JEE-Main-03-04-2025 (Memory Based)
[MORNING SHIFT]
Maths

Question: Let A be 3 × 3 matrix such that det(A) = 5. If det(3adj(2Aadj(2A))) = $2\alpha.3\beta.5\gamma$, then $(\alpha+\beta+\gamma)$ is equal to

Options:

(a) 25

(b) 26

(c) 27

(d) 28

Answer: (c)

$|A| = 5,$

$|3adj(2Aadj(2A))| = 2^{\alpha}\cdot 3^{\beta}\cdot 5^{\gamma}$

$3^3|adj(2A(adj(2A)))| = 2^{\alpha}\cdot 3^{\beta}\cdot 5^{\gamma}$

$3^3|2A(adj(2A))| = 2^{\alpha}\cdot 3^{\beta}\cdot 5^{\gamma}$

$3^3\times 2^6|A|^2|adj(2A)|^2 = 2^{\alpha}\cdot 3^{\beta}\cdot 5^{\gamma}$

$3^3\times 2^6\times 5^2|2A|^4 = 2^{\alpha}\cdot 3^{\beta}\cdot 5^{\gamma}$

$3^3\times 2^6\times 5^2\times(2)^{12}|A|^4 = 2^{\alpha}\cdot 3^{\beta}\cdot 5^{\gamma}$

$3^3\times 2^{18}\times 5^2\times 5^4 = 2^{\alpha}3^{\beta}5^{\gamma}$

$2^{18}\times 3^3\times 5^6 = 2^{\alpha}3^{\beta}5^{\gamma}$

$\alpha = 18, \beta = 3, \gamma = 6$

$\alpha+\beta+\gamma = 27$

Question: The sum of all rational number in $\left(2+\sqrt{3}\right)^8$ is

Options:

(a) 18117

(b) 18817

(c) 17280

(d) 1800

Answer: (b)

$\left(2+\sqrt{3}\right)^8$

$T_{r+1} = {}^8C_r 2^{8-r}\left(\sqrt{3}^r\right)$

$0 \le r \le 8$

For rotational terms $r = 0, 2, 4, 6, 8$

$T_1 = {}^8C_0 2^8\left(\sqrt{3}\right)^0 = 256$

$T_3 = {}^8C_2 2^6\left(\sqrt{3}^2\right) = 5376$

$T_5 = {}^8C_4 2^4\left(\sqrt{3}\right)^4 = 10080$

$T_7 = {}^8C_6 2^2\left(\sqrt{3}\right)^6 = 3024$

$T_9 = {}^8C_8 2^0\left(\sqrt{3}\right)^8 = 81$

$=18817$

Question: If the sum $\sum_{r=1}^{9}\left(\frac{r+3}{2^r}\right){}^9C_r - 9 = \alpha\,.\left(\frac{3}{2}\right)^9 - \beta,$ **then the value of** $(\alpha+\beta)^2$ **is equal to**

Options:

(a) 9

(b) 81

(c) 27

(d) 36

Answer: (b)

$\sum_{r=1}^{9} \frac{r+3}{2^r}.9C_r$

$\sum_{r=1}^{9} \frac{r\,{}^9C_r}{2^r} + 3\sum_{r=1}^{9} {}^9C_r\left(\frac{1}{2}\right)^r$

$\sum_{r=1}^{9} \frac{9\,{}^8C_{r-1}}{2^r} + 3\left(1+\frac{1}{2}\right)^9 - {}^9C_0\left(\frac{1}{2}\right)^0$

$\frac{9}{2}\sum_{r=1}^{9} {}^8C_{r-1}\left(\frac{1}{2}\right)^{r-1} + 3\left(\frac{3}{2}\right)^{9-3}$

$\frac{9}{2}\left(1+\frac{1}{2}\right)^8 + 3\left(\frac{3}{2}\right)^{9-3}$

$\frac{9}{2}\left(\frac{3}{2}\right)^8 + 3\left(\frac{3}{2}\right)^{9-3}$

$\left(\frac{3}{2}\right)^8\left[\frac{4}{2} + \frac{3\times 3}{2}\right] - 3$

$\left(\frac{3}{2}\right)^8 \frac{18}{2} - 3$

$(6)\left(\frac{3}{2}\right)^9 - 3$

$\alpha = 6; \beta = 3$

$(\alpha+\beta)^2 = (6+3)^2 = 81$

Question: Let S_n **= 1 + 3 + 11 + 25 + 45 + …….. Then sum upto** 20^{th} **term equals to**

