



Sol.  $S = 1 + 3 + 11 + 25 + \dots + T_n$   
 $S = 1 + 3 + 11 + \dots + T_{n-1} + T_n$   
 $T_n = 1 + 2 + 8 + 14 + \dots + (T_n - T_{n-1})$

$$T_n = 1 + \frac{n-1}{2} [4 + (n-2)6]$$

$$= 1 + \left(\frac{n-1}{2}\right) [6n - 8]$$

$$= 1 + (n-1)(3n-4)$$

$$= 1 + 3n^2 - 4n - 3n + 4$$

$$T_n = 3n^2 - 7n + 5$$

$$S_n = \sum T_n = 3 \sum n^2 - 7 \sum n + \sum 5$$

$$= \frac{3n(n+1)(2n+1)}{6} - \frac{7n(n+1)}{2} + 5n$$

now  $n = 20$

$$= \frac{3 \times 20 \times 21 \times 41}{6} - \frac{7 \times 20 \times 21}{2} + 5 \times 20 = 7240$$

5. Evaluate  $\int x^3 \sqrt{1-x^2} dx$

(1)  $\frac{-1}{15} (1-x^2)^{3/2} (3x^2+2) + C$

(2)  $\frac{1}{3} (1+x^2)^{3/2} - \sqrt{1-x^2} + C$

(3)  $\frac{2}{3} (1-x^2)^{3/2} (3x^2+2) + C$

(4)  $\frac{1}{3} (1-x^2)^{3/2} + \sqrt{1-x^2} + C$

Answer (1)

Sol.  $\int x^3 \sqrt{1-x^2} dx$

Put  $1-x^2 = t^2$

$-2x dx = 2t dt$

$$-\int t^2 (1-t^2) dt = -\left[\frac{t^3}{3} - \frac{t^5}{5}\right] + C$$

$$\frac{t^5}{5} - \frac{t^3}{3} + C$$

$$= \frac{(1-x^2)^{5/2}}{5} - \frac{(1-x^2)^{3/2}}{3} + C$$

$$= \frac{(1-x^2)^{3/2}}{15} [3(1-x^2) - 5] + C$$

$$= \frac{-(1-x^2)^{3/2}}{15} (3x^2+2) + C$$

6. A relation  $R = \{(x, y); x, y \in A = \{-3, -2, -1, 0, 1, 2, 3\} \text{ such that } x^2 + 2y \leq 4\}$ . If the number of ordered pairs in relation  $R$  be  $r$  and number of ordered pairs required to add in  $R$  so that it becomes reflexive relations is  $m$ , then  $r + m$  is equal to

(1) 26

(2) 28

(3) 24

(4) 23

Answer (2)

Sol.  $x^2 + 2y \leq 4$

$A = \{-3, -2, -1, 0, 1, 2, 3\}$

$x^2 \leq 4 - 2y$

For  $y = -3$

$x^2 \leq 4 - (2(-3))$

$x^2 \leq 10$

$\Rightarrow x \in \{-3, -2, -1, 0, 1, 2, 3\}$

For  $y = -2$

$x^2 \leq 4 - 2(-2)$

$x^2 \leq 8$

$\Rightarrow x \in \{-2, -1, 0, 1, 2\}$

For  $y = -1$

$x^2 \leq 4 - (2(-1))$

$x^2 \leq 6$

$\Rightarrow x \in \{-2, -1, 0, 1, 2\}$

For  $y = 0$

$x^2 \leq 4$

$\Rightarrow x \in \{-2, -1, 0, 1, 2\}$

For  $y = 1$

$x^2 \leq 2$

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$$\Rightarrow x \in \{-1, 0, 1\}$$

For  $y = 2$

$$x^2 \leq 0$$

$$\Rightarrow x \in \{0\}$$

For  $y = 3$

$$x^2 \leq -2 \Rightarrow \text{No value of } x$$

Total number of ordered pair in relation  $R$  is,  $r = 26$

For it to be reflexive we have to add  $\{(3, 3), (2, 2)\}$

$$\Rightarrow m = 2$$

$$\Rightarrow r + m = 28$$

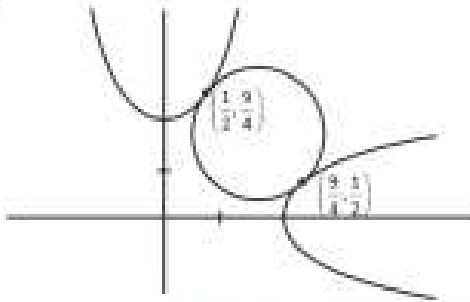
7. The radius of circle touching both parabolas  $y = x^2 + 2$  and  $x = y^2 + 2$  is

(1)  $\frac{7\sqrt{2}}{2}$  (2)  $\frac{7\sqrt{2}}{6}$

(3)  $\frac{7\sqrt{2}}{8}$  (4)  $\frac{7\sqrt{2}}{4}$

Answer (4)

Sol.



The circle will have its centre at  $x = y$  line and since these parabolas are symmetric about the line  $y = x$ . The slope will be of tangents at closest points.

$$\Rightarrow y^2 = x - 2 \quad \Rightarrow 2y \frac{dy}{dx} = 1 \quad \Rightarrow y = \frac{1}{2}$$

$$\Rightarrow \text{Point will be } \left(\frac{9}{4}, \frac{1}{2}\right)$$

Similarly on  $x^2 = y - 2$

$$\Rightarrow 2x = \frac{dy}{dx} = 1 \quad \Rightarrow x = \frac{1}{2}$$

$$\Rightarrow \left(\frac{1}{2}, \frac{9}{4}\right)$$

Circle's diameter will be equal to shortest distance

$$2r = \sqrt{\left(\frac{9}{4} - \frac{1}{2}\right)^2 + \left(\frac{1}{2} - \frac{9}{4}\right)^2} = \sqrt{\frac{7}{4} \times 2} = \sqrt{\frac{7}{2}}$$

$$\Rightarrow r = \frac{1}{2} \sqrt{\frac{7}{2}} = \sqrt{\frac{7}{8}} = \frac{7\sqrt{2}}{4}$$

8. Let  $3x + 2 \tan x = \pi$ ,  $x \in [-2\pi, 2\pi] - \left\{ \pm \frac{\pi}{2}, \pm \frac{3\pi}{2} \right\}$

Then number of values of  $x$  satisfying the above condition is

(1) 4

(2) 5

(3) 6

(4) 7

Answer (2)

Sol.  $3x + 2 \tan x = \pi$

$$2 \tan x = \pi - 3x$$

$$\tan x = \frac{\pi - 3x}{2}$$

5 solution

$\therefore$  In  $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$  well get 1 solution as  $\tan x$  is increasing in

$$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$$

$$\left. \begin{aligned} \left(\frac{-3\pi}{2}, \frac{\pi}{2}\right) &\rightarrow 1 \text{ solution} \\ \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) &\rightarrow 1 \text{ solution} \\ \Rightarrow \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) &\rightarrow 1 \text{ solution} \\ \left(\frac{3\pi}{2}, 2\pi\right) &\rightarrow 1 \text{ solution} \\ \left(-2\pi, \frac{-3\pi}{2}\right) &\rightarrow 1 \text{ solution} \end{aligned} \right\} 5$$

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