

PART : MATHEMATICS

1. Let α, β are roots of quadratic equation and $P_n = \alpha^n + \beta^n$, $P_{10} = 123$, $P_9 = 76$, $P_8 = 47$ and $P_1 = 1$ the quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

(1) $x^2 + x - 1 = 0$

(2) $x^2 - x + 1 = 0$

(3) $x^2 + x + 1 = 0$

(4) $x^2 - x - 1 = 0$

Ans. (1)

Sol. $P(x) = ax^2 + bx + c$, $P_1 = 1 \Rightarrow \alpha + \beta = 1 \Rightarrow -\frac{b}{a} = 1 \Rightarrow a = -b$

$aP_{10} + bP_9 + cP_8 = 0$

$123a + 76b + 47c = 0$

$47a + 47c = 0$

$a = -c = -b$

So, equation $x^2 - x - 1 = 0$.

$\alpha + \beta = 1, \alpha\beta = -1$

Sum = $\frac{1}{-1}$ and Prod = $\frac{1}{-1}$

$x^2 - (-1)x + (-1) = x^2 + x - 1 = 0$.

2. The total number of 10 digits numbers formed by only $\{0, 1, 2\}$ where 1 should be used atleast 5 times and 2 should be used exactly three times, is

Ans. (2892)

0 digit	1 digit	2 digit
2	5	3
1	6	3
0	7	3

Sol.

numbers when 0 is used two times

$= \frac{9!}{4!3!2!} + \frac{9!}{2!5!2!}$

when 0 is used 1 time.

$\frac{10!}{1!6!3!} - \frac{9!}{6!3!}$

$= \frac{10!}{7!3!}$

Total = 2892

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3. If $a_1, a_2, a_3, \dots$ is an A.P. and $\sum_{k=1}^{12} a_{2k-1} = -\frac{72}{5} a_1$ and $\sum_{k=1}^n a_k = 0$ then value of n is
 (1) 8 (2) 9 (3) 10 (4) 11

Ans. (4)

Sol. $\sum_{k=1}^{12} a_{2k-1} = -\frac{72}{5} a_1$
 $\frac{12}{2} [2a_1 + 11(2d)] = -\frac{72}{5} a_1$
 $12a_1 + 132d = -\frac{72}{5} a_1$
 $60a_1 + 660d = -72a_1$
 $132a_1 + 660d = 0$
 $2a_1 + 10d = 0$
 $a_1 + 5d = 0$
 $\sum_{k=1}^n a_k = 0$
 $\frac{n}{2} [2a_1 + (n-1)d] = 0$
 $2a_1 + (n-1)d = 0$
 $-10d + (n-1)d = 0$
 $-10 + n - 1 = 0$
 $n = 11$

4. If $\lfloor 50 \rfloor$ is exactly divisible by 3^n then find maximum value of n .

(1) 20 (2) 22 (3) 18 (4) 16

Ans. (2)

Sol. $E(3) = \left\lfloor \frac{50}{3} \right\rfloor + \left\lfloor \frac{50}{3^2} \right\rfloor + \left\lfloor \frac{50}{3^3} \right\rfloor = 16 + 5 + 1 = 22$
 $\therefore$ Exponent of 3 in $\lfloor 50 \rfloor$ is 22.
 $\therefore \lfloor 50 \rfloor = 3^{22} \cdot \lambda \quad \therefore n = 22.$

5. Let $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, $f(x)$ has maxima at $x = p$ and minima at $x = q$ such that $p^2 = q$ then find $f(3)$.

(1) 37 (2) 38 (3) 40 (4) 45

Ans. (1)

Sol. $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$
 $f'(x) = 6x^2 - 18ax + 12a^2$
 $f'(x) = 6(x^2 - 3ax + 2a^2) = 6(x-a)(x-2a)$

 $\therefore x = p = a$ and $x = q = 2a$
 Given $p^2 = q$
 $\therefore a^2 = 2a$
 $\Rightarrow a = 0$ or 2
 $\Rightarrow a = 2 \quad (a \neq 0)$
 $\therefore f(3) = 37$

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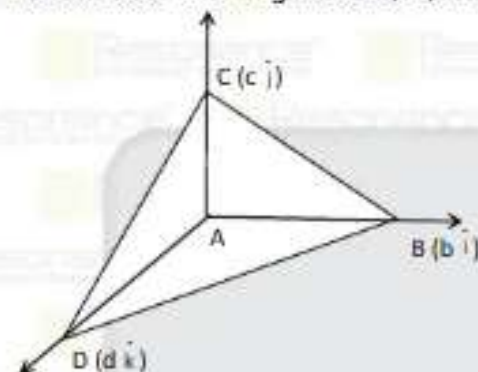
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6. Let ABCD is a tetrahedron in which $\angle BAC, \angle CAD, \angle BAD$ is at right angle and area of faces. ABC, BAD & CAD are 5, 7 and 6 respectively then area of face BCD is –

