KARNATAKA SCHOOL EXAMINATION AND ASSESSMENT BOARD

II PUC EXAM-1, MARCH 2025

Subject: 35-Mathematics SCHEME OF VALUATION MAX. MARKS: 80

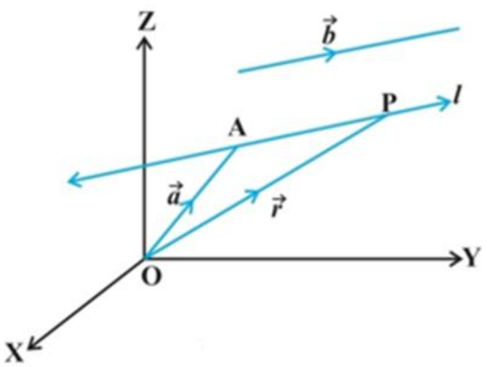
Instructions:

a) Any answer by alternate method should be valued and suitably awarded.
b) All answers (including extra, struck off and repeated) should be valued. Answers with maximum marks must be considered.

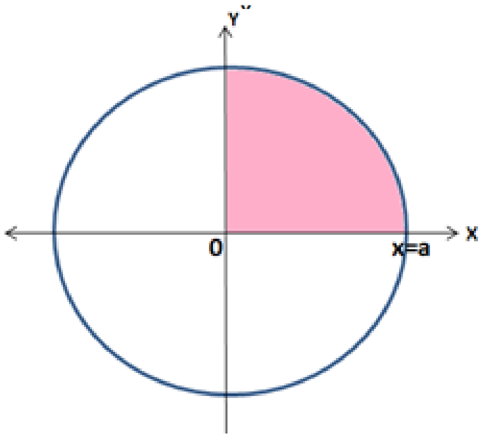
Qn No :	PART A I	Marks
1	a) or writing $(a, a) \in R$ for all $a \in A$	1
2	c) or writing $\frac{\pi}{4}$	1
3	d) or writing A-iii, B-i, C-ii	1
4	b) or writing $\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$	1
5	d) or writing $\|A\|^2$	1
6	b) or writing -2	1
7	a) or writing Statement 1 is true and statement 2 is false	1
8	d) or writing 8	1
9	a) or writing $-e^x \cos x$	1
10	d) or writing $not\ defined$	1
11	b) or writing $\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$	1
12	c) or writing $\frac{\pi}{4}$	1
13	b) or writing $x = 0, z = 0$	1
14	a) or writing $\frac{1}{3}$	1
15	c) or both [A] and [R] are true	1
	II	
16	1	1
17	-1	1
18	6	1
19	0	1
20	$\frac{5}{9}$	1

