Resonance®
Educating for better tomorrow

JEE (MAIN) 2025

MEMORY BASED QUESTIONS & TEXT SOLUTION

SHIFT-2

DATE & DAY: 03rd April 2025 & Thursday

PAPER-1

Duration: 3 Hrs.
Time: 03:00 PM – 06:00 PM

SUBJECT: MATHEMATICS

REGISTERED & CORPORATE OFFICE (CIN: U80302RJ2007PLC024029):
CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Rajasthan) - 324005
0744-2777777 | 73400 10345 | contact@resonance.ac.in | www.resonance.ac.in | Follow Us: @ResonanceEdu | @Resonance_Edu

PART : MATHEMATICS

1. If $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}=P$, then 96 ℓnp is

(1) 12 (2) 18 (3) 32 (4) 48

Ans. **(3)**

Sol. $e^{\lim_{x\to 0}\left(\frac{\tan x}{x}-1\right)\frac{1}{x^2}}=e^{1/3}$

$96\,\ell n e^{1/3}=96\times\frac{1}{3}=32$

2. Let $A=\{-3,-2,-1,0,1,2,3\}$. A relation R is defined such that x R y iff $y=\max.\{x,1\}$. Number of elements required to make it reflexive is ℓ, number of elements required to make it symmetric is m and number of elements in the relation R is n. The value of $\ell+m+n$ is

Ans. **(15)**

Sol. $R=\{(-3,1),(-2,1),(-1,1),(0,1),(1,1),(2,2),(3,3)\}$

$n=7$

Ref. Add $\rightarrow\{(-3,-3),(-2,2),(-1,1),(0,0)\}\quad\rightarrow 4=\ell$

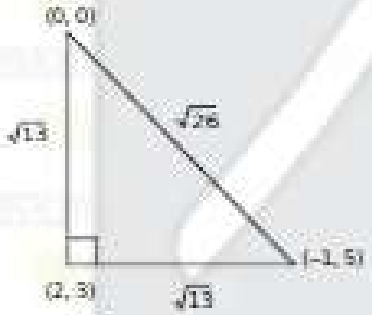
Symmetry Add. $\{(1,-3),(1,-2),(1,0),(1,-1)\}\rightarrow m=4$

$\ell+m+n=4+4+7=15$

3. Let a circle with radius r passes through four distinct point (0, 0), (k, 3k), (2, 3) & (−1, 5) such that $k\neq 0$ then $10k+2r^2$ is equal to

Ans. **(27)**

Sol.



$2r=\sqrt{26}$

$4r^2=26$

$2r^2=13$

equation of ⊙ is $x(x+1)+(y)(y-5)=0$

$k(k+1)+3k(3k-5)=0$

$k+1+9k-15=0$

$10k=14$

$10k+2r^2=14+13=27$

4. $S=1+\frac{1+3}{1!}+\frac{1+3+5}{2!}+\ldots\infty$. The value of S is equal to

Ans. **(5)**

Sol. $S=1+\frac{2^2}{1!}+\frac{3^2}{2!}+\frac{4^2}{3!}+\ldots$

$\sum_{r=0}^{\infty}\frac{r^2}{(r-1)!}$

$r-1=n$

$r=n+1$

$\sum_{n=0}^{\infty}\frac{(n+1)^2}{n!}$

$\sum_{n=0}^{\infty}\frac{n^2+2n+1}{n!}$

$\sum_{n=0}^{\infty}\frac{n}{(n-1)!}+\frac{2}{(n-1)!}+\frac{1}{n!}$

$\sum_{n=0}^{\infty}\left(\frac{1}{(n-2)!}+\frac{3}{(n-1)!}+\frac{1}{n!}\right)$

$e+3e+e=5e$

5. Let $I=\int_0^{\pi}\frac{8x}{4\cos^2 x+\sin^2 x}dx$ is equal to

(1) π^2 (2) $2\pi^2$ (3) $4\pi^2$ (4) $3\pi^2$

Ans. **(2)**

Sol. $\because$ $I=\int_0^{\pi}\frac{8x}{4\cos^2 x+\sin^2 x}dx$ ..... (1)

$I=\int_0^{\pi}\frac{8(\pi-x)}{4\cos^2 x+\sin^2 x}dx$ ..... (2)

(1) + (2)

