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# JEE (MAIN) 2025

MEMORY BASED QUESTIONS & TEXT SOLUTION

## SHIFT-1

**DATE & DAY:** 04th April 2025 & Friday

**PAPER-1**

**Duration:** 3 Hrs.
**Time:** 09:00 – 12:00 IST

## SUBJECT: MATHEMATICS





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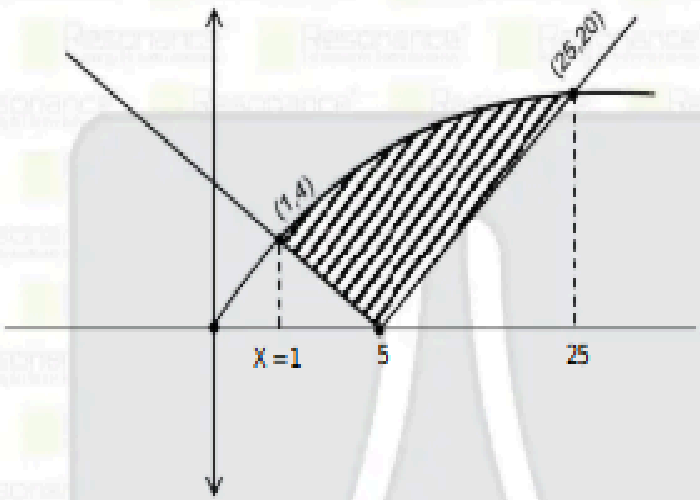
# PART : MATHEMATICS

**1.** Area bounded by curves $|x - 5| < y \leq 4\sqrt{x}$ is A, then the value of 3A is equal to

(1) 368 (2) 81 (3) 225 (4) 96

**Ans.** (1)

**Sol.**




$y = x - 5$ & $y = 4\sqrt{x}$

$4\sqrt{x} = x - 5$

$x - 4\sqrt{x} - 5 = 0$

$\sqrt{x} = 5$, $\sqrt{x} = -1$

$x = 25$

Solving $y = 5 - x$ & $y = 4\sqrt{x}$

$x + 4\sqrt{x} - 5 = 0$

$\sqrt{x} = 1 \Rightarrow x = 1$

Required area

$= \int_1^{25} 4\sqrt{x}\,dx - \frac{1}{2} \times 4 \times 4 - \frac{1}{2} \times 20 \times 20$

$= \left(4.x^{3/2}.\frac{2}{3}\right)_1^{25} - 8 - 200$

$= \frac{8}{3}(25 \times 5 - 1) - 208$

$= \frac{8}{3} \times 124 - 208$

$A = \frac{992 - 624}{3}$

Now $3A = 368$

**2.** In the expansion of $\left(2^{\frac{1}{3}}+3^{-\frac{1}{3}}\right)^n$, If $\frac{T_{15}\text{ from begining}}{T_{15}\text{ from end}}=\frac{1}{6}$, then the value of ${}^nC_3$ is equal to :

(1) 2200 (2) 2300 (3) 2400 (4) none of these

**Ans.** **(2)**

**Sol.** $\therefore$ $T_{15}$ from beginning $= {}^nC_{14}\left(2^{\frac{1}{3}}\right)^{n-14}\left(3^{-\frac{1}{3}}\right)^{14}$

$T_{15}$ from end $= {}^nC_{14}\left(3^{-\frac{1}{3}}\right)^{n-14}\left(2^{\frac{1}{3}}\right)^{14}$

$$\frac{{}^nC_{14}\,2^{\frac{n-14}{3}}.3^{-\frac{14}{3}}}{{}^nC_{14}\,2^{\frac{14}{3}}.3^{-\frac{(n-14)}{3}}}=\frac{1}{6}$$

$$2^{\frac{n-28}{3}}.3^{\frac{n-28}{3}}=\frac{1}{6}=6^{-1}$$

$$\frac{n-28}{3}=-1 \qquad n=25$$

$$\therefore \quad {}^nC_3={}^{25}C_3=\frac{25\times24\times23}{3\times2\times1}=2300$$

