JEE-Main-02-04-2025 (Memory Based)
[MORNING SHIFT]
Maths

Question: Find the maximum value of n such that 50! is divisible by 3^n.
Options:
(a) 22
(b) 16
(c) 5
(d) 1
Answer: (22)

$$\left[\frac{50}{3}\right]+\left[\frac{50}{9}\right]+\left[\frac{50}{27}\right]$$
$$=16+5+1=22$$
$$n=22$$

Question: Let $P_n=\alpha^n+\beta^n$, $P_{10}=123$, $P_9=76$, $P_8=47$ and $P_1=1$, the quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.
Options:
(a) $x^2+x-1=0$
(b) $x^2-2x+1=0$
(c) $x^2+x-2=0$
(d) $x^2-x-2=0$
Answer: (a)

$$ax^2+bx+c=0 \quad b=-a$$
$$x^2-x+c=0$$
$$123-76+c(47)=0 \quad (P_{10}-P_9+cP_8=0)$$
$$c=-1$$
$$x^2-x-1=0 \quad \text{(Equation with roots } \alpha \text{ \& } \beta)$$
$$\text{Req eq}: x^2+x-1=0 \quad \text{(Equation with roots } \tfrac{1}{\alpha} \text{ and } \tfrac{1}{\beta})$$

Question: The total number of 10 digits sequences formed by only {0, 1, 2} where 1 should be used at least 5 times and 2 should be used exactly three times, is
Options:
(a) 3250
(b) 3680
(c) 3480
(d) 3840
Answer (c)

$^{10}C_3 \times \left[{}^7C_5 + {}^7C_6 + {}^7C_7\right]$

$= \frac{10 \times 9 \times 8}{6}[21+7+1]$

$= 120 \times 29$

$= 3480$

Question: Let a_1, a_2, a_3, ... is an A.P. and $\sum_{k=1}^{12} a_{2k-1} = -\frac{72}{5}a_1$ and $\sum_{k=1}^{n} a_{k=0}$.**Then the value of n is**

Options:

(a) 10

(b) 13

(c) 11

(d) 8

Answer: (c)

$a_1 + a_3 + a_5 - - + a_{23} = -\frac{72}{5}a_1$

$\frac{12}{2}[2a_1 + 11.2d] = -\frac{72}{5}a_1$

$10a_1 + 110\,d = -12\,a_1$

$22a_1 + 110\,d = 0$

$a_1 + 5d = 0$

$2a_1 + 10d = 0$

$\frac{n}{2}[2a + (n-1)d] = 0$

$n = 11$

Question: Number of solutions in [-2π,2π] for equation

$2\sqrt{2}\cos^2\theta + \left(2-\sqrt{6}\right)\cos\theta - \sqrt{3} = 0.$

Options:

(a) 4

(b) 2

(c) 6

(d) 8

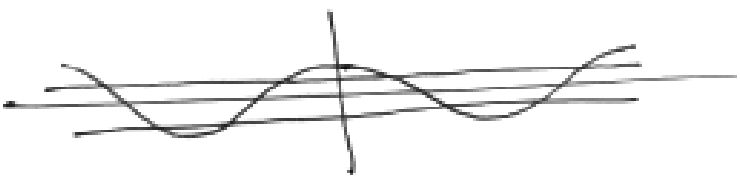
Answer: (d)

$2\sqrt{2}c^2 + \left(2-\sqrt{6}\right)c - \sqrt{3} = 0$

$2c\left(\sqrt{2}c+1\right) - \sqrt{3}\left(\sqrt{2}c+1\right) = 0$

$c = \frac{\sqrt{3}}{2}, \frac{-1}{2}$

= 8



Question:Given the equation of a hyperbola $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ **and its directrix is** $x = \sqrt{\frac{10}{81}}$ **with a focus at (√10.0), then find the value of 9(e + l^2), where l is length of latus rectum is**

Options:

(a) 2697
(b) 2597
(c) 2487
(d) 2587
Answer: (d)

$$\frac{a}{e} = \sqrt{\frac{10}{81}}\ ae = \sqrt{10}$$

$$a^2 = \frac{10}{9},\ e^2 = \frac{9}{10} = 9$$

$$l = 2a\left(e^2 - 1\right) = 2 \times \sqrt{\frac{10}{9}}\,(8)$$

$$l^2 = 4 \times \frac{10}{9} \times 64$$

$$9\left(e + l^2\right) = 9\left[3 + \frac{2560}{9}\right] = 2587$$

Question: If a twice differentiable function f satisfies *f'(x)=f(x)* such that *f(0)= ½ f'(0)*. Then find $f''\left(\frac{\pi}{3}\right)$.
Options:
(a) $e^{\frac{\pi}{3}}$