$2I=\int_0^{\pi}\frac{8\pi}{4\cos^2 x+\sin^2 x}dx$

$I=4\pi\int_0^{\pi}\frac{1}{1+3\cos^2 x}dx$

$I=8\pi\int_0^{\pi/2}\frac{1}{1+3\cos^2 x}dx$

$I=8\pi\int_0^{\pi/2}\frac{\sec^2 x}{\sec^2 x+3}dx$

$I=8\pi\int_0^{\pi/2}\frac{\sec^2 x}{4+\tan^2 x}dx$ Put $\tan x=t$

$\sec^2 x\,dx=dt$

$$I = 8\pi \int_0^{\infty} \frac{dt}{t^2 + 2^2}$$

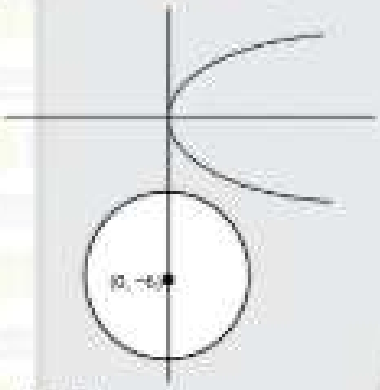
$$I = 4\pi \left(\tan^{-1}\left(\frac{t}{2}\right) \right)_0^{\infty} \quad \Rightarrow \quad U = 4\pi \times \frac{\pi}{2} = 2\pi^2$$

6. Minimum distance between $y^2 = 8x$ & $x^2 = y^2 + 12y + 35 = 0$ is

(1) $2\sqrt{2} + 1$ (2) $2\sqrt{2} - 1$ (3) 3 (4) $4\sqrt{2} - 1$

Ans. **(2)**

Sol. Let point P $(2t^2, 4t)$ lies on parabola



normal at P is

$$y - 4t = -\frac{4t}{4}\left(x - 2t^2\right)$$

It passes through centre of circle (0, –6)

$-6 - 4t = -t(-2t^2)$

$2t^3 + 4t + 6 = 0$

$t = -1$ is one of root

$(t + 1)(2t^2 - 2t + 6) = 0$

$t = -1$

Point P (2, –4) hence

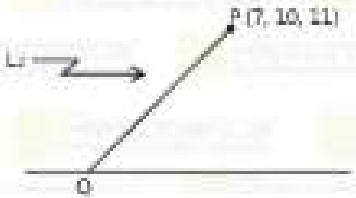
min. distance $= \sqrt{4 + 4} - 1 = 2\sqrt{2} - 1$

7. The distance of the point (7, 10, 11) from the line $\frac{x-4}{1} = \frac{y-4}{0} = \frac{z-2}{3}$ along the line $\frac{x-9}{2} = \frac{y-13}{3} = \frac{z-17}{6}$ is –

(1) 14 (2) 12 (3) 13 (4) 11

Ans. **(14)**

Sol.



$$L_1 : \frac{x-4}{1} = \frac{y-4}{0} = \frac{z-2}{3}$$

L_2 : $\frac{x-9}{2}=\frac{y-13}{3}=\frac{z-17}{6}$

$\because$ (7, 10, 11) is lying L_2

$\therefore$ required distance = PQ

$\because$ Q $(4+\lambda, 4, 3\lambda+2)$ $\quad\because$ Q line on L_2

$\therefore$ $\frac{4+\lambda-9}{2}=\frac{4-13}{3}=\frac{3\lambda+2-17}{6}$

$\frac{\lambda-5}{2}=-3=\frac{3\lambda-15}{6}$

$\Rightarrow$ $\lambda=-1$

$\therefore$ Q (3, 4, –1)