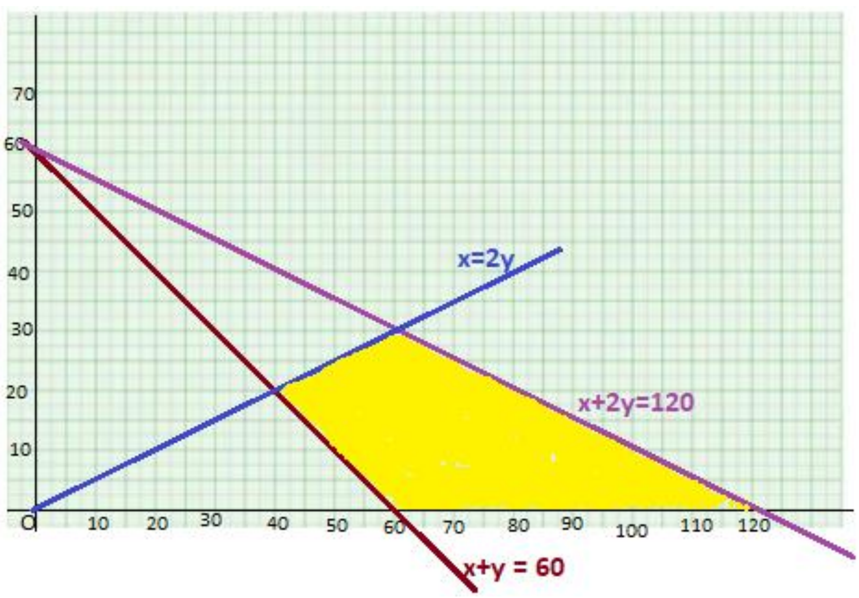
PART B		
21	Writing $\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$ **OR** $\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$	1
	Getting $-4x + 2y = 0$ **OR** $4x - 2y = 0$ **OR** $2x - y = 0$	1
22	Writing $\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$	1
	Getting $\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}} \Longrightarrow \frac{dy}{dx} + \sqrt{\frac{y}{x}} = 0$	1
23	Writing volume of the sphere, $V = \frac{4}{3}\pi r^3$ and $\frac{dV}{dr} = 4\pi r^2$	1
	Getting $\frac{dV}{dr} = 4\pi 10^2 = 400\pi\, cm^3/cm$ (**Note:** Units are not compulsory)	1
24	Writing $f'(x) = 12x^2 - 12x - 72.$ **OR** $f'(x) = 12(x-3)(x+2)$	1
	Getting $x \in (-2, 3)$	1
25	Put $\log(\sin x) = t \Rightarrow dt = \cot x\, dx$	1
	Getting $\int t\, dt = \frac{t^2}{2} + C = \frac{(\log(\sin x))^2}{2} + C$	1
26	Writing, $\frac{dy}{dx} = -a sinx + b\, cosx$ **OR** $\frac{d^2y}{dx^2} = -a cosx - b\, sinx$	1
	Getting $\frac{d^2y}{dx^2} = -y \Rightarrow \frac{d^2y}{dx^2} + y = 0$ Therefore $y = acosx + bsinx$ is solution of $\frac{d^2y}{dx^2} + y = 0$.	1
27	Getting $\vec{d} = 2\vec{a} - \vec{b} + 3\vec{c} = 3\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$	1
	Getting $\lvert\vec{d}\rvert = \sqrt{22}$ and unit vector = $\hat{d} = \frac{3\hat{\imath} - 3\hat{\jmath} + 2\hat{k}}{\sqrt{22}}$.	1
28	Writing $a_1a_2 + b_1b_2 + c_1c_2 = 0$ **OR** $(-3)(3k) + (2k)(1) + (2)(-5) = 0$	1
	Getting $k = -\frac{10}{7}$	1
29	Writing P(black ball in first draw) = P(E) $= \frac{10}{15}$ OR P(black ball in second draw) = P(F|E) $= \frac{9}{14}$	1
	Getting $P(E \cap F) = P(E).P(F\|E) = \frac{10}{15}.\frac{9}{14} = \frac{3}{7}$	1

PART-C		
30	Proving not reflexive (by giving suitable counter example) [Say a = 0.1 : $a \le a^3$ *is not true* (*here* $0 < a < 1$)]	1
	Proving not symmetric: By giving any suitable counter example [say $(1,2) \in R$ but $(2,1) \notin R \Rightarrow$ R is not symmetric.]	1
	Proving not transitive: By giving any suitable counter example [Say a = 9, b = 3 and c=2 , $9 \le 3^3$, $3 \le 2^3$ but $(9,2) \notin R$]	1
31	Writing LHS = $\sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5} = \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{4}{3}$	1
	Writing : $= \tan^{-1}\left(\frac{\frac{5}{12}+\frac{4}{3}}{1-\left(\frac{5}{12}\right)\left(\frac{4}{3}\right)}\right)$	1
	Getting: L.H.S = $\tan^{-1}\left(\frac{15+48}{36-20}\right) = \tan^{-1}\left(\frac{63}{16}\right)$	1
OR	Writing A = $\sin^{-1}\frac{5}{13}$, $B = \cos^{-1}\frac{3}{5} \Rightarrow \sin A = \frac{5}{13}, \cos B = \frac{3}{5}$	1
	Writing : $\tan A = \frac{5}{12}$, $\tan B = \frac{4}{3}$, $\tan(A+B) = \frac{tanA+tanB}{1-tanAtanB}$	1
	Getting: $\tan(A+B) = \frac{63}{16} \Rightarrow$ A+B = $\tan^{-1}\left(\frac{63}{16}\right)$	1
	OR any other Alternate method allot appropriate marks	
32	Getting: $(A+A') = \begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix}$ OR $\frac{1}{2}(A+A') = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$	1
	Getting: $(A-A') = \begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$ OR $\frac{1}{2}(A-A') = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$	1
	Getting: $\frac{1}{2}(A+A') + \frac{1}{2}(A-A') = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} = A$	1
33	Getting: $\frac{dx}{d\theta} = a(-\sin t + \frac{1}{\tan\left(\frac{t}{2}\right)}.\sec^2\left(\frac{t}{2}\right).\frac{1}{2}) = a\frac{\cos^2 t}{\sin t}$	1
	Getting: $\frac{dy}{d\theta} = a \cos t$	1
	Getting: $\frac{dy}{dx} = \frac{a \cos t}{a\left(\frac{\cos^2 t}{\sin t}\right)}$ OR $\frac{dy}{dx} = tant$	1
34		
	Writing: $P = xy^3$ and $P = (60-y)y^3$ OR $P = x(60-x)^3$	1
	Getting: $\frac{dP}{dy} = 60 \times 3y^2 - 4y^3$ OR $\frac{dP}{dx} = -3x(60-x)^2 + (60-x)^3$	1
	Getting: $x = 15$ and $y = 45$ and showing $\frac{d^2p}{dy^2} < 0$	1

