JEE-Main-04-04-2025 (Memory Based)
[EVENING SHIFT]
Maths

Question: $\cot^{-1}\left(\frac{7}{4}\right)+\cot^{-1}\left(\frac{19}{4}\right)+\cot^{-1}\left(\frac{39}{4}\right)+\ldots\infty$

Options:

(a) $\cot^{-1}(2)$

(b) $\cot^{-1}\left(\frac{1}{2}\right)$

(c) $\cot^{-1}\left(\frac{1}{3}\right)$

(d) $\cot^{-1}(3)$

Answer: (b)

$T_n=\tan^{-1}\left(\frac{1}{1+\left(n^2-\frac{1}{4}\right)}\right)=\tan^{-1}\left(\frac{\left(n+\frac{1}{2}\right)-\left(n-\frac{1}{2}\right)}{1+\left(n+\frac{1}{2}\right)\left(n-\frac{1}{2}\right)}\right)$

$=\tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\left(n-\frac{1}{2}\right)$

$S_n=\sum_{r=1}^{n}T_r=\tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\left(\frac{1}{2}\right)$

$S_n=\tan^{-1}(\infty)-\tan^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{2}-\tan^{\infty}\left(\frac{1}{2}\right)$

$=\cot^{-1}\left(\frac{1}{2}\right)$

Question: $\sum_{k=1}^{n}\left(\alpha^k+\frac{1}{\alpha^k}\right)^2=20,$ α **is one of the root of $x^2+x+1=0$ then n = ?**

Answer: (n = 20)

$x^2+x+21=0 <^{w}_{w^2}$

$x=\frac{-1\pm\sqrt{1-4}}{2}$

$x=-\frac{1\pm\sqrt{3}\hat{i}}{2}$

$w^3=1$

$1+w+w^2=0$

$w+w^2=-1$

$$\sum\left(w^{k}+\frac{1}{w^{k}}\right)^{2}=20$$

$$\sum_{k=1}^{n}\left(w^{2k}+\frac{1}{w^{2k}}+2\right)=20$$

$$\sum_{k=1}^{n}\left(w^{2k}+w^{k}+2\right)=20$$

(w^2+w+2)	w^4+w^2+2
-1	$w+w^2$
$\Rightarrow 1$	$-1+2$

n = 20

Question: Let $L_1:\frac{x-1}{3}=\frac{y}{4}=\frac{x}{5}$ **and** $L_2:\frac{x-p}{2}=\frac{y}{3}=\frac{z}{4}.$ **If the shortest distance between L_1 and L_2 is** $\frac{1}{\sqrt{6}}.$ **Then possible value of p is**

Options:

(a) 3

(b) 2

(c) 5

(d) 7

Answer: ($\sqrt{6}$)

$L_1 = \frac{x-1}{3} = \frac{y}{4} = \frac{x}{5}, L_2 = \frac{x-p}{2} = \frac{y}{3} = \frac{z}{4}$

$$\begin{vmatrix} 1-p & 0 & 0 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{vmatrix} = \frac{1}{\sqrt{6}}$$

$$\begin{vmatrix} 1-p & 0 & 0 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{vmatrix} = \frac{1}{\sqrt{6}}$$

$$\left(3\hat{i} + 4\hat{j} + 5\widehat{k}\right) \times \left(2\hat{i} + 3\hat{j} + 4\widehat{k}\right)$$

$\hat{i} \quad \hat{j} \quad \widehat{k}$

$3 \quad 4 \quad 5$

$2 \quad 3 \quad 4$

$= \hat{i}(1) - \hat{j}(2) + \widehat{k}(1)$

$\left|\bar{b}_1 \times \bar{b}_2\right| = \sqrt{1+1+4}$

$$\sqrt{6}$$

Question: Let the mean & variance of observation 2, 3, 3, 4, 5, 7, a , b is 4 and 2, then mean deviation about mode of the observation is
Answer: (1)

$\sigma^2 = 2$

$\frac{2+3+3+4+5+7+a+b}{8} = 4$

$24 + a + b = 32$

$a + b = 8$

$\frac{2^2+3^2+3^2+4^2+5^2+7^2+a^2+b^2}{8} - 4^2 = 2$

$\frac{38+25+49+a^2+b^2}{8} = 18$

$a^2 + b^2 = 144 - 112$

$a^2 + b^2 = 32$

$2, 3, 3, 4, 4, 4, 5, 7$

$\text{mode} = 4$

$di : xi - m$

$di 2, 1, 1, 0, 0, 0, 1, 3$

$\overline{x} = \frac{8}{8} = 1$

Question: If the sum of first 20 terms of series
$\frac{4.1}{4+3.1^2+1^4} + \frac{4.2}{4+3.2^2+2^4} + \frac{4.3}{4+3.3^2+3^4} + \frac{4.4^5}{4+4.4^2+4^4} + \ldots$ **is** $\frac{m}{n}$,
where m, n are co-primes, then m + n is equal to