- (1) $\sqrt{55}$ (2) $\sqrt{220}$ (3) $\sqrt{110}$ (4) $\sqrt{340}$

Ans. (3)

Sol. Let Consider A at origin and B, C, D at x, y & z axes



now

$$\text{ar } \triangle ABC = \frac{1}{2} bc = 5$$

$$\text{ar } \triangle ABD = \frac{1}{2} bd = 7$$

$$\triangle ACD = \frac{1}{2} cd = 6$$

Now area of $\triangle BCD$

$$= \frac{1}{2} |\overrightarrow{BC} \times \overrightarrow{BD}| =$$

$$\overrightarrow{BC} \times \overrightarrow{BD} = (c\hat{j} - b\hat{i}) \times (d\hat{k} - b\hat{i})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -b & c & 0 \\ -b & 0 & d \end{vmatrix}$$

$$= \hat{i}(cd) - \hat{j}(-bd) + \hat{k}(bc)$$

$$\text{are BCD} = \frac{1}{2} \sqrt{(cd)^2 + (bd)^2 + (bc)^2}$$

$$= \frac{1}{2} \sqrt{100 + 196 + 144}$$

$$= \frac{1}{2} \sqrt{440}$$

$$= \sqrt{110}$$

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7. Three distinct numbers are selected from set $\{1, 2, 3, \dots, 40\}$, then find probability that they form an increasing G.P.

(1) $\frac{1}{520}$ (2) $\frac{3}{520}$ (3) $\frac{7}{520}$ (4) $\frac{11}{520}$

Ans. (1)

Sol. Given set is $\{1, 2, 3, \dots, 40\}$

$$h(s) = {}^{40}C_3$$

a, b, c is G.P. $\therefore b^2 = ac$

$a=1$ then $c=4, 9, 16, 25, 36$

$a=2$ then $c=8, 18, 32$

$a=3$ then $c=12, 27$

$a=4$ then $c=9, 16, 25, 36$

$a=5$ then $c=20$

$a=6$ then $c=24$

$a=7$ then $c=28$

$a=8$ then $c=32$

$a=9$ then $c=36$

Total cases $= 19 = h(A)$

$$\therefore P(A) = \frac{h(A)}{h(s)}$$

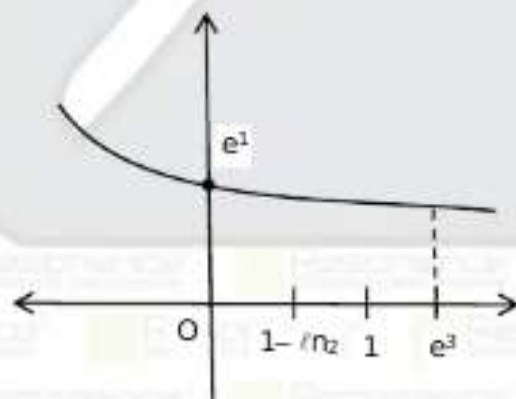
$$= \frac{19}{{}^{40}C_3} = \frac{19 \times 3 \times 2 \times 1}{40 \times 39 \times 38}$$

$$= \frac{1}{520}$$

8. $\int_0^{e^3} \left[\frac{1}{e^{x-1}} \right] dx = \alpha - \ln 2$ then α is equal to (where $[x]$ denotes greatest integer less than or equal to x).

(1) 2 (2) 3 (3) 4 (4) 5

Ans. (1)



Sol.

$$\int_0^{1/\ln 2} 2 dx + \int_{1/\ln 2}^1 1 dx + \int_1^{e^3} 0 dx$$

$$= (2x)_0^{1/\ln 2} + (x)_1^{1/\ln 2} + 0$$

$$= 2 - 2/\ln 2 + 1 - (1 - 1/\ln 2)$$

$$= 2 - 1/\ln 2 = \alpha - \ln 2$$

$$\text{So, } \alpha = 2$$

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9. The term independent of x in the expansion of $\left(\frac{x+1}{x^{\frac{2}{3}}+1-x^{\frac{1}{3}}} - \frac{x-1}{x-\sqrt{x}} \right)^{10}$ is

- (1) 252 (2) 60 (3) 210 (4) 45

Ans. (3)