35	$$\frac{2x}{x^2+3x+2}=\frac{2x}{(x+1)(x+2)}=\frac{A}{x+1}+\frac{B}{x+2}$$	1
	Getting: $A=\ -2,\ \ B=4$	1
	Getting $\int\frac{-2}{x+1}dx+\int\frac{4}{x+2}dx=-2\log\lvert x+1\rvert+4\log\lvert x+2\rvert+c$ (without c deduct 1 mark)	1
OR	Writing I = $\int\frac{2x}{x^2+3x+2}dx=\int\frac{2x+3-3}{x^2+3x+2}dx$	1
	Getting: I = $\int\frac{d(x^2+3x+2)}{x^2+3x+2}dx-\int\frac{3}{(x+\frac{3}{2})^2-\frac{1}{4}}dx$	1
	Getting: $I=\log(x^2+3x+2)-\ 3\log\left(\frac{x+\frac{3}{2}-\frac{1}{2}}{x+\frac{3}{2}+\frac{1}{2}}\right)$+c $=\log(x^2+3x+2)-3\log\left(\frac{x+1}{x+2}\right)$+c OR $-2\log\lvert x+1\rvert+4\log\lvert x+2\rvert+c$	1
36	Writing : $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=-\hat{\imath}+3\hat{\jmath}-\hat{k}$ and $\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=-\hat{\imath}+2\hat{\jmath}+\hat{k}$	1
	Getting: $\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 3 & -1\\ -1 & 2 & 1\end{vmatrix}=5\hat{\imath}+2\hat{\imath}+\hat{k}$	1
	Writing : Area of $\triangle ABC=\frac{1}{2}\lvert\overrightarrow{AB}\times\overrightarrow{AC}\rvert=\frac{\sqrt{30}}{2}$ square units (unit not Compulsory)	1
37	Writing correct figure (Writing x, y and z axes necessary) 	1
	Writing $\overrightarrow{AP}=\lambda\vec{b}$ where λ is a scalar	1
	Getting $\vec{r}=\vec{a}+\lambda\vec{b}$	1
38	Writing: $P(E_1)=\frac{1}{2}$, $P(E_2)=\frac{1}{2}$ and $P(A\lvert E_1)=1$, $P(A\lvert E_2)=\frac{1}{2}$	1
	Writing: $P(E_1\lvert A)=\frac{P(E_1)\,P(A\lvert E_1)}{P(E_1)\,P(A\lvert E_1)+P(E_2)\,P(A\lvert E_2)}$	1
	Getting : $P(E_1\lvert A)=\frac{2}{3}$	1

