MHT-CET 2023 Question Paper - Maths

12^{th} May 2023 (Shift – I)

1. If the matrix A = $\begin{bmatrix} 1 & 2 \\ -5 & 1 \end{bmatrix}$ and $A^{-1} = xA + yI$, when I is a unit matrix of order 2, then the value of $2x + 3y$ is
   (A) $\frac{8}{11}$ (B) $\frac{4}{11}$
   (C) $\frac{-8}{11}$ (D) $\frac{-4}{11}$

2. $\int \frac{x^2+1}{x(x^2-1)} dx =$
   (A) $\log x(x^2 - 1) + c$, where c is a constant of integration.
   (B) $\log\left(\frac{x^2-1}{x}\right) + c$, where c is a constant of integration.
   (C) $\log(x^2 - 1) + c$, where c is a constant of integration.
   (D) $\log\left(\frac{x^2+1}{x}\right) + c$, where c is a constant of integration.

3. Let $\overline{A}$ be a vector parallel to line of intersection of planes P_1 and P_2 through origin, P_1 is parallel to the vectors $2\hat{j}+3\hat{k}$ and $4\hat{j}-3\hat{k}$ and P_2 is parallel to $\hat{j}-\hat{k}$ and $3\hat{i}+3\hat{j}$, then the angle between $\overline{A}$ and $2\hat{i}+\hat{j}-2\hat{k}$ is
   (A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$
   (C) $\frac{\pi}{6}$ (D) $\frac{3\pi}{4}$

4. Let PQR be a right angled isosceles triangle, right angled at P(2, 1). If the equation of the line QR is $2x + y = 3$, then the equation representing the pair of lines PQ and PR is
   (A) $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$
   (B) $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$
   (C) $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$
   (D) $3x^2 - 3y^2 - 8xy - 10x - 15y - 20 = 0$

5. The derivative of f(tan x) w.r.t. g(sec x) at $x = \frac{\pi}{4}$ where $f'(1) = 2$ and $g'(\sqrt{2}) = 4$ is
   (A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$
   (C) 1 (D) 0

6. If λ is the perpendicular distance of a point P on the circle $x^2 + y^2 + 2x + 2y - 3 = 0$, from the line $2x + y + 13 = 0$, then maximum possible value of λ is
   (A) $2\sqrt{5}$ (B) $3\sqrt{5}$
   (C) $4\sqrt{5}$ (D) $\sqrt{5}$

7. The integral
   $$\int \frac{\sin^2 x \cos^2 x}{\left(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x^2\right)} dx \text{ is}$$
   (A) $\frac{1}{3(1+\tan^3 x)} + c$, where c is a constant of integration.
   (B) $\frac{-1}{3(1+\tan^3 x)} + c$, where c is a constant of integration.
   (C) $\frac{1}{1+\cot^3 x} + c$, where c is a constant of integration.
   (D) $\frac{-1}{1+\cos^3 x} + c$, where c is a constant of integration.

8. If $\frac{dy}{dx} = y + 3$ and $y(0) = 2$, then $y(\log 2) =$
   (A) 5 (B) 7
   (C) 13 (D) –2

9. The solution set of $8\cos^2\theta + 14\cos\theta + 5 = 0$, in the interval $[0, 2\pi]$, is
   (A) $\left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$ (B) $\left\{\frac{\pi}{3}, \frac{4\pi}{3}\right\}$
   (C) $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$ (D) $\left\{\frac{2\pi}{3}, \frac{5\pi}{3}\right\}$

10. If the line $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles, then p =
    (A) $\frac{70}{11}$ (B) $\frac{11}{70}$
    (C) $\frac{-70}{11}$ (D) $\frac{-11}{70}$

11. If T_n denotes the number of triangles which can be formed using the vertices of regular polygon of n sides and $T_{n+1} - T_n = 21$, then n =
    (A) 5 (B) 7
    (C) 6 (D) 4

12. If $g(x) = 1 + \sqrt{x}$ and $f(g(x)) = 3 + 2\sqrt{x} + x$, then $f(f(x))$ is
(A) $x^2 + 4x + 6$ (B) $x^4 + x^2 + 6$
(B) $x^2 + x + 6$ (D) $x^4 + 4x^2 + 6$

13. The function $f(x) = \sin^4 x + \cos^4 x$ is increasing in
(A) $0 < x < \frac{\pi}{8}$ (B) $\frac{\pi}{4} < x < \frac{\pi}{2}$
(C) $\frac{3\pi}{8} < x < \frac{5\pi}{8}$ (D) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

14. If the variance of the numbers –1, 0, 1, k is 5, where k > 0, then k is equal to
(A) $2\sqrt{\frac{10}{3}}$ (B) $2\sqrt{6}$
(C) $4\sqrt{\frac{5}{3}}$ (D) $\sqrt{6}$