Options:

(a) 6200

(b) 7200

(c) 7240

(d) 6240

Answer: (c)

Question: Evaluate $\int x^3\sqrt{1-x^2}dx$

Options:

(a) $-\frac{1}{15}\left(1-x^2\right)^{\frac{3}{2}}\left(3x^2+2\right)+C$

(b) $\frac{1}{3}\left(1+x^2\right)^{\frac{2}{3}}-\sqrt{1-x^2}+C$

(c) $\frac{2}{3}\left(1-x^2\right)^{\frac{3}{2}}\left(3x^2+2\right)+C$

(d) $\frac{1}{3}\left(1-x^2\right)^{\frac{2}{3}}+\sqrt{1-x^2}+C$

Answer: (a)

$$\int x^3\sqrt{1-x^2}dx$$

$$=1-x^2=t^2$$

$$=-2x\ dx=2t\ .\ dt$$

$$=\int\left(1-t^2\right)t^2dt$$

$$=\int\left(t^4-t^2\right)dt$$

$$=\frac{1}{5}t^5-\frac{1}{3}t^3+C=\frac{t^3}{15}\left[3t^2-5\right]$$

$$=\frac{-1}{15}\left(1-x^2\right)^{\frac{3}{2}}\left(3x^2+2\right)+C$$

Question: A relation R = {(x, y) : y ∈ A = {-3, -2, -1, 0, 1, 2, 3} such that x^2 + 2y ≤ 4}. Then the number of ordered pairs in relation R be and number of ordered pairs required to add in R so that it becomes reflexive relations is m, then r + m is equal to

Options:

(a) 26

(b) 28

(c) 24

(d) 23

Answer: (b)

$A=\{\pm3,\pm2,\pm1,0\}$

$R=\left\{(x,y):x,y\in A\&x^2+2y\leq4\right\}$

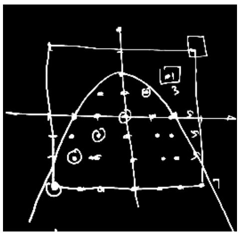
$$R=\left\{\begin{array}{l}(0,2),(0,-2),(0,-3),(0,1),(0,0),(0,-1),(1,1),(1,0),\\(1,-1),(1,-2),(1,-3)(2,0),(2,-1),(2,-2),(2,-3)),\\(3,-3),(-1,1),(-1,0),(-1,-1),(-1,-2),(-1,-3)\\(-2,0)(-2,-1),(-2,-2),(-2,-3),(-3,-3)\end{array}\right\}$$

$r=26$

same as$\{(2,2)(3,3)\}$

$r=26,m=2$

$r+m=28$



$y = 2 - \frac{1}{2}x^2$

$$1 + 3 + 5 + 5 + 5 + 7 = 26$$

$$r = 26$$

$$m = 2$$

$$r + m = 28$$

Question: The radius of the smallest circle touching both parabolas y = x^2 + 2 and x = y^2 + 2 is

Options:

(a) $\frac{7\sqrt{2}}{2}$

(b) $\frac{7\sqrt{2}}{6}$

(c) $\frac{7\sqrt{2}}{8}$

(d) $\frac{7\sqrt{2}}{4}$

Answer: (c)

Question: $3x + 2\tan x = \pi, x \in [-2\pi, 2\pi] = \left[\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}\right]$. **Then number of value of x satisfy is**

Options:

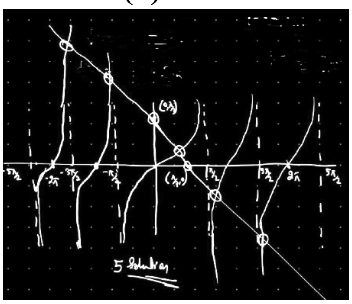
(a) 4

(b) 5

(c) 6

(d) 7

Answer: (b)



$3x + 2\tan = \pi$

$\tan = \frac{\pi - 3x}{2}$ $[-2\pi, 2\pi]$

$\tan = \frac{\pi}{2} - \frac{3x}{2}$ $y = \frac{\pi}{2} - \frac{3}{2}r$

Question: Let $\int_0^x g(t)dt = x - \int_0^x tg(t)dt, x \geq 0$ **and** $\frac{dy}{dx} - y\tan x = 2(x+1)\sec x g(x)$ **satisfying the condition y(0) = 0. Then** $y\left(\frac{\pi}{3}\right)$ **is.**