PART D		
39	Let $x_1, x_2 \in A = R - \{3\}$ such that $f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$	1
	$\Rightarrow (x_1-2)(x_2-3) = (x_2-2)(x_1-3)$ $\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$	1
	$\Rightarrow 3x_2 - 2x_2 = 3x_1 - 2x_1 \Rightarrow x_1 = x_2$. ∴ f is one-one.	1
	Take y ∈ B and let f(x) = y $\Rightarrow \frac{x-2}{x-3} = y$	1
	Getting $x = \frac{2-3y}{1-y} \in A$ $\therefore f$ is onto.	1
	OR	
	$f(x) = \frac{x-2}{x-3} = 1 + \frac{1}{x-3}$	1
	$f(x_1) = f(x_2) \Rightarrow 1 + \frac{1}{x_1-3} = 1 + \frac{1}{x_2-3} \Rightarrow \frac{1}{x_1-3} = \frac{1}{x_2-3}$	1
	Writing $\Rightarrow x_1 = x_2$. ∴ f is one-one.	1
	Take y ∈ B and let f(x) = y $\Rightarrow y = 1 + \frac{1}{x-3}$	1
	Writing $y - 1 = \frac{1}{x-3} \Rightarrow x = 3 + \frac{1}{y-1} \in A$ ∴ f is one-one.	1
40	Getting AB $= \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$	1
	writing $(AB)^{\prime} = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$ .... (1)	1
	writing $A^{\prime} = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$ and $B^{\prime} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ **OR** $B^{\prime}A^{\prime} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$	1
	Getting $B^{\prime}A^{\prime} = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$ .... (2)	1
	Comparing (1) and (2) $(AB)^{\prime} = B^{\prime}A^{\prime}$	1
41	Writing $A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $and\ B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$ **OR** Getting $\|A\| = 50 \neq 0$ ***Note:*** *Award a mark, if student writes directly* $\|A\| = 50$.	1
	Getting adj(A) $= \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$ ***Note:*** *If any 4 cofactors are correct award 1 mark..*	2
	Writing $X = A^{-1}B = \frac{1}{\|A\|}(adjA)B$ **OR** $X = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$	1
	Getting x =5, y =8, z =8	1

42	y = $(\tan^{-1} x)^2$ Diff. w.r.to x, $y_1 = 2\tan^{-1} x . \frac{1}{1+x^2}$,	1
	multiply by $(1+x^2)$ $\Rightarrow$ $(1+x^2)y_1 = 2\tan^{-1} x$	1
	diff. again w. r. t. x, and getting $(1+x^2)y_2 + 2xy_1 = \frac{2}{(1+x^2)}$,	1
	multiply by $(1+x^2)$ **OR** writing $(1+x^2)[(1+x^2)y_2 + 2xy_1] = 2$	1
	Writing $(1+x^2)^2 y_2 + 2x(1+x^2)y_1 = 2$	1
43	Taking x=a$\tan\theta$ $\Rightarrow \tan^{-1}\frac{x}{a} = \theta$ and dx=a$sec^2\theta d\theta$	1
	Getting $\int \frac{dx}{x^2+a^2} = \int \frac{asec^2\theta d\theta}{a^2tan^2\theta + a^2} = \int \frac{asec^2\theta d\theta}{a^2(tan^2\theta+1)}$ $= \int \frac{asec^2\theta d\theta}{a^2sec^2\theta} = \frac{1}{a}\int 1d\theta = \frac{1}{a}(\theta) + c$	1
	Getting $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$+c	1
	Writing $x^2 - 6x + 13 = (x-3)^2 + 2^2$	1
	Getting $\int \frac{dx}{x^2-6x+13} = \frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + c$	1
44	Writing correct figure 	1
	Writing y = $\sqrt{a^2 - x^2}$ **OR** Writing Area=4 $\int_0^a ydx$ **OR** Area = 4 times shaded area	1
	Writing Area =4 $\int_0^a \sqrt{a^2 - x^2}\, dx$	1
	Getting Area = 4 $\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$	1
	Getting Area = πa^2 square units **Note:** Units are not compulsory	1