Options:

(a) 422

(b) 423

(c) 420

(d) 421

Answer: (d)

$a_n = \frac{4_n}{4+3n^2+n^4}$

$a_n = \frac{4n}{n^2+4n^2+4-n^2} = \frac{4n}{(n^2+2)^2-n^2}$

$a_n = \frac{4n}{(n^2+n+2)(n^2-n+2)}$

$= 2\left[\frac{(n^2+n+2)-(n^2-n+2)}{(n^2+n+2)(n^2-n+2)}\right]$

$a_n = 2\left[\frac{1}{n^2-n+2} - \frac{1}{n^2+n+2}\right]$

$\left[S_n = 2\left[\frac{1}{2} - \frac{1}{n^2+n+2}\right]\right]$

put $n = 20$

$\text{Ans} = 421$

Question: Let the three sides of a triangle ABC is given by vectors $2\hat{i} - \hat{j} + \widehat{k}, \hat{i} - 3\hat{i} - 5\widehat{k}\ \&\ 3\hat{i} - 4\hat{j} - 4\widehat{k},$ **let G be the centroid of triangle ABC. Then** $6\left(\left|\overrightarrow{AG}\right|^2 + \left|\overrightarrow{BG}\right|^2 + \left|\overrightarrow{CG}\right|^2\right)$ **is ______**

Answer: (164)

$\overrightarrow{AB} = \left(2\hat{i} - \hat{j} + \widehat{k}\right)$

$\overrightarrow{BC} = \hat{i} - 3\hat{j} - 5\widehat{k}$

$\overrightarrow{CA} = 3\hat{i} - 4\hat{j} - 4\widehat{k}$

$G; \left(2, \frac{-8}{3}, \frac{-8}{3}\right)$

$|GA| = \sqrt{0 + \frac{25}{9} + \frac{121}{9}}$

$|GA|^2 = \frac{146}{9}$

$|GB|^2 = 1^2 + \frac{1}{9} + \frac{49}{9}$

$= \frac{9+1+49}{9}$

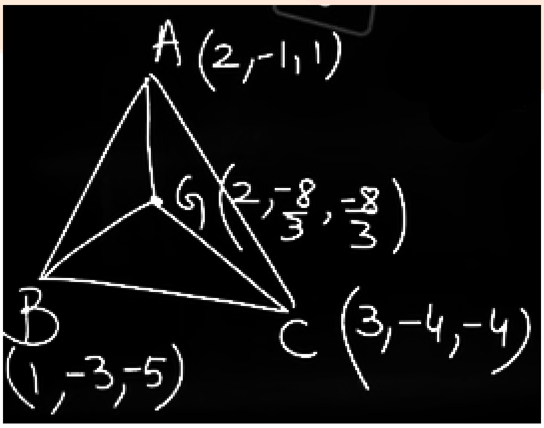
$\frac{59}{9}$

$|Gl|^2 = 1^2 + \frac{16}{9} + \frac{16}{9}$

$= \frac{9+32}{9} = \frac{41}{9}$

$\therefore 6\left(\frac{146}{9} + \frac{59}{9} + \frac{41}{9}\right)$

$= 164$



Question: Consider two sets A & B containing three numbers in A.P. Let the sum and the product of the elements of A be 36 and p respectively and the sum and the product

of B be 36 and q respectively. Let d & D be the common difference of AP's in A & B respectively such that $D = d + 3, d > 0.\text{If } \frac{p+q}{p-q} = \frac{19}{5}$ **then p - q = ?**

Answer: (= 6, -20.4)

$$A\{12 - d, 12, 12 + d\}$$

$$B\{12 - D, 12, 12 + D\}$$

$$p = (12 - d)12(12 + d)$$

$$q = (12 - D)12(12 + D)$$

$$\left[\frac{p}{q} = \frac{(12-d)(12+d)}{(12-D)(12+d)} = \frac{144-d^2}{144+D^2}\right] = \frac{12}{7}$$

