JEE-Main-03-04-2025 (Memory Based)
[EVENING SHIFT]
Maths

Question: If $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}} = p,$ **then 96** ***l*****n p is**
Answer: (32)

$$p = \lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$$

$$p = \lim_{x\to 0} e^{\left(\frac{\tan x}{x}-1\right)\cdot\frac{1}{x^2}}$$

$$= \lim_{x\to 0} e^{\frac{\tan x - x}{x^3}} = \lim_{x\to 0} e^{\frac{x+\frac{x^3}{3}+\frac{2}{15}x^5+\lim - x}{x^3}}$$

$$p = e^{\frac{1}{3}} \Rightarrow 96\ln p = 32$$

Question: Let A = {-3, -2, -1, 0, 1, 2, 3}. A relation R is defined such that xRy if y = max{x, 1}. Number of elements required to make it reflexive is l, number of elements required to mark it symmetric as m and number of elements in the relation R is n. Then value of l + m + n is equal to
Answer: (15)

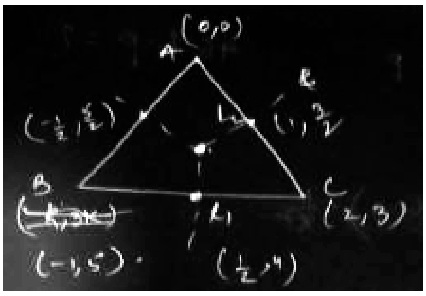
$$r = \{(-3,1),(-2,1)(-1,1),(0,1),(1,1)(2,2)(3,3)\}$$

$$\text{for } \ell{:}(-3,3),(-2,-2),(-1,-1)(0,0) \to 4$$

$$m = (1,3),(1,-2)(1,-1)(1,0) \to 4$$

$$l + m + n = 4 + 4 + 7 = 15$$

Question: Let a circle C with radius r passes through four distinct points (0, 0), (k, 3k), (2, 3) and (-1, 5), such that k ≠ 0, then (10k + 2r2) is equal to
Options:
(a) 35
(b) 34
(c) 27
(d) 32
Answer: (c)



$mBc = \frac{3-5}{2+1} = \frac{-2}{3}$ $\qquad$ $mAc = \frac{3}{2}$

$ml_1 = \frac{3}{2}$ $\qquad$ $ml_2 = \frac{-2}{3}$

equation $l_1 \Rightarrow y - 4 = \frac{3}{2}\left(x - \frac{1}{2}\right)$ $\quad$ equation $l_2 \Rightarrow y - \frac{3}{2} = \frac{-2}{3}(x-1)$

$2y - 8 = 3x - \frac{3}{2}$ $\qquad$ $3y - \frac{9}{2} = -2x + 2$

$2y - 3x = \frac{13}{2}$ $\qquad$ $3y + 2x = \frac{13}{2}$

$4y - 6x = 13$ $\qquad$ $6y + 4x = 13$

$x = -\frac{1}{2}, y = \frac{5}{2}$

for r=$\left(-\frac{1}{2}, \frac{5}{2}\right) \& (2,3) \Rightarrow r = \sqrt{65}$

$\therefore 2r^2 = 13$

for k; equation if circle

$\left(x + \frac{1}{2}\right)^2 + \left(3k - \frac{5}{2}\right)^2 = \frac{13}{2}$

$(k, 3k)$should satisfy

$\left(k + \frac{1}{2}\right)^2 + \left(3k - \frac{5}{2}\right)^2 = \frac{13}{2}$

$k^2 + \frac{1}{4} + k + 9k^2 + \frac{25}{4} - 15k = \frac{13}{2}$

$10k^2 - 14k + \frac{25}{4} = \frac{13}{2}$

$\therefore 10k = 14$

$10k + 24^2$

$= 14 + 13$

$= 27$

Question: $\ell = \int_0^{\pi} \frac{8x}{4\cos^2 x + \sin^2 x} dx$ **equals to**

Options:

(a) π^2

(b) $4\pi^2$

(c) $2\pi^2$

(d) $\frac{3}{2}\pi^2$

Answer: (c)

$$I=\int_0^{\pi}\frac{8x}{4\cos^2x+\sin^2x}dx------(1)$$

$$I=\int_0^{\pi}\frac{8(\pi-x)}{4\cos^2x+\sin x}dx-------(2)$$

$$2I=\int_0^{\pi}\frac{8\pi}{4\cos^2x+\sin^2x}dx$$

$$I=8\pi\int_0^{\frac{\pi}{2}}\frac{\sec^2xdx}{\tan^2x+4}=8\pi\int_0^{\infty}\frac{dt}{t^2+2^2}$$

$$=8\pi\cdot\tfrac{1}{2}\left(\tan^{-1}\tfrac{t}{2}\right)_{0=}^{\infty}=4\pi\left(\tfrac{\pi}{2}-0\right)=2\pi^2$$

Question: $S=1+\dfrac{1+3}{1!}+\dfrac{1+3+5}{2!}+...\infty$ **The value of S is equal to**

Options:

(a) 4e-2

(b) 4e

(c) 5e

(d) 7e

Answer: (c)

$$S=1+\tfrac{1+3}{1!}+\tfrac{1+3+5}{2!}+------$$

$$=\sum_{n=1}^{8}\frac{n^2}{(n-1)!}=\sum_{n=1}^{8}\frac{(n-1)(n+1)+1}{(n-1)!}$$

$$=\sum_{n=1}^{8}\frac{n+1}{(n-2)!}+\sum_{n=1}^{8}\frac{1}{(n-1)!}$$

$$=\sum_{n=1}^{8}\frac{n-2}{(n-2)!}+\sum_{n=1}^{8}\frac{3}{(n-2)!}+e$$

$$=e+3e+e=5e$$

Question: Let y= f(x) be the solution of the differential equation $\dfrac{dy}{dx}+3y\tan^2x+3y=\sec^2x$ **such that** $f(0)=\dfrac{e^3}{3}+1$, then $f\left(\dfrac{\pi}{4}\right)$

Options:

(a) $(1+e^{-3})$

(b) $\frac{2}{3}\left(1+\frac{1}{e^3}\right)$

(c) $\frac{1}{3}\left(1-\frac{1}{e^3}\right)$

(d) $\frac{1}{3}\left(1+\frac{1}{e^3}\right)$

Answer: (b)

$\frac{dy}{dx} + 3y\sec^2 x + \sec^2 x$

$If = e^{\int 3\sec^2 x dx} = e^{3\tan x}$

$ye^{3\tan x} = \int e^{3\tan x}\sec^2 x dx$

$ye^{3\tan x} = \frac{3\tan x}{3} + C$

At $x = 0, r = \frac{e^3}{3} + 1 \Rightarrow c = \frac{e^3}{3} + \frac{2}{3}$

At x$=\frac{\pi}{4}, ye^3 = \frac{e^3}{3} + \frac{e^3}{3} + \frac{2}{3} = \frac{2}{3}(e^3 + 1)$

$y = \frac{2}{3}\left(1 + \frac{1}{e^3}\right)$

Question: Area bounded by $|x - y| \leq y \leq 4\sqrt{x}$ is equal to (in square units)

Options:

(a) $\frac{2048}{3}$

(b) $\frac{1024}{3}$

(c) $\frac{512}{3}$

(d) $\frac{128}{3}$

Answer: (b)

Question: If $(1 + x + x^2)^{10} = 1 + a_1 + a_2x^2 + \ldots$, then $(a_1 + a_3 + a_5 + \ldots + a_{19}) - 11a_2$ equals to

Answer: (55)

$(1 + x + x^2)^{10} = (1 + a_1x + a_2x^2 + \ldots\ldots. + a_{20}x^{20})\text{then}(a_1 + a_3 + \ldots.a_{19})11a_2$