Sol. $\left(\left(x^{\frac{1}{3}}+1 \right) - \left(\frac{\sqrt{x}+1}{\sqrt{x}} \right) \right)^{10}$

$$\left(x^{\frac{1}{3}} - \frac{1}{x^{\frac{1}{2}}} \right)^{10}$$

$$T_{r+1} = {}^{10}C_r x^{\frac{r}{3}} \cdot \left(-\frac{1}{x^{\frac{1}{2}}} \right)^{10-r}$$

$$\frac{r}{3} - \left(\frac{10-r}{2} \right) = 0$$

$$2r - 30 + 3r = 0$$

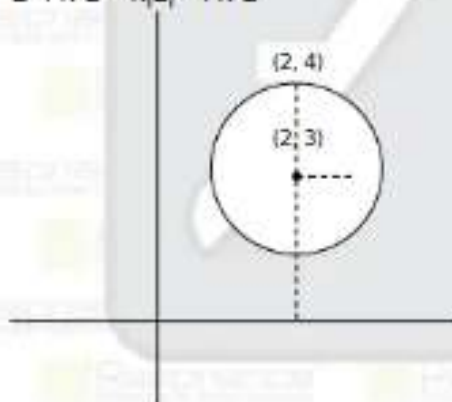
$$r = 6$$

$${}^{10}C_6 = {}^{10}C_4 = 210$$

10. If $|z| = 1$ and $\frac{2+k^2z}{\bar{z}+k} = kz$ where $k \in \mathbb{R}$ then the maximum value of $k + k^2i$ from $|z - (2+3i)| = 1$ is

Ans. (2)

Sol. $2 + k^2z = k|z|^2 + k^2z$



$$k = 2$$

$$2 + 4i = 2$$

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11. Area bounded by region $\{(x, y) : |4 - x^2| \leq y \leq x^2 \mid x \geq 0 \mid y \leq 4\}$ is

(1) $\frac{1}{3}[40\sqrt{2} - 48]$

(2) $\frac{1}{3}[40\sqrt{2} + 48]$

(3) $\frac{1}{2}[40\sqrt{2} - 48]$

(4) $\frac{1}{2}[40\sqrt{2} + 48]$

Ans. (1)

Sol. Solving $4 - x^2 = x^2$

$$x = \pm \sqrt{2}$$

Required region is as $x \geq 0$

$$= \int_{\sqrt{2}}^2 x^2 - (4 - x^2) dx + \int_2^{2\sqrt{2}} 4 - (x^2 - 4) dx$$

$$= \int_{\sqrt{2}}^2 (2x^2 - 4) dx + \int_2^{2\sqrt{2}} (8 - x^2) dx$$

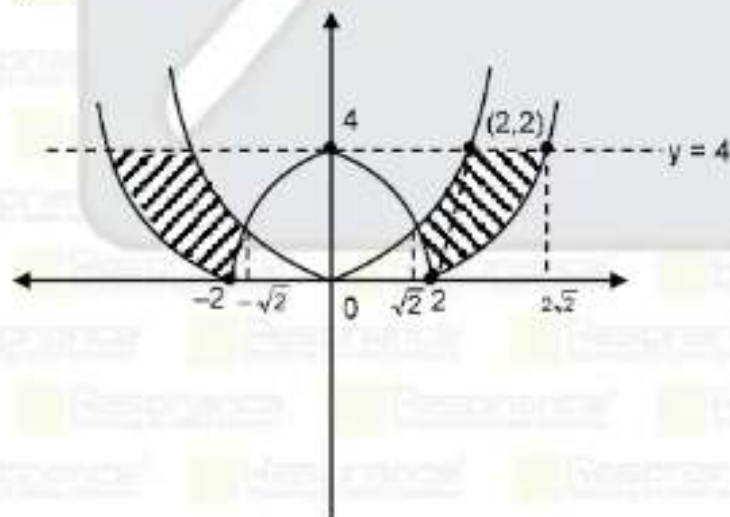
$$= \left[\frac{2x^3}{3} - 4x \right]_{\sqrt{2}}^2 + \left[8x - \frac{x^3}{3} \right]_2^{2\sqrt{2}}$$

$$= \left[\left(\frac{16}{3} - 8 \right) - \left(\frac{4\sqrt{2}}{3} - 4\sqrt{2} \right) \right] + \left[\left(16\sqrt{2} - \frac{16\sqrt{2}}{3} \right) - \left(16 - \frac{8}{3} \right) \right]$$

$$= \left[\frac{-8}{3} + \frac{8\sqrt{2}}{3} \right] + \left[\frac{32\sqrt{2}}{3} - \sqrt{2} \right]$$

$$= \frac{1}{3}[8\sqrt{2} - 8 + 32\sqrt{2} - 40]$$

$$= \frac{1}{3}[40\sqrt{2} - 48]$$



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- 12.** If $\lim_{x \rightarrow 0} \frac{x^2 \sin(\alpha x) - (\gamma - 1)e^{x^2}}{\sin(2x) - \beta x} = 3$ then the value of $\beta + \gamma - \alpha$ is

Ans. (1) -1 (2) 7 (3) 4 (4) 3

Sol.
$$\lim_{x \rightarrow 0} \frac{x^2(\alpha x - \frac{\alpha^3 x^3}{3!} + \dots) - (\gamma - 1)[1 + x^2 + \frac{x^4}{2!} + \dots]}{2x - \frac{(2x)^3}{3!} + \dots - \beta x}$$