15. $\lim_{x\to 0} \frac{\cos 7x^\circ - \cos 2x^\circ}{x^2}$ is
(A) $\frac{-45}{2}\pi^2$ (B) $\frac{-45}{2}\pi$
(C) $\frac{-\pi^2}{1440}$ (D) $\frac{-\pi^2}{2880}$

16. If $\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, $0 \le \alpha \le \frac{\pi}{2}$, then the value of $\cos 2\theta$ is
(A) $\cos 2\alpha$ (B) $\sin\alpha$
(C) $\cos\alpha$ (D) $\sin 2\alpha$

17. The contrapositive of "If x and y are integers such that xy is odd, then both x and y are odd" is
(A) If both x and y are odd integers, then xy is odd.
(B) If both x and y are even integers, then xy is even.
(C) If x or y is an odd integer, then xy is odd.
(D) If both x and y are not odd integers, then the product xy is not odd.

18. The decay rate of radio active material at any time t is proportional to its mass at that time. The mass is 27 grams when t = 0. After three hours it was found that 8 grams are left. Then the substance left after one more hour is
(A) $\frac{27}{8}$ grams (B) $\frac{81}{4}$ grams
(C) $\frac{16}{3}$ grams (D) $\frac{16}{9}$ grams

19. If $x = -1$ and $x = 2$ are extreme points of $f(x) = \alpha \log x + \beta x^2 + x$, α and β are constants, then the value of $\alpha^2 + 2\beta$ is
(A) –3 (B) 3
(C) $\frac{3}{2}$ (D) 5

20. $\bar{u}, \bar{v}, \bar{w}$ are three vectors such that $|\bar{u}| = 1$, $|\bar{v}| = 2$, $|\bar{w}| = 3$. If the projection of $\bar{v}$ along $\bar{u}$ is equal to projection of $\bar{w}$ along $\bar{u}$ and $\bar{v}, \bar{w}$ are perpendicular to each other, then $|\bar{u} - \bar{v} + \bar{w}| =$
(A) 4 (B) $\sqrt{7}$
(C) $\sqrt{14}$ (D) 2

21. $\int_0^4 |2x - 5|\, dx =$
(A) $\frac{13}{2}$ (B) $\frac{15}{2}$ (C) $\frac{17}{4}$ (D) $\frac{17}{2}$

22. The approximate value of sin (60° 0′ 10″) is (given that $\sqrt{3} = 1.732$, $1' = 0.0175^c$)
(A) 0.08660243 (B) 0.0008660243
(C) 0.8660243 (D) 0.008660243

23. The p.m.f of random variate X is
$$P(X) = \begin{cases} \frac{2x}{n(n+1)}, & x = 1, 2, 3, \ldots\ldots, n \\ 0 & \text{, otherwise} \end{cases}$$
Then E(X) =
(A) $\frac{n+1}{3}$ (B) $\frac{2n+1}{3}$
(C) $\frac{n+2}{3}$ (D) $\frac{2n-1}{3}$

24. If the area of the triangle with vertices (1, 2, 0), (1, 0, 2) and (0, x , 1) is $\sqrt{6}$ square units, then the value of x is
(A) 1 (B) 2 (C) 3 (D) 4

25. The differential equation $\cos(x + y)\, dy = dx$ has the general solution given by
(A) $y = \sin(x + y) + c$, where c is a constant.
(B) $y = \tan(x + y) + c$, where c is a constant
(C) $y = \tan\left(\frac{x+y}{2}\right) + c$, where c is a constant
(D) $y = \frac{1}{2}\tan(x + y) + c$, where c is a constant

26. An experiment succeeds twice as often as it fails. Then the probability, that in the next 6 trials there will be atleast 4 successes, is
(A) $\frac{1}{729}$ (B) $\frac{496}{729}$
(B) $\frac{233}{729}$ (D) $\frac{491}{729}$

27. A plane is parallel to two lines whose direction ratios are 1, 0, –1 and –1, 1, 0 and it contains the point (1, 1, 1). If it cuts the co-ordinate axes at A, B, C then the volume of the tetrahedron OABC (in cubic units) is
(A) $\frac{9}{4}$ (B) $\frac{9}{2}$ (C) 9 (D) 27

28. The area of the region bounded by the curves $y = e^x$, $y = \log x$ and lines $x = 1$, $x = 2$ is
(A) $(e - 1)^2$ sq. units
(B) $(e^2 - e + 1)$ sq. units
(C) $(e^2 - e + 1 - 2\log 2)$ sq. units
(D) $(e^2 + e - 2\log 2)$ sq. units

29. $y = (1 + x)(1 + x^2)(1 + x^4) \dots \dots \dots (1 + x^{2n})$, then the value of $\frac{dy}{dx}$ at $x = 0$ is
(A) 0 (B) −1
(C) 1 (D) 2

