JEE (MAIN) 2025

MEMORY BASED QUESTIONS & TEXT SOLUTION

SHIFT-1

DATE & DAY: 03rd April 2025 & Thursday

PAPER-1

Duration: 3 Hrs.
Time: 09:00 – 12:00 IST

SUBJECT: MATHEMATICS

PART : MATHEMATICS

1. g(x) is a function such that $\int_0^x g(t)dt = x - \int_0^x t\,g(t)dt$ & $\frac{dy}{dx} - y\tan x = 2(x+1)\,g(x)\sec x$ such that $y(0) = 0$ then $y\left(\frac{\pi}{3}\right) =$

(1) $\frac{\pi}{3}$ (2) $\frac{2\pi}{3}$ (3) $\frac{4\pi}{3}$ (4) $\frac{5\pi}{3}$

Ans. **(3)**

Sol. $\int_0^x g(t)dt = x - \int_0^x t\,g(t)dt$

diff. w.r.t x both side

$g(x) = 1 - x\,g(x)$

$(1+x)\,g(x) = 1 \Rightarrow \quad g(x) = \frac{1}{1+x}$

$\therefore \quad \frac{dy}{dx} - y\tan x = 2\sec x$

I.F. $= e^{-\int \tan x\,dx} = e^{-\ln\sec x} = \cos x$

$\therefore$ Solution is

$y\cos x = \int 2\sec x \cdot \cos x\,dx + c$

$y\cos x = 2x + c$

$y(0) = 0 \therefore 0 = 0 + c \Rightarrow c = 0$

$\therefore \quad y = 2x$ sex x

$\therefore \quad y\left(\frac{\pi}{3}\right) = 2 \cdot \frac{\pi}{3} \cdot \sec\frac{\pi}{3} = \frac{4\pi}{3}$

2. Let A be 3 × 3 matrix such that det (A) = 5 if det (2 adj (3A. adj(2A))) = $2^{\alpha} \times 3^{\beta} \times 5^{\gamma}$ then $\alpha + \beta + \gamma$ is equal to

Ans. **(27)**

Sol. det (2 adj (3A. adj (2A))

$= 2^3$ |adj (3A. adj (2A))|

$= 8\,|3A.\ \text{adj}\,(2A)|^2$

$= 8 \times 3^6\,|A\ \text{adj}\,(2A)|^2$

$= 2^3 \times 3^6\,|A|^2\,|\text{adj}\,(2A)|^2$

$= 2^3 \times 3^6 \times 5^2\,|2A|^4$

$= 2^3 \times 3^6 \times 5^2\ 2^{12}|A|^4$

$= 2^{15} \times 3^6 \times 5^2 \times 5^4$

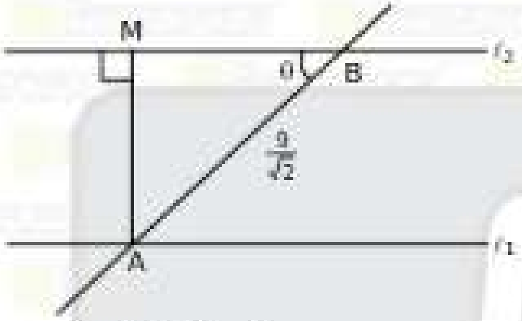
$= 2^{15} \times 3^6 \times 5^6$

$\alpha + \beta + \gamma = 27$

3. A line with slope =1 cuts line $L_1 : 2x + y - 6 = 0$ at A & $L_2 : 4x + 2y - p = 0$, $p > 0$ at B such that $AB = \frac{9}{\sqrt{2}}$. A perpendicular AM from A drawn to line ℓ_2 meets at M then $\frac{AM}{BM}$ is equal to

Ans. (3)

Sol.



$\tan\theta = \left|\frac{-2-1}{1+(-2)(1)}\right| = \frac{3}{1}$

$\sin\theta = \frac{3}{\sqrt{10}}$

$AM = \frac{3}{\sqrt{10}} \times \frac{9}{\sqrt{2}} = \frac{27}{\sqrt{20}} = \frac{27}{2\sqrt{5}}$