**3.** The sum of 40 terms of $1^2+3+5^2+7+9^2+\ldots\ldots$ is equal to

(1) 39600 (2) 41880 (3) 43500 (4) 38500

**Ans.** **(2)**

**Sol.** $\sum_{r=1}^{20}(4r-3)^2+\sum_{r=1}^{20}(4r-1)$

$=\sum(16r^2-24r+9+4r-1)$

$=\sum_{r=1}^{20}(16r^2-20r+8)$

$=\frac{16\times20\times21\times41}{6}-\frac{20\times20\times21}{2}+160$

$=16\times70\times41-10\times420+160$

$=41880$

**4.** If roots of $x^2-4x-n=0$ are integral, $n\in N$ and $n\in[20,100]$ then number of values of 'n'

**Ans.** **(6)**

**Sol.** D = Perfect square

$16+4n=\ell^2$

$$n=\frac{\ell^2-16}{4}$$

| $\ell$ | n |
|---|---|
| 6 | 5 |
| 8 | 12 |
| 10 | 21 |

Values of n are 21, 32, 45, 60, 77, 86

**5.** If the arithmetic mean of binomial coefficient in the expansion of $(x+y)^{2n-3}$ is equal to 16, then the length of perpendicular drawn from the point $(2n-1, n^2-4n)$ to the line $x+y=8$, is equal to

(1) $3\sqrt{2}$ (2) $4\sqrt{2}$ (3) $5\sqrt{2}$ (4) $6\sqrt{2}$

**Ans.** **(1)**

**Sol.** $\therefore \dfrac{{}^{2n-3}C_0 + {}^{2n-3}C_1 + {}^{2n-3}C_2 + .... + {}^{2n-3}C_{2n-3}}{2n-2} = 16$ .

$2^{2n-3} = 16\,(2n-2) = 2^5\,(n-1)$

$\Rightarrow \quad 2^{2n-8} = (n-1)$

$\Rightarrow \quad n = 5$

$\therefore \quad (2n-1,\ n^2-4n) = (9, 5)$

$\therefore \quad$ Required length $= \left|\dfrac{9+5-8}{\sqrt{1+1}}\right| = 3\sqrt{2}$

**6.** Let $10\sin^4\theta + 15\cos^4\theta = 6$

Then value of

$\dfrac{27\text{cosec}^6\theta + 8\sec^6\theta}{16\sec^8\theta}$ is equal to

(1) $\dfrac{3}{5}$ (2) $\dfrac{2}{5}$ (3) $\dfrac{35}{16}$ (4) $\dfrac{19}{16}$

**Ans.** **(2)**

**Sol.** Let $\sin^2\theta = t$

$10t^2 + 15\,(1-t)^2 = 6$

$25t^2 - 30t + 9 = 0$

$t = \dfrac{3}{5}$

$\text{Sin}^2\theta = \dfrac{3}{5}$

$\text{Cos}^2\theta = \dfrac{2}{5}$

Now

$\dfrac{27\text{cosec}^6\theta + 8\sec^6\theta}{16\sec^8\theta}$

$= \dfrac{27\left(\dfrac{125}{27}\right) + 8\left(\dfrac{125}{8}\right)}{16 \times \dfrac{625}{16}}$

$= \dfrac{125+125}{625} = \dfrac{10}{25} \Rightarrow \dfrac{2}{5}$

**7.** The value of $\int_{-1}^{1}\left(\frac{\left(1+\ \ |x|-x\right)e^{x}+\left(\ \ |x|-x\right)e^{-x}}{e^{x}+e^{-x}}\right)dx$ is equal to

(1) $2-\frac{2}{3}$ (2) $2+\frac{2\ \ 2}{3}$ (3) $1+\frac{2\ \ 2}{3}$ (4) $1+\frac{2}{3}$