$x = 1, 3^{10} = (1 + a_1 + a_2 + a_3 + \ldots\ldots + a_{20})$

$x = -1, 1 = 1 - a_1 + a_2 - a_3 + \ldots\ldots\ldots. + a_{20}$

$3^{10} - 1 = 2(a_1 + a_3 + \ldots\ldots.a_{19})$

$(a_1 + a_3 + \ldots\ldots\ldots. + a_{19}) = \left(\frac{3^{10}-1}{2}\right) - 11\ldots.55$

$= \frac{3^{10}-1-1210}{2}$

$= \frac{3^{10}-1211}{2}$

$Also, 10(1 + x + x^2)^9(2x + 1) = a_1 + 2a_2x + \ldots\ldots\ldots + 20x^{19}a^{20}$

$10a(1 + x + x^2)^8(2x + 1)^2 + 10(1 + x + x^2)^9(2) = 2a_2$

$90 \cdot 1 + 20 = 2a_2$

$a_2 = 55$

Question: Let $A(z_1)$, $B(z_2)$ and $C(z_3)$ are the vertices of an equilateral triangle. If z0 is the centroid of triangle ABC and $|z_1 - z_2| = 1$, then the value of $\sum_{i=1}^{3} |z_1 - z_0|^2$ is equal to

Options:

(a) 1

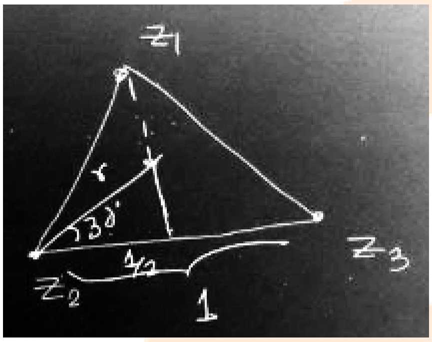
(b) 2
(c) 3
(d) 9
Answer: (a)

$$|z_1 - z_2| = 1$$

$$\sum_{i=1}^{3} |z_i - z_0|^2$$

$$= |z_1 - z_0|^2 + |z_2 - z_0|^2 + |z_3 - z_0|^2$$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 3 = 1$$



$$\cos 30^9 = \frac{1}{2r}$$

$$\frac{\sqrt{3}}{2} = \frac{1}{2r}$$

$$r = \frac{1}{\sqrt{3}}$$

Question: If f(x) = ||x+ 2| - 2|x||, then number of points of local maxima and local minima is
Options:
(a) 5
(b) 3
(c) 2
(d) 7
Answer: (c)

$f(x)=||x+2|-2|x||$

$C_1 x<-1$ $\quad$ $C_2: -2 \leq x<0$ $\quad$ $C_3 x \geq 0$

$f(x)=-x+2$ $\quad$ $f(x)=3x+2$ $\quad$ $f(x)=x-2$

$at\ x=-2,$ $\quad$ at $x=0$ $\quad$ at $x=2$

$f'(x)=-1$ $\quad$ $f'(x)=3$ $\quad$ $f'(x)=-1$

$\therefore$ Minima at $x=2$

maxima at $x=0$

minima at $x=2$

Question: x(x - 2) (12 - k) = 2 has both roots same. The distance of $\left(k, \frac{k}{2}\right)$ from the line 3x +4y + 5= 0 is

Options:

(a) 24

(b) 14

(c) 15

(d) 20

Answer: (c)

$x(x-2)(12+k)=2$ as same roots

D=0

$\left(x^2-2x\right)(12-k)$

$(12-k)x^2-2(12-k)x-2=0$

$D=0$

$4(12-k)^2+8(12-k)=0$

$(12-k)^2+2(12-k)=0$

$(12-k)(12-k+12)=0$

$k=12, 14$

$k=12$ rejected

k=14

$\therefore \left(k, \frac{k}{2}\right)=(14,7)$

$3x+4y+5$

$d=\left|\frac{3\times r14+4\times 7+5}{\sqrt{3^2+4^2}}\right|=\left|\frac{42+28+5}{5}\right|$