dis. between ℓ_2 & ℓ_2 $AM = \left|\frac{\frac{-p}{2}+6}{\sqrt{5}}\right|$

$\frac{6-\frac{p}{2}}{\sqrt{5}} = \pm\frac{27}{2}$

$6 + \frac{27}{2} = \frac{p}{2}$

$p = 39$ as $p > 0$

$BM = \sqrt{\frac{81}{2} - \left(\frac{27}{2\sqrt{5}}\right)^2}$

$= \sqrt{\frac{810-729}{20}} = \sqrt{\frac{81}{20}} = \frac{9}{2\sqrt{5}}$

Now $\frac{AM}{BM} = \frac{\frac{27}{2\sqrt{5}}}{\frac{9}{2\sqrt{5}}} = 3$

4. Let $f(x) = \int x^3\sqrt{3-x^2}\,dx$ such that $5f(\sqrt{2}) = -4$ then $f(1) =$

(1) $-\frac{3\sqrt{2}}{5}$ (2) $-\frac{4\sqrt{2}}{5}$ (3) $-\frac{6\sqrt{2}}{5}$ (4) $\frac{6\sqrt{2}}{5}$

Ans. **(3)**

Sol. $f(x) = \int x^3\sqrt{3-x^2}\,dx = \int x^2\sqrt{3-x^2}\,x\,dx$

Let $3 - x^2 = t \therefore -2x\,dx = dt$

$\therefore f(x) = \int (3-t)\sqrt{t}\left(-\frac{1}{2}dt\right) + c$

$= \frac{1}{2}\int (t^{3/2} - 3t^{1/2})dt + c$

$= \frac{1}{2}\frac{t^{5/2}}{5/2} - \frac{3}{2}\frac{t^{3/2}}{3/2} + c$

$= \frac{t^{5/2}}{5} - t^{3/2} + c$

$= \frac{(3-x^2)^{5/2}}{5} - \frac{(3-x^2)^{3/2}}{1} + c$

Put $x = \sqrt{2}$

$\therefore f(\sqrt{2}) = -\frac{4}{5} = \frac{1}{5} - 1 + c$

$-\frac{4}{5} = -\frac{4}{5} + c \Rightarrow c = 0$

$\therefore f(x) = \frac{(3-x^2)^{5/2}}{5} - (3-x^2)^{3/2}$

$\therefore f(x) = \frac{2^{5/2}}{5} - 2^{3/2}$

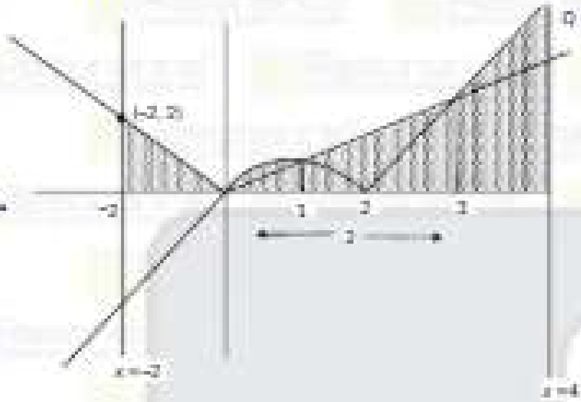
$= \sqrt{2}\left(\frac{4}{5} - 2\right)$

$= \sqrt{2}\left(-\frac{6}{5}\right) = -\frac{6\sqrt{2}}{5}$

5. Area between x –axis and curve $y = \max.\{|x|, x|x-2|\}$ from $x = -2$ to $x = 4$ is

Ans. **(12)**

Sol.



$$\text{Required area} = \frac{1}{2}\times 2\times 2 + \int_0^1 (2x - x^2)dx$$

$$+\frac{1}{2}(2)(1+3) + \int_3^4 (x^2 - 2x)dx$$

$$= 2 + \left(x^2 - \frac{x^3}{3}\right)_0^1 + 4 + \left(\frac{x^3}{3} - x^2\right)_3^4$$

$$= 2 + \left(1 - \frac{1}{3}\right) + 4 + \left(\frac{64}{3} - 16\right) - (9-9)$$

$$= \frac{8}{3} + 4 + \frac{16}{3} = 12$$

6. Sum of all rational terms in expansion of $\left(2+\sqrt{3}\right)^8$

Ans. **(18817)**

Sol. $T_{r+1} = {}^8C_r.\ 2^{8-r}\ 3^{r/2}$

for rational terms r = 0, 2, 4, 6, 8.

sum $= {}^8C_0.\ 2^8.\ 3^0 + {}^8C_2.\ 2^6.\ 3^1 + {}^8C_4.\ 2^4.\ 3^2 + {}^8C_6.\ 2^2 \times 3^3 + {}^8C_8 \times 2^0 \times 3^4$

$= 256 + 28 \times 64 \times 3 + 70 \times 16 \times 9 + 28 \times 4 \times 27 + 81$