$\therefore$ $PQ=\sqrt{16+36-144}=\sqrt{196}=14$

8. If $(1+x+x^2)^{10}=1+a_1x+a_2x^2+\ldots\ldots\ldots\ldots$

Then $a_1+a_3+a_5+\ldots\ldots a_{19}-11a_2$ is equal to

Ans. **(28919)**

Sol. $x=1$ $\quad 3^{10}=1+a_1+a_2+a_3+\ldots\ldots+a_{19}+a_{20}$

$X=-1$ $\quad 1-a_1+a_2-a_3+\ldots\ldots\ldots-a_{19}+a_{20}$

Substrate

$3^{10}-1 \quad 2(a_1+a_3+a_5+\ldots\ldots a_{19})$

$a_1+a_3+a_5+\ldots\ldots a_{19}=\frac{3^{10}-1}{2}$

$(1+x+x^2)^{10}={}^{10}C_0+{}^{10}C_1(x+x^2)+{}^{10}C_2(x+x^2)+\ldots\ldots$

Coefficient of $x^2={}^{10}C_1+{}^{10}C_2=a_2$

$a_2=55$

So, $a_1+a_3+\ldots\ldots a_{19}-11a_2=\frac{3^{10}-1}{2}-11\times55$

$=29524-605$

$=28919$

9. Let $y=f(x)$ be the solution of the differential equation $\frac{dy}{dx}+3y\tan^2x+3y=\sec^2x$.

Such that $f(0)=e^3+\frac{1}{3}$. Then $f\left(\frac{\pi}{4}\right)$ is equal to

(1) $\frac{2}{3}$ (2) $\frac{5}{4}$ (3) $\frac{4}{3}$ (4) $\frac{5}{3}$

Ans. **(3)**

Sol. $\frac{dy}{dx}+(3\sec^2x)\,y=\sec^2x$

$IF=e^{\int 3\sec^2x\,dx}=e^{3\tan x}$

Sol. $y \cdot e^{3\tan x} = \int e^{3\tan x} \sec^2 x dx + c$

$Y\, e^{3\tan x} = \frac{e^{3\tan x}}{3} + c$

$x = 0$ and $y = e^3 + \frac{1}{3}$

$e^3 + \frac{1}{3} = \frac{1}{3} + C$

$C = e^3$

$y\, e^{3\tan x} = \frac{e^{3\tan x}}{3} + e^3$

$x = \frac{\pi}{4}$

$y\, e^3 = \frac{e^3}{3} + e^3$

$y = \frac{4}{3}$

10. If $f(x) + 3f\left(\frac{24}{x}\right) = 4x,\ x \neq 0$, then the value of $f(3) + f(8)$ is equal to

Ans. **(11)**

Sol. Put $x = 3$

$f(3) + 3f(8) = 12$ ____(1)

Put $x = 8$

$f(8) + 3f(3) = 32$ ____(2)

(1) + (2)

$4(f(3) + f(8)) = 44$

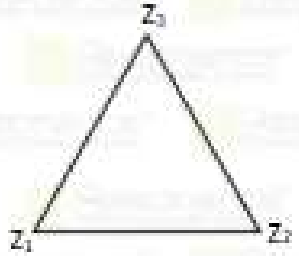
$f(3) + f(8) = 11$

11. If Z_0 is the centroid of equilateral triangle ABC having vertices Z_1, Z_2 & Z_3. The value of $\sum_{k=1}^{3} (Z_k - Z_0)^2$ is

Ans. **(0)**

Sol. $(Z_1 - Z_0)^2 + (Z_2 - Z_0)^2 + (Z_3 - Z_0)^2$

$\sum Z_1^2 - 2Z_0 \sum Z_1 + 3Z_0^2$



$Z_1 + Z_2 + Z_3 = 3Z_0$ & $\sum Z_1^2 - Z_1 Z_2 = 0$

$\sum Z_1^2 + 2\sum Z_1 Z_2 = 9Z_0^2$

$\sum Z_1^2 = 3Z_0^2$

now $(Z_1 - Z_0)^2 + (Z_2 - Z_0)^2 + (Z_3 - Z_0)^2$

$= 3Z_0^2 - 2Z_0(3Z_0) + 3Z_0^2 = 0$

12. Let $f(x) = ||x+2| - 2|x||$. If m denotes number of minima and n denotes number of maxima, then value of $(m+n)$

Ans. **(3)**

Sol. $f(x) = ||x+2| - 2|x||$

$|x+2| - 2|x| \geq 0 \quad \Rightarrow \quad |x+2| \geq 2|x|$

$(x+2)^2 \leq 4x^2$

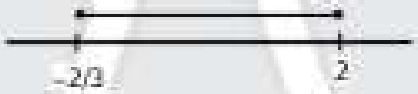
$x^2 + 4x + 4 \leq 4x^2$

$3x^2 - 4x - 4 \geq 0$

$3x^2 - 6x + 2x - 4 \geq 0$

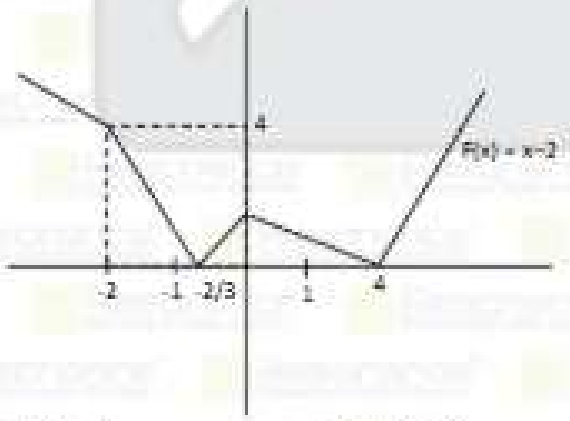
$3x(x-2) + 2(x-2) \geq 0$

$(x-2)(3x+2) \geq 0$



$\therefore \quad f(x) = \begin{cases} |x+2| - 2|x|; & x \in \left[-\frac{2}{3}, 2\right] \\ |x+2| + 2|x|; & x \in \left(-\infty, -\frac{2}{3}\right) \cup (2, \infty) \end{cases}$