30. A and B are independent events with $P(A) = \frac{1}{4}$ and $P(A \cup B) = 2P(B) - P(A)$, then P(B) is
(A) $\frac{1}{4}$ (B) $\frac{3}{5}$
(C) $\frac{2}{3}$ (D) $\frac{2}{5}$

31. If $a > 0$ and $z = \frac{(1+i)^2}{a+i}$, $(i = \sqrt{-1})$ has magnitude $\frac{2}{\sqrt{5}}$ then $\bar{z}$ is equal to
(A) $-\frac{2}{5}+\frac{4}{5}i$ (B) $\frac{2}{5}-\frac{4}{5}i$
(C) $-\frac{2}{5}-\frac{4}{5}i$ (D) $\frac{2}{5}+\frac{4}{5}i$

32. The angle between the tangents to the curves $y = 2x^2$ and $x = 2y^2$ at (1, 1) is
(A) $\tan^{-1}\left(\frac{15}{8}\right)$ (B) $\tan^{-1}\left(\frac{7}{8}\right)$
(C) $\tan^{-1}\left(\frac{3}{4}\right)$ (D) $\tan^{-1}\left(\frac{1}{4}\right)$

33. If $x = \operatorname{cosec}\left(\tan^{-1}\left(\cos\left(\cot^{-1}\left(\sec\left(\sin^{-1} a\right)\right)\right)\right)\right)$, $a \in [0, 1]$
(A) $x^2 - a^2 = 3$ (B) $x^2 + a^2 = 3$
(C) $x^2 - a^2 = 2$ (D) $x^2 + a^2 = 2$

34. The distance of the point P(−2, 4, −5) from the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ is
(A) $\frac{\sqrt{37}}{10}$ (B) $\sqrt{\frac{37}{10}}$
(C) $\frac{37}{\sqrt{10}}$ (D) $\frac{37}{10}$

35. The value of $\sin(\cot^{-1} x)$ is
(A) $\frac{1}{\sqrt{1+x^2}}$ (B) $\sqrt{1+x^2}$
(C) $\frac{1}{x\sqrt{1+x^2}}$ (D) $x\sqrt{1+x^2}$

36. The values of a and b, so that the function

$$f(x) = \begin{cases} x + a\sqrt{2}\sin x, & 0 \le x \le \frac{\pi}{4} \\ 2x\cot x + b, & \frac{\pi}{4} \le x \le \frac{\pi}{2} \\ a\cos 2x - b\sin x, & \frac{\pi}{2} < x \le \pi \end{cases}$$

is continuous for $0 \le x \le \pi$, are respectively given by
(A) $-\frac{\pi}{12}, \frac{\pi}{6}$ (C) $-\frac{\pi}{6}, -\frac{\pi}{12}$
(C) $\frac{\pi}{6}, \frac{\pi}{12}$ (D) $\frac{\pi}{6}, -\frac{\pi}{12}$

37. Two adjacent sides of a parallelogram ABCD are given by $\overline{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overline{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of parallelogram so that AD becomes AD′. If AD′ makes a right angle with side AB, then the cosine of the angle α is given by
(A) $\frac{8}{9}$ (B) $\frac{\sqrt{17}}{9}$
(C) $\frac{1}{9}$ (D) $\frac{4\sqrt{5}}{9}$

38. $\int \frac{\operatorname{cosec} x \, dx}{\cos^2\left(1 + \log \tan \frac{x}{2}\right)} =$
(A) $\tan\left(1 + \log\left(\tan\frac{x}{2}\right)\right)$ + c, where c is constant of integration
(B) $\tan(1 + \log(\tan x))$ + c, where c is constant of integration
(C) $\tan\left(\log\left(\tan\frac{x}{2}\right)\right)$ + c, where c is constant of integration.
(D) $\tan\left(\tan\frac{x}{2}\right)$ + c, where c is constant of integration.

39. The co-ordinates of the points on the line $2x - y = 5$ which are the distance of 1 unit from the line $3x + 4y = 5$ are
(A) $\left(\frac{30}{11}, \frac{-5}{11}\right), \left(\frac{20}{11}, \frac{15}{11}\right)$
(B) $\left(\frac{-30}{11}, \frac{5}{11}\right), \left(\frac{-20}{11}, \frac{15}{11}\right)$
(C) $\left(\frac{30}{11}, \frac{5}{11}\right), \left(\frac{20}{11}, \frac{-15}{11}\right)$
(D) $\left(\frac{-30}{11}, \frac{5}{11}\right), \left(\frac{-20}{11}, \frac{-15}{11}\right)$