$= 16\,[16 + 336 + 360 + 189] + 81$

$= 16 \times 1171 + 81$

$= 18817$

7. $\sum_{r=1}^{9} \frac{r+3}{2^r} {}^9C_r =$

(1) $\frac{3^{10}}{2^8}+3$ (2) $\frac{3^{10}}{2^8}-3$ (3) $\frac{3^{10}}{2^8}-5$ (4) $\frac{3^{10}}{2^8}+5$

Ans. (2)

Sol. $=\sum_{r=1}^{9} \frac{r}{2^r} \frac{9}{r} {}^8C_{r-1} + \sum_{r=1}^{9} \frac{3}{2^r} \times {}^9C_r$

$= \frac{9}{2}\sum_{r=1}^{9} {}^8C_{r-1} \frac{1}{2^{r-1}} + 3\sum_{r=1}^{9} \frac{{}^9C_r}{2^r}$

$= \frac{9}{2}\left(1+\frac{1}{2}\right)^8 + 3\left[\left(1+\frac{1}{2}\right)^9 - 1\right]$

$= \frac{9}{2} \times \frac{3^8}{2^8} + 3 \times \frac{3^9}{2^9} - 3$

$= \frac{3^{10}}{2^9} + \frac{3^{10}}{2^9} - 3$

$= \frac{3^{10}}{2^8} - 3$

8. $y = \begin{vmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 26 & 27 & 26 \\ 1 & 1 & 1 \end{vmatrix}$ then $\frac{d^2y}{dx^2} + y =$

(1) 0 (2) 1 (3) –1 (4) 2

Ans. (3)

Sol. $y = \begin{vmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 26 & 27 & 26 \\ 1 & 1 & 1 \end{vmatrix}$

$= \sin x\,(27 - 26) - \cos x(26 - 26) + (\sin x + \cos x + 1)\ (26 - 27)$

$= \sin x - 0 - \sin x - \cos x - 1$

$y = -\cos x - 1$

$\therefore \frac{dy}{dx} = \sin x \qquad \therefore \frac{d^2y}{dx^2} = \cos x$

$\therefore \frac{d^2y}{dx^2} = \cos x - \cos x - 1 = -1$ ans

9. Let α , β are roots of $x^2 + \sqrt{3}x - 16 = 0$ & γ, δ are roots of $x^2 + 3x - 1 = 0$ and let $P_n = \alpha^n + \beta^n$ & $Q_n = \gamma^n + \delta^n$ the find value of $\frac{P_{25} + \sqrt{3}P_{24}}{2P_{23}} + \frac{Q_{24} - Q_{22}}{Q_{23}}$

Ans. **(5)**

Sol. α, β are routs of $x^2 + \sqrt{3}x - 16 = 0$

$\therefore$ $\alpha^n + \sqrt{3}\ \alpha^{n-1} - 16\,\alpha^{n-2} = 0$

$\beta^n + \sqrt{3}\ \beta^{n-1} - 16\,\beta^{n-2} = 0$

$P_n + \sqrt{3}\ P_{n-1} - 16\,P_{n-2} = 0$

$\therefore$ $P_{25} + \sqrt{3}\ P_{24-1} - 16\,P_{23} = 0$

$\Rightarrow$ $\frac{P_{25} + \sqrt{3}P_{24}}{2P_{23}} = 8$

Similarity

$Q_n + 3Q_{n-1} - Q_{n-2} = 0$

$\therefore$ $Q_{24} + 3Q_{23} - Q_{22} = 0$

$\Rightarrow$ $\frac{Q_{24} - Q_{22}}{Q_{23}} = -3$

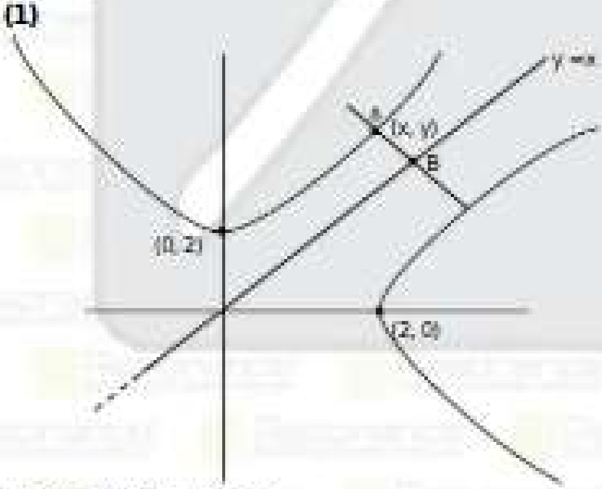
$\therefore$ $\frac{P_{25} + \sqrt{3}P_{24}}{2P_{23}} + \frac{Q_{24} - Q_{22}}{Q_{23}} = 8 - 3 = 5$

10. The radius of smallest circle touches both parabolas externally $x^2 + 2 = y$ & $y^2 + 2 = x$

(1) $\frac{7}{4\sqrt{2}}$ (2) $\frac{7}{2\sqrt{2}}$ (3) $\frac{7}{3\sqrt{2}}$ (4) $\frac{7}{5\sqrt{2}}$

Ans. **(1)**

Sol.