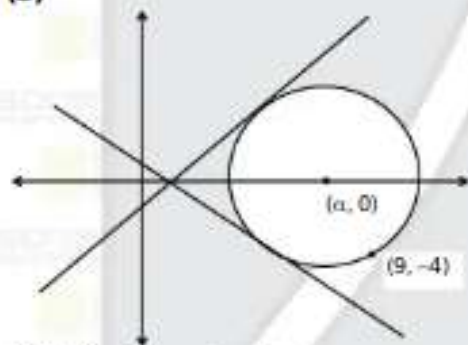
$$\lim_{x \rightarrow 0} \frac{-(\gamma - 1) - (\gamma - 1)x^2 + \alpha x^3 + \frac{(\gamma - 1)}{2!} x^4 + \dots}{(2 - \beta)x - \frac{8}{6}x^3 + \dots} = 3$$

$$\gamma = 1, \beta = 2, \frac{\alpha}{-4/3} = 3, \alpha = -4$$

$$\beta + \gamma - \alpha = 2 + 1 - (-4) = 7$$

- 13.** Two circles touches line $x + y = 3$ & $x - y = 3$ also passes through $(9, -4)$ then absolute difference of radius of two circles is

Ans. (3)



Sol.

Both circle touches lines
 $x + y = 3$ and $x - y = 3$
 so centre lies on x-axis. Let centre $(a, 0)$ let radius r
 $(x - a)^2 + y^2 = r^2$
 it passes through $(9, -4)$
 $(9 - a)^2 + 16 = r^2$ (1)
 also it touches $x + y = 3$

so $\left| \frac{\alpha + 0 - 3}{\sqrt{2}} \right| = r$

$$\alpha - 3 = \pm \sqrt{2} r$$

$$a = \pm\sqrt{2}r + 3$$

put in (1)

$$(9 - (\pm\sqrt{2}r + 3))^2 = r^2 - 16$$

$$(36 + 2r^2 \pm 12\sqrt{2}r) = r^2 - 16$$

$$r^2 + 12\sqrt{2}r + 52 = 0$$

Let radius r_1 & r_2

$$(r_1 - r_2) = \sqrt{(12\sqrt{2})^2 - 4 \times 52} = \sqrt{288 - 208} = \sqrt{80}$$

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14. Let P is any point on ellipse $\frac{x^2}{18} + \frac{y^2}{9} = 1$ if S_1 and S_2 are foci of ellipse then product of maximum and minimum value of product of length PS_1 & PS_2 is –

Ans. (162)

Sol. Let P $(3\sqrt{2} \cos\theta, 3\sin\theta)$
 $PS_1 \& PS_2 = (a - ex_1)(a - ex_2)$
 $= a^2 - e^2 x_1^2$
 $= 18 - \frac{1}{2} \times (18 \cos^2\theta)$
 $= 18 - 9 \cos^2\theta$

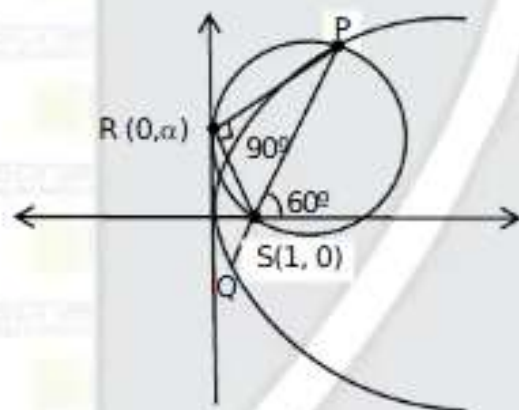
So maximum & minimum values are 18 & 9
 $= 18 \times 9 = 162$

15. Focal Chord PQ of the parabola $y^2 = 4x$ makes an angle 60° with positive x-axis where P in Ist quadrant. A circle is drawn with SP as diameter touches the tangent at the vertex of the parabola at $(0, \alpha)$ the α^2 is (where S is focus of parabola)

(1) 1 (2) 2 (3*) 3 (4) 4

Ans. (3)

Sol.



Let $P(t^2, 2t)$
 $m_{SP} = \frac{2t-0}{t^2-1} = \tan 60^\circ$

$\Rightarrow \sqrt{3}t^2 - 2t - \sqrt{3} = 0$

$\Rightarrow (\sqrt{3}t+1)(t-\sqrt{3}) = 0$

$\Rightarrow t = \sqrt{3}$

$\therefore$ Point P is $(3, 2\sqrt{3})$

$\therefore$ tangent at P is

$2\sqrt{3}y = 2(x+3)$

at R $x = 0$

$\therefore \alpha = \sqrt{3}$

$\therefore \alpha^2 = 3$

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