$$\frac{p+q}{p-q} = \frac{19}{5}$$

$$\frac{p+q+p-q}{p+1-(p-q)} = \frac{19+5}{19-5}$$

$$\frac{2p}{2q} = \frac{24}{14}$$

$$\frac{p}{q} = \frac{12}{7}$$

$$7(144) - 7d^2 = 12(144) - 12D^2$$

$$12D^2 - 7d^2 = 144(5)$$

$$12\left(d^2 + 9 + 6d\right) - 7d^2 - (144)5$$

$$5d^2 + 72d + 108 - 720 = 0$$

$D = d + 3$
$= 6 + 3 = 9$
$p = (12 - d)12(12 + s)$
$= (12 - 6)12(12 + 6) = 6 \times 12 \times 18 = 1206$
$q = (12 - D)12(12 + D)$
$= (12 - 9)12(12 + 9) = 3 \times 12 \times 21 = 756$
$\therefore p - q = 1296 - 756 = 540$
$5d^2 + 72d - 612 = 0$
$\Rightarrow d = \frac{-72 \pm \sqrt{72^2 + 4 \times 5 \times 612}}{2 \times 5}$
$= \frac{-72 \pm \sqrt{17424}}{10}$
$= \frac{-72 \pm 132}{10}$
$= \frac{60}{10}, -\frac{204}{10}$
$= 6, -20.4$

Question: Let the matrix $A \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ **satisfy** $A^n = A^{n-2} + A^2 - I$ **for** $n \geq 3$ **then the sum of all the elements of** A^{50} **is ______**
Answer: (53)

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$A^2 - I = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 1+1 & 1 & 0 \\ 1+1 & 0 & 1 \end{bmatrix}$

$A^6 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$A^6 = \begin{bmatrix} 1 & 0 & 0 \\ 2+1 & 1 & 0 \\ 2+1 & 0 & 1 \end{bmatrix}$

$A^3 = A + A^2 - I$
$A^4 = A^2 + A^2 - I$
$A^5 = A^3 + A^2 - I$
$A^{50} = A^{48} + A^2 - I$

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 24 & 1 & 0 \\ 24 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$

$\therefore$ Sum of all elements $= 53$

Question: Let A = {-3, -2, -1, 0, 1, 2, 3} and xRy $\Rightarrow$ **2x - y** $\in$ **{0,1}. If l is number of elements in given relation, m and n are minimum number of elements to be added to make it reflexive and symmetric respectively. Then** l **+ m + n equals to**
Answer: (17)

$A=\{-3,-2,-1,0,1,2,3\}$
$xRy \Rightarrow 2x-y \in \{0,1\}$
$2x-y=0$ $\quad\quad\quad$ $2x-y=1$

x y	x y
-1 -2	-1 -3
0 0	0 -3
1 2	1 1
	2 3

Relation
$\{(-1,2),(0,0),(1,2),(-1,3),(0,-1),(1,1),(2,3)\}$
$l=$ no. of cells in $R=7$
Reflexive $=(-3,-3),(2,-2)(-1,-1),(2,2)(3,3)$: 5 cells reqired
Symmetric $=(-2,-1),(2,1),(-3,-1),(-1,0),(3,2)$: 5 cells required
$l+m+n=7+5+5=17$

Question: Let f(x) and g(x) satisfies the functional equation 2g(x) + 3g (1/x) = x and 2f (x) + 3f (1/x) = x^2 + 5. If $\alpha=\int_1^2 f(x)dx$ and $\beta=\int_1^2 f(x)dx$ **is equal to**

Options:

(a) $\dfrac{27+6\ell n_2}{10}$

(b) $\dfrac{27-6\ell n_2}{10}$

(c) $\dfrac{3}{5}\ell n_2$

(d) $\dfrac{3}{5}\ell n_2+\dfrac{7}{30}$

Answer: (a)

Question: If 12, 15C1 + 22 . 15C2 + ... +152 . 15C15 is equal to 2n × 3m × 5k, then m + n + k is equal to
Answer: (2^{17}. 3 . 5)

$$\sum_{r=1}^{15} r^2 \cdot {}^{15}C_r$$

$$\Rightarrow 15 \sum_{r=1}^{15} r \cdot {}^{14}C_{r-1}$$

$$\Rightarrow 15 \sum_{r=1}^{15} (r-1+1)\ {}^{14}C_{r-1}$$

$$= 15 \sum_{r=1}^{15} (r-1)^{14}C_{r-1} + 15 \sum_{r=1}^{15} {}^{14}C_{r-1}$$

$$= 15 \sum_{r=0}^{14} r \cdot {}^{14}C_r + 15.2^{12}$$

$$= 15 \cdot 14 \cdot \sum {}^{13}C_{r-1} + 15 \cdot 2^{14}$$

$$= 15 \cdot 14 \cdot 2^{13} + 15.2^{14}$$

$$= 15 \cdot 2^{13}(14+2) = 15 \cdot 2^{13} \cdot 16$$

$$= 2^{17} \cdot 3 \cdot 5$$