$x^2 + 2 = y$ & $y^2 + 2 = x$

are inverse of each other

Let $A(x_1, y_1)$ lies on $x^2 + 2 = y$

then normal at A is

$y - y_1 = -\frac{1}{2x_1}(x - x_1)$

Slope of normal $= -\frac{1}{2x_1} = -1$

$x_1 = \frac{1}{2}$

Put in $x^2 + 2 = y$

$\frac{1}{4} + 2 = y$

$y = \frac{9}{4}$

radius AB $= \left|\frac{\frac{1}{2} - \frac{9}{4}}{\sqrt{2}}\right|$

$= \frac{7}{4\sqrt{2}}$

11. Let positive numbers $a_1, a_2, a_3, \ldots$ are in G.P. such that $a_3 a_5 = 729$ & $a_2 + a_4 = \frac{111}{4}$, then $a_1 + a_2 + a_3 =$

(1) $\frac{41}{8}$ (2) $\frac{43}{8}$ (3) $\frac{45}{8}$ (4) $\frac{47}{8}$

Ans. **(2)**

Sol. Let $a_1, a_2, a_3, \ldots$ are in G.P.

$a_3 a_5 = 729 \Rightarrow ar^2 . ar^4 = 729$

$\Rightarrow a^2 r^6 = 729 \Rightarrow ar^3 = 27$

& $a_2 + a_4 = \frac{111}{4} \Rightarrow ar + ar^3 = \frac{111}{4}$

$ar = \frac{111}{4} - 27 = \frac{111 - 108}{4} = \frac{3}{4}$

$\therefore \frac{ar^3}{ar} = 27 \times \frac{4}{3} = 9 \times 4$

$r^2 = 36 \quad \therefore r = 6 \quad \therefore a = \frac{3}{4} \times \frac{1}{6} = \frac{1}{8}$

$\therefore a = \frac{1}{8}, r = 6$

$\therefore a_1 + a_2 + a_3 = a + ar + ar^2 = a(1 + r + r^2)$

$= \frac{1}{8}(1 + 6 + 36) = \frac{43}{8}$

12. The sum of 1 +3 +11 +25 +..... upto 20 terms is

Ans. **(7240)**

Sol. S = 1 + 3 + 11 + 25 ....

$t_n = an^2 + bn + c$

$t_1 = a + b + c = 1$

$t_2 = 4a + 2b + c = 3$

$t_3 = 9a + 3b + c = 11$

$\therefore 5a + b = 8$

$3a + b = 2$

$2a = 6 \qquad \therefore a = 3 \qquad \therefore b = -7, c = 5$

$\therefore t_n = 3n^2 - 7n + 5$

$\therefore S_n = \sum t_n = 3 . \sum n^2 - 7 \sum n + \sum 5$

$S_n = 3.\frac{n(n+1)(2n+1)}{6} - 7.\frac{n(n+1)}{2} + 5n$

$\therefore S_{20} = 3.\frac{20.21.41}{6} - \frac{7}{2}.20.21 + 100$

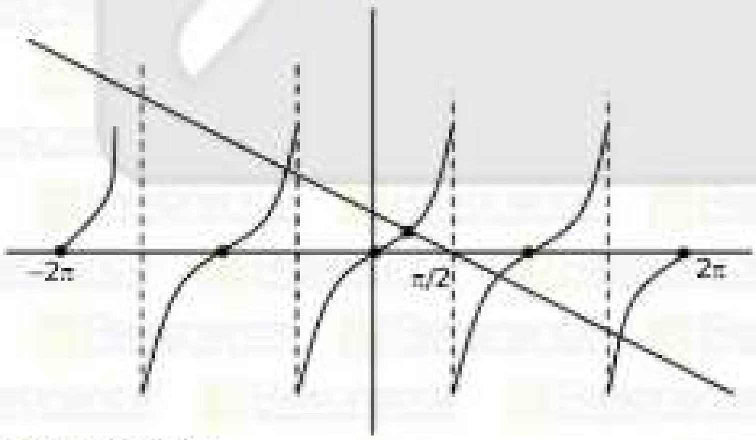
$= 8610 - 1470 + 100$

$= 7240$

13. Number of solutions of equation $2x + 3\tan x = \pi$, $\{x : x \neq (2n-1)\frac{\pi}{2}, x \in [-2\pi, 2\pi]\}$ is

Ans. **(5)**

Sol. $\tan x = \frac{\pi - 2x}{3}$



Total 5 solutions

14. Domain of $f(x) = \ell n\left(\frac{3-2x}{5+4x}\right) + \sin^{-1}\left(\frac{4-3x}{2-x}\right)$ is [a, b) then $a^2 + b^2$ is