45	Writing $\frac{dy}{dx}$ + $\sec^2x \cdot y$ = $\tan x.\sec^2x$ **OR** $P = \sec^2x$, $Q = \tan x \sec^2x$	1
	Getting I.F = $e^{\int P\,dx}$ = $e^{\int \sec^2 x}$ = $e^{\tan x}$	1
	Writing y(I.F) = $\int Q(I.F)dx$+c **OR** $y\,e^{\tan x} = \int \tan x.\sec^2x\; e^{\tan x}\,dx + c$	1
	Put $\tan x = t \Rightarrow \sec^2x\,dx = dt \Rightarrow ye^{\tan x} = \int t\,e^t\,dt + c$	1
	Getting $ye^{\tan x} = e^{\tan x}(\tan x - 1) + c$ **OR** $y = (\tan x - 1) + c.e^{-\tan x}$ (without c deduct 1 marrk)	1
PART E		
46	Let $I = \int_0^a f(x)dx$ Putting $x = a - t$, then $dx = -dt$ $x = 0 \implies t = a \quad and \quad x = a \implies t = 0$	1
	Getting I $= -\int_a^0 f(a-t)dt$	1
	Getting I $= \int_0^a f(a-x)dx$	1
	Writing Let $I = \int_0^{\pi/4} \log(1 + \tan x)dx$ replace x by $\frac{\pi}{4} - x$ $I = \int_0^{\pi/4} \log\left(1 + \frac{1+\tan x}{1-\tan x}\right)dx$	1
	$I = \int_0^{\pi/4} \log\left(\frac{2}{1+\tan x}\right)dx = \int_0^{\pi/4} (\log(2) - \log(1 + \tan x))dx$	1
	$2I = (\log 2)\frac{\pi}{4} \implies I = \frac{\pi}{8}\log 2$	1
OR	 Drawing the graph of (any two lines 1mark) all 3 lines award 2 marks	2

<table>
<tr><td></td><td>Getting corner points $(60,0),(120,0),(40,20)\ and\ (60,30)$</td><td>1</td></tr>
<tr><td></td><td>

Corner points	Z=5x+10y
(60, 0)	300
(120, 0)	600
(40, 20)	400
(60, 30)	600

</td><td>1</td></tr>
<tr><td></td><td>Writing the minimum value of Z is 300 at $(60,0)$.</td><td>1</td></tr>
<tr><td></td><td>The maximum value of Z is 600 at all the points on the line segment joining $(60,0)\ and\ (120,0)$</td><td>1</td></tr>
<tr><td>47</td><td>Getting $A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$</td><td>1</td></tr>
<tr><td></td><td>Proving $A^2 - 5A + 7I = O$</td><td>1</td></tr>
<tr><td></td><td>Getting $7A^{-1} = 5I - A\ \ or\ \ A^{-1} = \frac{1}{7}(5I - A))$</td><td>1</td></tr>
<tr><td></td><td>Getting $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ OR $A^{-1} = \begin{bmatrix} \frac{2}{7} & -\frac{1}{7} \\ \frac{1}{7} & \frac{3}{7} \end{bmatrix}$</td><td>1</td></tr>
<tr><td>OR</td><td>Writing condition for continuity $\lim_{x \to \frac{\pi}{2}}\left(f(x)\right) = f\left(\frac{\pi}{2}\right)$</td><td>1</td></tr>
<tr><td></td><td>Put $\pi - 2x = t \Rightarrow x = \frac{\pi}{2} - \frac{t}{2}$ When $x = \frac{\pi}{2}$, $t = 0$</td><td>1</td></tr>
<tr><td></td><td>Getting $\lim_{t \to 0}\frac{k\cos(\frac{\pi}{2} - \frac{t}{2})}{t} = \lim_{t \to 0}\frac{k\sin\frac{t}{2}}{2 \cdot \frac{t}{2}} = \frac{k}{2}$</td><td>1</td></tr>
<tr><td></td><td>Getting $\frac{k}{2} = 3 \Rightarrow k = 6$. [Any other alternate method award marks]</td><td>1</td></tr>
<tr><td></td><td>PART F</td><td></td></tr>
<tr><td>7</td><td>a) or writing Statement 1 is true and statement 2 is false</td><td>1</td></tr>
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