Options:

(a) $\frac{2\pi}{3}$

(b) $\frac{4\pi}{3}$

(c) π

(d) 2π

Answer: (b)

Question: $f(x) = \begin{vmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{vmatrix}$. **Then, the value of f'(x) + f(x) is**

Options:

(a) -1

(b) 28

(c) 27

(d) 1

Answer: (d)

$$\begin{vmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{vmatrix}$$

$$f' = \begin{vmatrix} \cos x & -\sin x & \cos x - \sin x \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{vmatrix}$$

$$f'' = \begin{vmatrix} -\sin x & -\cos x & -\sin x - \cos x \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{vmatrix}$$

$$f'' + f = \begin{vmatrix} 0 & 0 & 1 \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{vmatrix}$$

$= 27 - 28$

$= -1$

Question: Let α, β **are the roots of the equation** $x^2 + \sqrt{3}x - 16 = 0$ **and** γ, δ **are the roots of the equation** $x^2 + 3x - 1 = 0$**. If** $Q_n = \alpha^n + \beta^n \ \forall \ n \in N$ **and** $P_n = \gamma^n + \delta^n \ \forall \ n \in N$ **then the value of** $\frac{Q_{25} + \sqrt{3}Q_{24}}{2Q_{23}} + \left(\frac{P_{25} - P_{23}}{P_{24}}\right)$

Options:

(a) 5
(b) 6
(c) 7
(d) 8
Answer: (a)
α, β are roots of the equation $x^2 + \sqrt{3}x - 16 = 0$ and γ, δ are the roots of the equation $x^2 + 3x - 1 = 0$.
If $Q_n = \alpha^n + \beta^n \ \forall \ n \in N$ and $P_n = \gamma^n + \delta^n \ \forall \ n \in N$ then

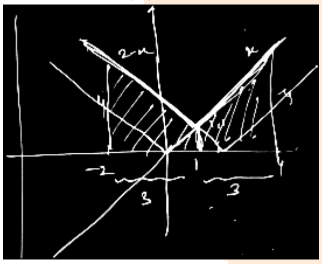
$$\frac{Q_{25}+\sqrt{3}Q_{24}}{2Q_{23}} + \frac{P_{25}-P_{23}}{P_{24}}$$

$$Q_{25} + \sqrt{3}Q_{24} - 16Q_{23} = 0 \qquad P_{25} + 3P_{24} - P_{23} = 0$$

$$\frac{Q_{25}+\sqrt{3}Q_{24}}{2Q_{23}} \qquad \frac{P_{25}-P_{23}}{P_{24}} = \frac{3P_{24}}{P_{24}} = -3$$

$$8 + 3 = 11$$

Question: If y = max{|x|, x, |x - 2|}, then the area under the curve from x = -2 to x = 4 is (in square units)
Options:
(a) 15
(b) 20
(c) 12
(d) 8
Answer: (a)



$$2 - x = x$$

$$x = 1$$

$$2 \cdot \frac{1}{2} \cdot 3 \cdot 5 = 15$$

Question: Let a_1, a_2, a_3 …. Are in GP, where $a_3.a_5 = 729$ and $a_2 + a_4 = \frac{111}{4}$, then $24(a_1 + a_2 + a_3)$
Options:
(a) 131
(b) 130
(c) 129
(d) 128
Answer: (c)

Question: Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$. Let $\bar{c}$ **is a unit vector such that** $\vec{a} \times \vec{c} = \vec{b} \times \vec{c}$ If $\vec{c} = \lambda\hat{i} + \mu\hat{j}$ **and** $\vec{d}$ **is a vector perpendicular to** $\vec{c}$ and $\vec{a}$, then $\left|\lambda\vec{c} + \mu\vec{d}\right|^2$ **is equal to**

Options:

(a) $\frac{6}{25}$

(b) $\frac{61}{25}$

(c) $\frac{41}{25}$

(d) $\frac{36}{25}$

Answer: (b)

Question: Let a line passing through (4,1,3) intersects the lines $l_1 : \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-3}{5}$ **at** (α, β, γ) **and** l_2 **: x - 1 = y = -z + 4 t (a, b, c) then find** $\begin{vmatrix} 63 & 21 & -21 \\ \alpha & \beta & \gamma \\ a & b & c \end{vmatrix}$ **is equal to**

Options:

(a) 102

(b) 204

(c) 63

(d) 21

Answer: (b)