$= \begin{cases} -x+2; & 0 \leq x \leq 2 \\ 3x+2; & -\frac{2}{3} \leq x < 0 \\ -3x-2; & -2 \leq x < -\frac{2}{3} \\ -x+2; & x < -2 \\ x-2; & x \geq 2 \end{cases}$



$\therefore m = 2$

$n = 1$

$\therefore m + n = 3$

13. If the equation $x(x+2)(12-k)=2$ has equal roots then shortest distance between $P\left(K,\frac{K}{2}\right)$ from $3x+4y+5=0$

Ans. **(15)**

Sol. $x^2+2x-\frac{2}{12-K}=0$

Roots one equal

$D=0$

$4+\frac{8}{12-K}=0$

$48-4K+8=0$

$K=14$

Now S.D. P(14,7) from line

$=\left|\frac{42+28+5}{5}\right|$

$=15$

14. Area bounded by region $|x-y|\le y\le 4\sqrt{x}$

(1) $\frac{1024}{3}$ (2) $\frac{1042}{3}$ (3) $\frac{512}{3}$ (4) $\frac{256}{3}$

Ans. **(64)**

Sol. **Case – I** When $x\ge y$

Then $x-y\le y \;\&\; y\le 4\sqrt{x}$

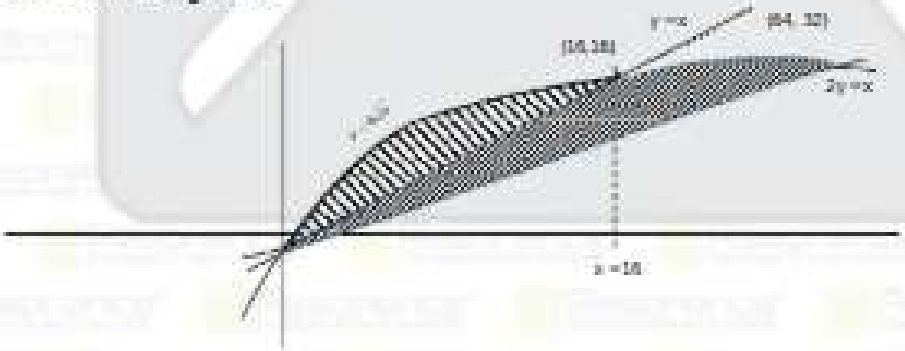
$x\le 2y$

Case – II When $x\le y$

$-x+y\le y$

$x\ge 0$

So common region is



Required area

$\int_0^{64} 4\sqrt{x}\,dx-\frac{1}{2}(64)(32)$

$=\left(\frac{2\times 4}{3}x^{3/2}\right)_0^{64}-1024$

$=\frac{1024}{3}$

15. Let $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$, $\vec{b}=3\hat{i}+3\hat{j}+3\hat{k}$, $\vec{c}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}\times\vec{d}=\vec{c}\times\vec{d}$ and $\vec{a}\cdot\vec{d}=4$ then $|\vec{a}\times\vec{d}|^2$ is

(1) $2\sqrt{2}$ (2) $4\sqrt{2}$ (3) $3\sqrt{2}$ (4) $8\sqrt{2}$

Ans. **(2)**

Sol. $(\vec{b}-\vec{c})\times\vec{d}=0$

$\vec{b}-\vec{c}=\lambda\vec{d}$

$\vec{b}-\vec{c}+\lambda\vec{d}$

Now taking dot with $\vec{a}$

$\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}+\lambda\vec{a}\cdot\vec{d}$

$(3-6+3)=(2+2+2)+\lambda(4)$

$\lambda=\frac{-3}{2}$

Hence $\vec{d}=\frac{2}{3}(\vec{b}-\vec{c})$

$=\frac{-2}{3}(\hat{i}+4\hat{j}+\hat{k})$

$=\frac{-2}{3}(\hat{i}+4\hat{j}+\hat{k})$

$\vec{a}\times\vec{d}=\frac{-2}{3}\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-2&1\\1&4&1\end{vmatrix}$

$=\frac{-2}{3}\quad(-6\hat{i}+6\hat{k})$

Now $|\vec{a}\times\vec{d}|^2=\frac{2}{3}\sqrt{36+36}=4\sqrt{2}$