$=\left|\frac{75}{5}\right|=15$

Question: The shortest distance between the parabola y^2 = 8x and the circle $x^2 + y^2$ + 12y + 35 = 0 is

Options:

(a) $\left(2\sqrt{2}-1\right)$ units

(b) $\left(\sqrt{2}-1\right)$ units

(c) $\left(2\sqrt{2}+1\right)$ units

(d) $\left(\sqrt{2}+1\right)$ units

Answer: (a)

Shortest distance = Normal

$y^2 = 8x$

$4a = 8$

$a = 2$

$x^2 + y^2 + 12y + 35 = 0$

$centre(0, -6)$

$y^2 = 8x$

$2yy' = 8$

$4y' = 4$

$y' = \frac{4}{y}$

stope of normal $= -\frac{y}{4} \rightarrow \frac{4t}{4} = -t$

$\frac{4t+6}{2+2} = -t$

$4t + 6 = -2t^3$

$2t^3 + 4t + 6 = 0$

$t = -1$

$P\left(2t^2, 4t\right)$

$P(2, -4)$

Centre $O\,(0, -6)$

radius $= 1$

Shortest distance = OP − radius

$=\sqrt{4+4} - 1$

$= 2\sqrt{2} - 1$

Question 14: The no of solutions of the equation

$$\left(4-\sqrt{3}\right)\sin x - 2\sqrt{3}\cos^2 x = \frac{-4}{1+\sqrt{3}},\ x \in \left[-2\pi, \frac{5\pi}{2}\right]$$

Answer: ([-2r,5$\frac{\pi}{2}$])

$$\left(4-\sqrt{3}\right)\sin x - 2\sqrt{3}\cos^2 x = \frac{-4}{1+\sqrt{3}}$$

$$\left(4-\sqrt{3}\right)s - 2\sqrt{3}\left(1-s^2\right) = -\frac{4\left(\sqrt{3}-1\right)}{2}$$

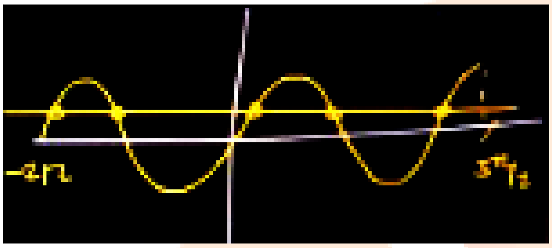
$$\left(4-\sqrt{3}\right)s - 2\sqrt{3} + 2\sqrt{3}s^2 = -2\sqrt{3}+2$$

$$2\sqrt{3}\,s^2 + \left(4-\sqrt{3}\right)s - 2 = 0$$

$$\sin x = \frac{\sqrt{3}}{2\sqrt{3}} \ \& \ \frac{-4}{2\sqrt{3}}$$

$$\therefore \ \sin x = \frac{1}{2}$$

from $\left[-2r, 5\frac{\pi}{2}\right]$



Question: The distance of the point (7, 10, 11) from the line $\dfrac{x-4}{1} = \dfrac{x-4}{0} = \dfrac{z-2}{3}$ **along the line** $\dfrac{x-9}{2} = \dfrac{y-13}{3} = \dfrac{z-17}{6}$ **is**

Answer: (PQ = 14 = d)

$\frac{x-4}{1} = \frac{y-4}{0} = \frac{z-2}{3} = \lambda \quad L_1$

$\frac{\lambda+4-9}{2} = \frac{4-1}{3} = \frac{3\lambda+z-17}{6} \quad L_2$

$\frac{\lambda-5}{2} = -3$

$\lambda = -1$

$\therefore Q(3, 4, -1)$

$PQ = \sqrt{4^2+6^2+12^2}$

$= \sqrt{16+36+144}$

$PQ = 14 = d$