40. The centroid of tetrahedron with vertices at A(–1, 2, 3), B(3, –2, 1), C(2, 1, 3) and D(–1, –2, 4) is

(A) $\left(\frac{3}{4}, \frac{-1}{4}, \frac{11}{4}\right)$ (B) $\left(\frac{5}{4}, \frac{-3}{4}, \frac{7}{4}\right)$

(C) $\left(\frac{-3}{4}, \frac{-1}{4}, \frac{11}{4}\right)$ (D) $\left(\frac{-5}{4}, \frac{-3}{4}, \frac{-7}{4}\right)$

41. If $\log(x + y) = 2xy$, then $\frac{dy}{dx}$ at $x = 0$ is

(A) 1 (B) –1
(C) 2 (D) –2

42. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Then the probability distribution of number of jacks is

(A)

$X = x$	0	1	2
$P(X = x)$	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

(B)

$X = x$	0	1	2
$P(X = x)$	$\frac{1}{169}$	$\frac{144}{169}$	$\frac{24}{169}$

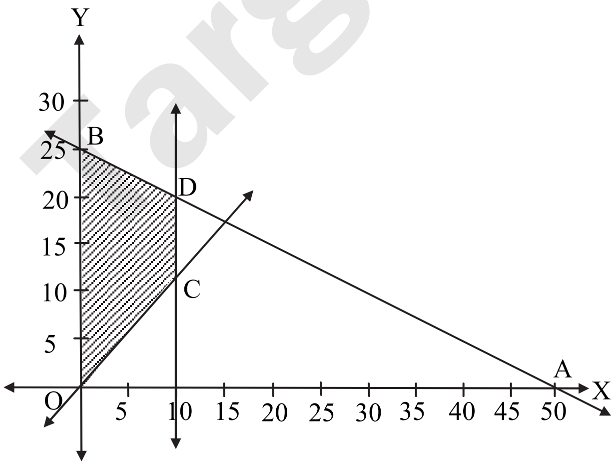
(C)

$X = x$	0	1	2
$P(X = x)$	$\frac{24}{169}$	$\frac{1}{169}$	$\frac{144}{169}$

(D)

$X = x$	0	1	2
$P(X = x)$	$\frac{144}{169}$	$\frac{1}{169}$	$\frac{24}{169}$

43. For a feasible region OCDBO given below, the maximum value of the objective function $z = 3x + 4y$ is



(A) 70 (B) 100
(C) 110 (D) 130

44. In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. If $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is

(A) $\frac{c}{3}$ (B) $\frac{c}{\sqrt{3}}$

(C) $\frac{3}{2}y$ (D) $\frac{y}{\sqrt{3}}$

45. If $\int \cos^{\frac{3}{5}} x \cdot \sin^3 x \, dx = \frac{-1}{m}\cos^m x + \frac{1}{n}\cos^n x + c$, (where c is the constant of integration), then (m, n) =

(A) $\left(\frac{18}{5}, \frac{8}{5}\right)$ (B) $\left(\frac{-8}{5}, \frac{18}{5}\right)$

(C) $\left(\frac{8}{5}, \frac{18}{5}\right)$ (D) $\left(\frac{-18}{5}, \frac{-8}{5}\right)$

46. If $\bar{a}, \bar{b}, \bar{c}$ are three vectors, $|\bar{a}| = 2$, $|\bar{b}| = 4$, $|\bar{c}| = 1$, $|\bar{b} \times \bar{c}| = \sqrt{15}$ and $\bar{b} = 2\bar{c} + \lambda\bar{a}$, then the value of λ is

(A) 2 (B) $2\sqrt{2}$ (C) 1 (D) 4

47. A ladder 5 meters long rests against a vertical wall. If its top slides downwards at the rate of 10 cm/s, then the angle between the ladder and the floor is decreasing at the rate of _______ rad./s when it's lower end is 4 m away from the wall.

(A) –0.1 (B) –0.025
(C) 0.1 (D) 0.025

48. The equation of the plane through (–1, 1, 2) whose normal makes equal acute angles with co-ordinate axes is

(A) $x + y + z - 3 = 0$ (B) $x + y + z - 2 = 0$
(C) $x + y - z - 2 = 0$ (D) $x - y + z - 3 = 0$

49. The inverse of the statement
"If the surface area increase, then the pressure decreases.", is

(A) If the surface area does not increase, then the pressure does not decrease.
(B) If the pressure decreases, then the surface area increases.
(C) If the pressure does not decreases, then the surface area does not increase.
(D) If the surface area does not increase, then the pressure decreases.

50. If general solution of $\cos^2\theta - 2\sin\theta + \frac{1}{4} = 0$ is $\theta = \frac{n\pi}{A} + (-1)^n \frac{\pi}{B}$, $n \in Z$, then A + B has the value

(A) 7 (B) 6 (C) 1 (D) –7