-4x+3$ at the point (4, 3).

● Solution:

- First, find the derivative: $\frac{dy}{dx} = 2x - 4$.
- Evaluate the derivative at $x = 4$: $\frac{dy}{dx} = 2(4) - 4 = 4$. This is the slope of the tangent.
- Use the point-slope form of a line: $y - y_1 = m(x - x_1)$.
- Substitute (4, 3) and $m = 4$: $y - 3 = 4(x - 4)$.
- Simplify: $y = 4x - 13$.

6. Solve the system of equations: $2x+y=7$ and $x-y=2$.

● Solution:

- Add the two equations: $3x = 9$, so $x = 3$.
- Substitute $x = 3$ into either equation. Using the second equation: $3 - y = 2$, so $y = 1$.
- Solution: $x = 3, y = 1$.

7. If $\sin(A) = \frac{3}{5}$, find $\cos(2A)$.

● Solution:

- Use the identity: $\cos(2A) = 1 - 2\sin^2(A)$.

- Substitute $\sin(A)=53$: $\cos(2A)=1-2(53)^2=1-2(259)=1-2518=257$.

Physics:

1. Mechanics:

- "A car accelerates from rest to a velocity of 20 m/s in 5 seconds. Calculate the acceleration."
 - Solution: Acceleration = (change in velocity) / (time) = (20 m/s) / (5 s) = 4 m/s².
- "A mass of 2kg is raised to a height of 10m. What is the potential energy gained?"
 - Solution: Potential energy (PE) = mgh = (2 kg) × (9.8 m/s²) × (10 m) = 196 Joules.

2. Electricity and Magnetism:

- "What is the equivalent resistance of two resistors of 10 ohms and 20 ohms connected in parallel?"
 - Solution: $1/R_{eq} = 1/R_1 + 1/R_2$. $1/R_{eq} = 1/10 + 1/20 = 3/20$. Therefore, $R_{eq} = 20/3$ ohms.
- "Describe the magnetic field around a current carrying wire."
 - Solution: The magnetic field forms concentric circles around the wire, with the direction determined by the right-hand rule.

3. Optics:

- "What is the focal length of a convex lens that forms a real image at 20cm when an object is placed at 10cm?"
 - Solution: Using the lens formula, $1/f = 1/v + 1/u$, where f is the focal length, v is the image distance, and u is the object distance. $1/f = 1/20 + 1/(-10)$. $1/f = 1/20 - 2/20 = -1/20$. This indicates that the problem was badly written, as a positive focal length is expected for a converging lens. If the image was created on the same side of the lens, then the image distance would be negative. If the question was written correctly, the object would have to be placed further from the lens than the image.
- "Explain the phenomena of total internal reflection."
 - Solution: Total internal reflection occurs when light travelling from a denser medium to a rarer medium strikes the interface at an angle greater than the critical angle. The light is then reflected into the denser medium.

4. Modern Physics:

- "Explain the photoelectric effect."
 - Solution: The photoelectric effect is the emission of electrons from a metal surface when light of suitable frequency shines on it. It demonstrates the particle nature of light.

5. A transformer has 100 turns in the primary coil and 500 turns in the secondary coil. If the primary voltage is 120V, what is the secondary voltage?

- Solution:
 - Use the transformer equation: $V_p V_s = N_p N_s$, where V is voltage and N is the number of turns.
 - $120 V_s = 100 \times 500$.
 - $V_s = 120 \times 5 = 600$ V.

6. A radioactive substance has a half-life of 10 days. How much of the substance will remain after 30 days if the initial amount is 100 grams?

- Solution:
 - Number of half-lives = 30 days / 10 days = 3.
 - Amount remaining = Initial amount / 2^n , where n is the number of half-lives.
 - Amount remaining = 100 grams / $2^3 = 100$ grams / 8 = 12.5 grams.

Chemistry:

1. Physical Chemistry:

- "What is the pH of a solution with a hydrogen ion concentration of 10^{-3} M?"
 - Solution: $\text{pH} = -\log[\text{H}^+] = -\log(10^{-3}) = 3$.
- "Describe the laws of thermodynamics."
 - Solution:
 - Zeroth Law: If two systems are each in thermal equilibrium with a third system, they are also in thermal equilibrium with each other.¹
 - First Law: Energy can neither be created nor destroyed, only converted from one form to another.
 - Second Law: The entropy of an isolated system always increases over time.
 - Third Law: The entropy of a perfect crystal at absolute zero is zero.

2. Inorganic Chemistry:

- "What are the properties of the elements in the p-block of the periodic table?"
 - Solution: P-block elements exhibit a wide range of properties, including metals, non-metals, and metalloids. They show variable oxidation states and form covalent and ionic compounds.
- "Explain the formation of coordinate bonds."
 - Solution: A coordinate bond is formed when one atom donates both electrons to the shared pair, creating a covalent bond.

3. Organic Chemistry:

- "What is the product of the reaction between ethanol and ethanoic acid?"
 - Solution: The reaction is esterification, producing ethyl ethanoate (an ester) and water: $\text{CH}_3\text{COOH} + \text{C}_2\text{H}_5\text{OH} \rightarrow \text{CH}_3\text{COOC}_2\text{H}_5 + \text{H}_2\text{O}$
- "Explain the difference between alkanes, alkenes and alkynes."

- Solution: Alkanes have only single bonds, alkenes have at least one double bond, and alkynes have at least one triple bond.

4. Question: What is the oxidation number of chromium in $K_2Cr_2O_7$?

- Solution:
 - Potassium (K) has an oxidation number of +1, and oxygen (O) has an oxidation number of -2.
 - Let the oxidation number of chromium (Cr) be x.
 - $2(+1)+2(x)+7(-2)=0$.
 - $2+2x-14=0$.
 - $2x=12$, so $x=+6$.

5. Question: Write the balanced chemical equation for the reaction between aluminium and hydrochloric acid.

- Solution:
 - $2Al+6HCl\rightarrow 2AlCl_3+3H_2$

6. What is the product of the dehydration of propan-2-ol?

- Solution:
 - Dehydration of propan-2-ol leads to the formation of propene.
 - $CH_3CH(OH)CH_3\rightarrow CH_3CH=CH_2+H_2O$

7. What are the products of the electrolysis of aqueous sodium chloride (NaCl)?

- Solution:
 - At the cathode: $2H_2O+2e^-\rightarrow H_2+2OH^-$.
 - At the anode: $2Cl^-\rightarrow Cl_2+2e^-$.
 - Overall: $2NaCl+2H_2O\rightarrow 2NaOH+H_2+Cl_2$.
 - Products are Hydrogen gas, Chlorine gas, and Sodium hydroxide.

8. What is the hybridization of the central atom in CH_4 (methane)?

- Solution:
 - Carbon has four bond pairs and no lone pairs.
 - The hybridization is sp^3 .

9. What is the major product of the addition of HBr to propene?

- Solution:
 - According to Markovnikov's rule, the hydrogen atom adds to the carbon atom with more hydrogen atoms, and the bromine atom adds to the carbon atom with fewer hydrogen atoms.
 - The product is 2-bromopropane.
 - $CH_3CH=CH_2+HBr\rightarrow CH_3CHBrCH_3$