(b) $\dfrac{e^{\frac{\pi}{3}}}{2}$

(c) $\dfrac{\sqrt{3}}{2}$

(d) $\dfrac{e^{\frac{2\pi}{3}}}{2}$

Answer: (b)

$$f'(x) = f(x)$$

$$\frac{dy}{dx} = y$$

$$dy = x + c$$

$$y = ke^x$$

$$f(x) = ke^x$$

$$f(0) = k = \frac{1}{2}$$

$$f(x) = e^{\frac{x}{2}}$$

$$f(0)=\frac{1}{2}=f\prime(0)$$
$$f\prime\prime(x)=ke^{x}$$
$$f\prime\prime\left(\frac{\pi}{3}\right)=ke^{\frac{\pi}{3}}$$
$$=\frac{e^{\frac{\pi}{3}}}{2}$$

Question: Let the system of equations,3x - y + βz = 3, 2x + αy + z =-3 and x+ y+4z = 4 has infinite solutions, then 22β -9α equals to

Options:

(a) 165

(b) 164

(c) 163

(d) 162

Answer: (b)

$$\begin{vmatrix} 3 & -1 & 3 \\ 2 & \alpha & -3 \\ 1 & 1 & 4 \end{vmatrix}=0$$

$$3(4\alpha+3)+1(11)+3(2-\alpha)=0$$
$$12\alpha+9+11+6-3\alpha=0$$
$$9\alpha=-26$$

$$\begin{vmatrix} 3 & 3 & \beta \\ 2 & -3 & 1 \\ 1 & 4 & 4 \end{vmatrix}=0$$

$$3(-12-4)-3(7)+\beta(11)=0$$

$$-48-21=-11\beta$$

$$\frac{-69}{-11}=\beta=\frac{69}{11},\ 22\beta=138$$

$$22\beta-9\alpha$$

$$=138+26$$

$$=164$$

Question: **PQ is focal chord of $y^2 = 4x$ making an angle of 60° with positive x-axis. P lies in first quadrant & S is its focus. Circle with PS as diameter touches y-axis at $(0, \alpha)$, find α^2**

Options:

(a) 1

(b) 2

(c) 3

(d) 4

Answer: (c)

$\alpha = t$

$\tan 60^o = \frac{t-0}{\frac{t^2+1}{2}-1} = \frac{2t}{t^2-1} = \sqrt{3}$

$2t = \sqrt{3}t^2 - \sqrt{3}$

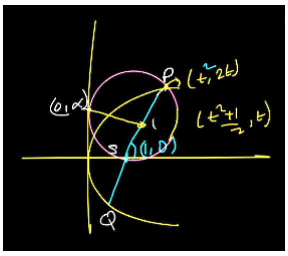
$\sqrt{3}t^2 - 2t - \sqrt{3} = 0$

$\left(t-\sqrt{3}\right)\left(\sqrt{3}\,t+1\right)=0$

$t=\sqrt{3}$

$\alpha=\sqrt{3}$

$\alpha^2=3$



Question: If $y \geq |4 - x^2|$, $y \leq x^2$, $y \leq 4$, $x > 0$, then find the area bounded by inequalities.

Answer: $(\frac{40\sqrt{2}}{3} - 16)$

$A = \int_{\sqrt{2}}^{2} x^2 - (4 - x^2) dx + \int_{2}^{3\sqrt{2}} (4 - (x^2 - 4)) dx$

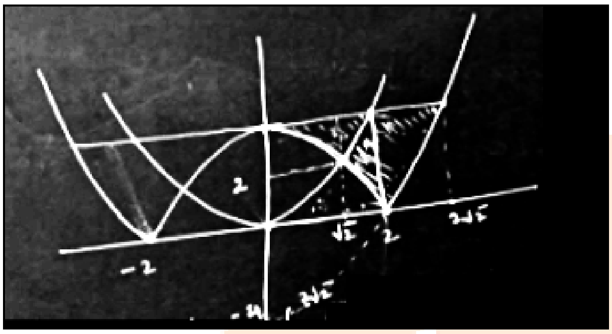
$= \int 2x^2 - 4\,dx \; + \; \int 8 - x^2 dx$

$= \frac{2x^3}{3} - 4x \Big|_{\sqrt{2}}^{2} \; + \; 8x - \frac{x^3}{3} \Big|_{2}^{2\sqrt{2}}$

$= \left(\frac{16}{3} - \frac{4\sqrt{2}}{3}\right) - 4\left(2 - \sqrt{2}\right) + \left(16\sqrt{2} - 16\right) - \left(\frac{16\sqrt{2} - 8}{3}\right)$

$= \frac{16}{3} - 8 - 16 + \frac{8}{3} + 4\sqrt{2} - \frac{4\sqrt{2}}{3} + 16\sqrt{2} - \frac{16\sqrt{2}}{3}$

$= \left(\frac{40\sqrt{2}}{3} - 16\right)$



Question: Let f(x) = $2x^3 + 9x^2a + 12a^2x + 1$. Local minima and local maxima occur at p & q respectively, such that $p^2 = q$. Then the value of f(3) is

Options:

(a) 30

(b) 35

(c) 32

(d) 37

Answer: (d)

$$6x^2 + 18ax + 12a^2$$

$$= 6\left[x^2 + 3ax + 2a^2\right] = 0$$

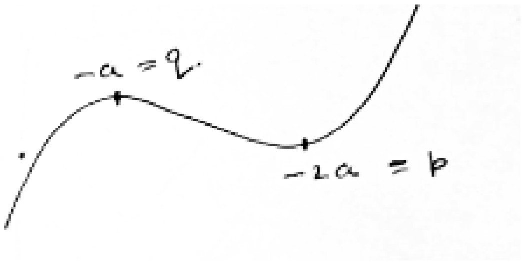
$$x = -a, \; -2a$$

$$a < 0 \quad 4a^2 = -a$$

$$a = \; -\frac{1}{4}$$

$$f(3) = 54 + 81\,a + 36a^2 + 1$$

$$55 - \frac{81}{4} + \frac{9}{4} = 55 - 18 = 37$$



Question: If $\int_0^{e^3} \left[\frac{1}{e^{x-1}}\right] dx = a - \log_e 2$, where [.] is Greatest integer function, then α^3 equals to

Options:

(a) 4

(b) 6

(c) 8

(d) 10

Answer: (c)

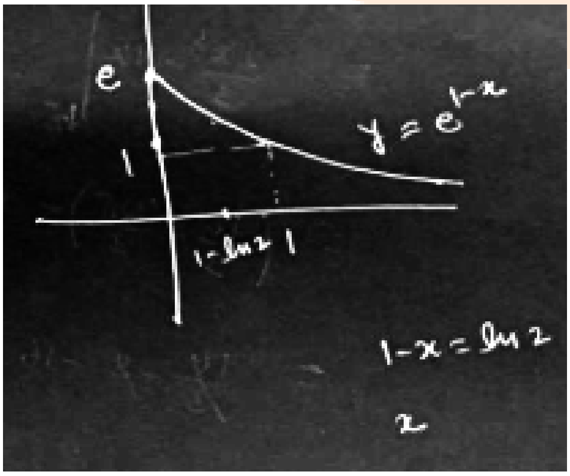
$$\int_0^{e^3} \left[\frac{1}{e^{x-1}}\right] dx$$

$$\int_0^{1-\ell nx} 2\,dx + \int_{1-\ell nx}^{1} 1\,dx$$

$$= 2(1 - \ln 2) + (\ln 2)$$

$$= 2 - \ln 2$$

$$\alpha = 2 \ \alpha^3 = 8$$



Question: If $\lim_{x \to 0} \frac{(\gamma - 1)e^{x^2} + x^2 \sin(\alpha x)}{\sin(2x) - \beta x} = 3,$ **then** $\alpha + 2\beta + \gamma$ **is equal to**

Options:

(a) 0

(b) 1

(c) 3

(d) 5

Answer: (b)

$$\lim_{x \to 0} \frac{(\gamma-1)e^{x^2}+x^2 \sin(\alpha x)}{\sin(2x)-\beta x} = 3$$

$$\to \lim_{x \to 0} \frac{\alpha x^3}{\sin 2x-\beta x} = 3 \qquad \gamma = 1$$

$$\to \quad \frac{3\alpha x^3}{2\cos 2x-\beta} = 3 \qquad \beta = 2$$

$$\frac{6\alpha x^2}{-4\sin 2x} = \frac{6\alpha}{-8} = 3 \to \alpha = -4$$

$$\alpha + 2\beta + \gamma = -4 + 4 + \gamma = 1$$

Question: The term independent of x in the binomial expression of $\left(\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}}\right)^{10}$ **is**

Options:

(a) 120

(b) 210

(c) 84

(d) 110

Answer: (b)

Term Independent at x

$$\left[\frac{x+1}{\left(x^{\frac{1}{3}}+1-x^{\frac{1}{3}}\right)} - \frac{x-1}{x-x^{\frac{1}{2}}}\right]^{10}$$

$$(x+1) = \left(x^{\frac{1}{3}}+1\right)\left(x^{\frac{2}{3}}+1-x^{\frac{1}{3}}\right)$$

$$\left(x^{\frac{1}{3}}+1\right) - \frac{(\sqrt{x}+1)(\sqrt{x}-1)}{\sqrt{x}(\sqrt{x}-1)}$$

$$\left(x^{\frac{1}{3}}-x^{\frac{1}{2}}\right)^{10}$$

$${}^{10}C_r\left(x^{\frac{1}{3}}\right)^{10-r} . \left(-x^{\frac{1}{3}}\right)^r$$

$$= \frac{10-r}{3} - \frac{r}{2} = 0$$

$$20 - 2r = 3r$$

$$4 = r$$

$${}^{10}C_4 = 210$$

Question: Let E be an ellipse such that $E : \frac{x^2}{18} + \frac{y^2}{9} = 1.$ **Let point P lies on E such that S and S' are foci of ellipse. Then find the sum of min (PS.PS') + max (PS.PS')**

Options:

(a) 18

(b) 36

(c) 9
(d) 27
Answer: (d)

$$\min(PS.PS\prime) + \max(PS.PS\prime)$$

$$PS.PS\prime = \left(a^2 - x^2e^2\right)$$

$$= \left(a^2 - x^2e^2\right) + a^2$$

$$= 2a^2 - e^2a^2$$

$$= 18\left(2 - e^2\right) = 18\left(2 - \frac{1}{2}\right) = 27$$