**Ans.** **(3)**

**Sol.** Using $\int_{-a}^{a}f(x)dx=\int_{0}^{a}f(x)+f(-x)dx$, we obtain

Given integral

$$=\int_{0}^{1}\frac{1+\ \ |x|-x\ e^{x}+\ \ |x|-x\ e^{-x}+1+\ \ |x|+x\ e^{-x}+\ \ |x|+x\ e^{x}}{e^{x}+e^{-x}}dx$$

$$=\int_{0}^{1}\frac{e^{x}+e^{-x}+\ \ |x|-x\left(e^{x}+e^{-x}\right)+\ \ |x|+x\left(e^{-x}+e^{x}\right)}{e^{x}+e^{-x}}dx$$

$$=\int_{0}^{1}\left(1+\ \ |x|-x+\ \ |x|+x\right)dx=\int_{0}^{1}(1+\ \ 2x)dx$$

$$=1+\ \ 2.\left[\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right]_{0}^{1}=1+\frac{2\ \ 2}{3}$$

**8.** If two foci of an ellipse are (2, 5) and (2, –3) and its e = 4/5, then the length of latus rectum of this ellipse is equal to

(1) $\frac{18}{5}$ (2) $\frac{9}{5}$ (3) $\frac{16}{5}$ (4) $\frac{12}{5}$

**Ans.** **(1)**

**Sol.** $\therefore$ $2ae=\ \ 64=8$

$\Rightarrow$ $ae=4$ $\Rightarrow a=5$ (as $e=4/5$)

$\because$ $ae=4$

$a^2-b^2=16$ $\Rightarrow b^2=9$

$\therefore$ $\ell\,(L.\,R)=\frac{2b^2}{a}=\frac{18}{5}$

**9.** Let $A=\{1,6,11,16,\ldots\ \text{upto}\ 2025\ \text{terms}\}$ & $B=\{9,16,23,\ldots 2025\ \text{terms}\}$ then $n(A\cup B)$ is ____.

(1) 3761 (2) 3650 (3) 3810 (4) 3619

**Ans.** **(1)**

**Sol.** Common first term = 16

and common difference of common term be 35

Last term of set A is $=1+2024\times 5$

$=10121$

Last term of set B $=9+2024\times 9$

$=14168+9$

$=14177$

Now Let number of common term be n.

$$16+(n-1)\times 35\le 10121$$
$$35n-19\le 10121$$
$$35n\le 10140$$
$$n\le 289.7$$

So 289 terms are common

Now $n(A\cup B)=2025+2025-289=3761$

10. There are 10 pens in which 3 are defective. 2 samples of pen are selected randomly. Let x be number of defective pen, then variance of probability distribution of number of defective pen is equal to

(1) $\frac{28}{75}$ (2) $\frac{37}{75}$ (3) $\frac{26}{75}$ (4) $\frac{23}{75}$

**Ans.** **(1)**

**Sol.**

| x | 0 | 1 | 2 |
|---|---|---|---|
| P(x) | $\frac{^7C_2}{^{10}C_2}$ | $\frac{^7C_1 \times {}^3C_1}{^{10}C_2}$ | $\frac{^3C_2}{^{10}C_2}$ |

Variance = $\sum P_i(x_i)^2 - \left(\sum P_i x_i\right)^2$

$= \left(0 + \frac{21}{45}\times 1^2 + \frac{3}{45}\times 2^2\right) - \left(0 + \frac{21}{45}\times 1 + \frac{3}{45}\times 2\right)^2$

$= \frac{33}{45} - \left(\frac{27}{45}\right)^2$

$= \frac{11}{15} - \left(\frac{3}{5}\right)^2$

$= \frac{55-27}{75}$

$= \frac{28}{75}$

11. A committee of 12 members is formed randomly out of 4 Engineers, 2 Doctors and 10 Professors . Find the probability that the committee has atleast 3 Engineers and atleast 1 Doctor.