(1) $\frac{13}{2}$ (2) $\frac{13}{8}$ (3) $\frac{13}{4}$ (4) $\frac{11}{2}$

Ans. **(3)**

Sol. $\frac{3-2x}{5+4x} > 0$

$x \in \left(-\frac{5}{4}, \frac{3}{2}\right)$ ____(1)

& $-1 \le \frac{4-3x}{2-x} \le 1$

$0 \le \frac{2-x+4-3x}{2-x}$ & $\frac{4-3x-2+x}{2-x} \le 0$

$0 \le \frac{6-4x}{2-x}$ & $\frac{2-2x}{2-x} \le 0$

$x \in \left(-\infty, \frac{3}{2}\right] \cup (2, \infty)$ $x \in [1, 2)$

by. (1) $\cap$ (2) $\cap$ (3)

Domain $x \in \left[1, \frac{3}{2}\right)$

Now $a^2 + b^2 = 1 + \frac{9}{4} = \frac{13}{4}$

15. A relation R is defined on Set A = {–3, – 2, –1, 0, 1, 2, 3}, R = {(x, y)} : $0 \le x^2 + 2y \le 4$}. If n (R) = m and minimum number of elements are added to R so that it becomes reflexive is n then m + n is

Ans. **(18)**

Sol. for reflexive x = y

$0 \le x^2 + 2x \le 4$

$x \in (-\infty, -2] \cup [0, \infty)$ and $x^2 + 2x - 4 \le 0$

$x \in [-1-\sqrt{5}, -1+\sqrt{5}]$

So R has (–3 , –3), (–2, –2) (0, 0) (1, 1)

So 3 elements has to add for reflexive hence n = 3

also when x = 0, y = 0, 1, 2 $\Rightarrow$ 3 elements

when x = ±1, y = 0, 1 $\Rightarrow$ 4 elements

when x = ±2, y = –2, –1, 0 $\Rightarrow$ 6 elements

when x = ±3, y = –3 $\Rightarrow$ 2 elements

So total elements m = 15

So m + n = 15 + 3 = 18

16. Let $f(x) = \begin{cases} (1+ax)^{\frac{1}{x}} & , \ x<0 \\ b+1 & , \ x=0 \\ \dfrac{(x+4)^{\frac{1}{2}}-2}{(x+c)^{\frac{1}{3}}-2} & , \ x>0 \end{cases}$ be a function. If f(x) is continuity at x = 0 then $e^a \times bc$ =

Ans. **(48)**

Sol. $f(x) = \begin{cases} (1+ax)^{\frac{1}{x}} & , \ x<0 \\ b+1 & , \ x=0 \\ \dfrac{(x+4)^{\frac{1}{2}}-2}{(x+c)^{\frac{1}{3}}-2} & , \ x>0 \end{cases}$

Hence f(0) = b + 1

$$\text{RHL} = \lim_{x\to 0^+} \frac{(x+4)^{\frac{1}{2}}-2}{(x+c)^{\frac{1}{3}}-2} \qquad \left(\text{for } \tfrac{0}{0} \text{ form } c^{1/3}-2=0,\ c=8\right)$$

$$= \lim_{x\to 0} \frac{2\left(1+\frac{x}{4}\right)^{\frac{1}{2}}-2}{\left(1+\frac{x}{8}\right)^{\frac{1}{3}}-2}$$

$$= \lim_{x\to 0} \frac{1+\frac{x}{8}-1}{1+\frac{x}{8}\cdot\frac{1}{3}-1} = \frac{\frac{1}{8}}{\frac{1}{8}\cdot\frac{1}{3}} = 3$$

$\therefore \quad b+1 = 3 \Rightarrow b = 2$

$$\text{LHL} = \lim_{x\to 0}(1+ax)^{1/x} = \lim_{x\to 0} e^{\frac{1+ax-1}{x}} = e^a = 3$$

$\therefore \quad e^a \,.\, bc = 3\,.\,2\,.\,8 = 48$