(1) $\frac{103}{182}$ (2) $\frac{129}{182}$ (3) $\frac{107}{182}$ (4) $\frac{109}{182}$

**Ans.** **(2)**

**Sol.** Engineers = 4, Doctors = 2, Professors = 10

$n(S) = {}^{16}C_{12} = {}^{16}C_4 = 14 \times 13 \times 10$

$\therefore$ n(A) =

| Eng. | Dr. | Pr. | |
|---|---|---|---|
| 3 | 1 | 8 | $= {}^4C_3 \times {}^2C_1 \times {}^{10}C_8 = 4 \times 2 \times 45 = 360$ |
| 4 | 1 | 7 | $= {}^4C_4 \times {}^2C_1 \times {}^{10}C_7 = 1 \times 2 \times 120 = 240$ |
| 3 | 2 | 7 | $= {}^4C_3 \times {}^2C_2 \times {}^{10}C_7 = 4 \times 1 \times 120 = 480$ |
| 4 | 2 | 6 | $= {}^4C_4 \times {}^2C_2 \times {}^{10}C_6 = 1 \times 1 \times 210 = 210$ |

$\therefore$ n(A) = 1290

$\therefore$ $P(\text{Reqd.}) = \frac{n(A)}{n(S)} = \frac{1290}{14\times 13\times 10} = \frac{129}{14\times 13} = \frac{129}{182}$

**12.** If $\lim_{x\to 1^+}\frac{(1-x)(1+\lambda\cos(x-1))+\mu\sin(1-x)}{(1-x)^3}=-1$, then $\lambda+\mu$ is equal to

(1) –1 (2) 1 (3) 2 (4) 3

**Ans.** **(1)**

**Sol.** $\lim_{h\to 0}\frac{(-h)[1+\lambda\cosh]+\mu(\sin(-h))}{-h^3}$

$$=\lim_{h\to 0}\frac{-h\left[1+\lambda\left(1-\frac{h^2}{2!}+...\right)\right]+\mu\left(-h+\frac{h^3}{3!}\right)}{-h^3}$$

Now $\frac{-h(1+\lambda+\mu)+h^3\left(\frac{\lambda}{2}+\frac{\mu}{6}\right)}{-h^3}=-1$

Now $\lambda+\mu=-1$ and $-\frac{\lambda}{2}-\frac{\mu}{6}=-1$

Now $\lambda+\mu=-1$

**13.** If $A=\begin{bmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{bmatrix}$, such that $A^2=A^T$, then find trace of $((A+I)^3+(A-I)^3-6A)$

**Ans.** **(6)**

**Sol.** $A^2=A^T\Rightarrow \begin{bmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{bmatrix}=\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}$

$\Rightarrow \begin{bmatrix}\cos2\theta & 0 & -\sin2\theta\\ 0 & 1 & 0\\ \sin2\theta & 0 & \cos2\theta\end{bmatrix}=\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}$

$\Rightarrow \cos2\theta=\cos\theta,\ \sin2\theta=-\sin\theta\Rightarrow 2\cos^2\theta-\cos\theta-1=0,\ \sin\theta\,(2\cos\theta+1)=0$

$\Rightarrow (\cos\theta-1)(2\cos\theta+1)=0,\quad \sin\theta\,(2\cos\theta+1)=0$

$\Rightarrow \cos\theta=1,\ \sin\theta=0$ or $\cos\theta=\frac{-1}{2}$

$\Rightarrow \theta=2n\pi$ or $\theta=2m\pi\pm\frac{2\pi}{3}$

$A=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ or $A=\begin{bmatrix}\frac{-1}{2}&0&-\frac{\sqrt3}{2}\\0&1&0\\\frac{\sqrt3}{2}&0&\frac{-1}{2}\end{bmatrix}$ or $A=\begin{bmatrix}\frac{-1}{2}&0&\frac{\sqrt3}{2}\\0&1&0\\-\frac{\sqrt3}{2}&0&\frac{-1}{2}\end{bmatrix}$

$\Rightarrow AA^T=I\Rightarrow A(A^2)=AA^T\Rightarrow A^3=I$

$(A+I)^3=A^3+3A^2+3A+I$

$(A-I)^3=A^3-3A^2+3A-I$

$((A+I)^3+(A-I)^3-6A)=2A^3=2I\Rightarrow$ Trace $=6$