JEE-Main-07-04-2025 (Memory Based)
[MORNING SHIFT]
Maths

Question: The remainder when $64^{64^{64}}$ is divided by 7 is equal to
Options:
(a) 1
(b) 2
(c) 3
(d) 4
Answer: (a)

$$64 = 1(\bmod 7)$$

$$64^{64^{64}} \equiv 1(\bmod 7)$$

$$r = 1.$$

Question: Let A be a set defined as A = {2, 3, 6, 9}. Find number of singular matrices of order 2 × 2 such that elements are from the set A.
Options:
(a) 4
(b) 10
(c) 12
(d) 15
Answer: (c)

$$A = \{2, 3, 6, 9\}.$$

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$|M| = ad - bc = 0 \Rightarrow ad = bc$$

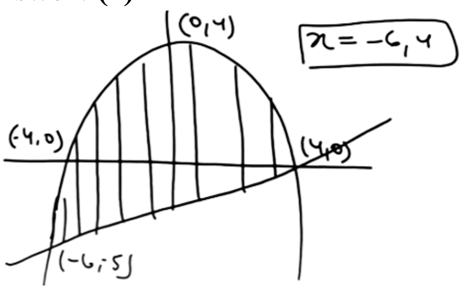
1. When only one element is used = 4
2. When exactly two elements are use = $^4C_2 . 2 \times 2 = 24$
3. When exactly three numbers used = 0
4. When all four used = $^4C_1$. 1 x 2 = 8
Total = 4+ 24 + 8 = 36

Question: Area bounded by the curves $y = 4 - \frac{x^2}{4}$ and $y = \frac{x-4}{2}$ (in square units) is
Options:
(a) $\frac{125}{3}$
(b) $\frac{20}{3}$

(c) $\frac{80}{3}$

(d) $\frac{120}{3}$

Answer: (a)



$$\int_{6}^{4}\left(\left(4-\frac{x^2}{4}\right)-\left(\frac{x-4}{2}\right)\right)dx$$

$$=\frac{125}{3}$$

Question: If x_1, x_2, x_3, x_4 are in GP, then we subtract 2, 4, 7, 8 from x_1, x_2, x_3, x_4 respectively, then the resultant number are in AP then the value of $\frac{1}{24}$ (x_1, x_2, x_3, x_4) is

Options:

(a) $\frac{2^4}{3^8}$

(b) $\frac{2^3}{3^9}$

(c) $\frac{2}{3^9}$

(d) $\frac{2}{3^8}$

Answer: (b)

$\Rightarrow x_1, x_2, x_3, x_4 = x_1, x_2r^1, x_1r^2, x_1r^3 = x_1^4r^6 = a^4r^6$

$(x, -1), (x_2-4), (x_3-7), (x_4-8)$ average in AP

$\Rightarrow (a-2)(ar-4)(ar^2-7), (ar^3-8)$

$(ar-4)-(a-2) = (ar^2-7)-(ar-4)$

$\Rightarrow ar-a-2 = ar^2-ar-3$

$\Rightarrow a(r-1)-ar(r-1) = -1$

$\Rightarrow a(r-1)[1-r] = -1$

$\Rightarrow a(r-1) = 1$

$\Rightarrow a(9) = 1$

$\boxed{a=\frac{1}{9}}$

$ar^2-7-(ar-4)$

$= (ar^3-8)-(ar^2-7)$

$\Rightarrow ar(r-1)-3 = ar^3-ar^2-1$

$\Rightarrow ar(r-1)-ar^2(r-1) = 2$

$\Rightarrow ar(r-1)[1-r = 2$

$\Rightarrow \frac{ar(r-1)^2}{a(r-1)^2} = \frac{-2}{1}$ $\boxed{r=-2}$

$\frac{1}{24}(a^4r^6)$

$=\frac{1}{24}\frac{1}{9^4}2^6$

Question: If $f(x) = \left[\frac{x^2}{2}\right] - [\sqrt{x}]\,\forall\, x \in [0.4]$ **where [.] denotes the greatest integer function, then number of point discontinuity of f(x) is**

Options:
(a) 12
(b) 8
(c) 6
(d) 4
Answer: (b)

$$f(x) = \left[\frac{x^2}{2}\right] - [\sqrt{x}]$$

$$\text{Check at } x = 0, 1, \sqrt{2}, 2, \sqrt{6}, \sqrt{8}, \sqrt{10}, \sqrt{12}, \sqrt{14}, 4$$

$$\text{Continous at } x = 0, 4$$

$$Ans = 8$$

Question: The integral $\displaystyle\int_0^{\pi} \frac{(x+3)\sin x}{1+3\cos^2 x}dx$ **is equal to**
Options:
(a) $\dfrac{\pi}{3\sqrt{3}}(\pi+6)$
(b) $\dfrac{\pi}{2\sqrt{3}}(\pi+4)$
(c) $\dfrac{\pi}{\sqrt{3}}(\pi+2)$
(d) $\dfrac{\pi}{\sqrt{3}}(\pi+1)$
Answer: (a)

$$I - \int_0^{\pi} \frac{(x+3)\sin x}{1+\cos^2 x}dx - - - - - - (1)$$

$$I = \int_0^{\pi} \frac{((\pi - x)+3)\sin x}{1+3\cos^2 x}dx - - - - - - (2)$$

$$2I = \int_0^{\pi} \frac{\sin x}{1+3\cos^2 x}(x+\pi+\pi-x+3)dx$$

$$2I = \left(\int_0^{\pi} \frac{\sin x}{1+3\cos^2 x}dx\right)(x+6) \Rightarrow 2I = \left(\int_0^{\pi/2} \frac{\sin x}{1+3\cos^2 x}dx\right)(\pi+6)$$

$$\text{let } cos = T \to -sindx = dt$$

$$I = \left(-\int_1^0 \frac{1}{1\left(\sqrt{35}\right)^2}dt\right)(\pi+6)$$

$$I = \left(\int_0^1 \frac{1}{1+\left(\sqrt{35}\right)}dt\right)(\pi+6)$$

$$I = \frac{(x+6)}{\sqrt{3}}\left(\tan^t\left(\sqrt{35}\right)\right)_0^1$$

$$= \left(\frac{\pi+6}{\sqrt{3}}\right)\frac{\pi}{3}$$

Question: If α and β are negative real roots of the quadratic equation x^2 - (p + 2)x +(2p +9) = 0 and p ∈ (α, β). Then the value of β - 2α is
Options:
(a) 11

(b) 13
(c) 7
(d) 5
Answer: (d)
O is greater than both the roots

$af(0) > 0$

$\Rightarrow 1 \cdot 2p + 1 > 0$

$\Rightarrow p > -\frac{9}{2}$

$D > 0$

$(p+2)^2 - 4(2p+1) > 0$

$p^2 + 4p + 4 - 8p - 36 > 0$

$p^2 - 4p - 32 > 0$

$(p-8)(p+4) > 0$

$p > 8$ or $p < -4$

$\alpha = -\frac{9}{2} \beta = -4$

$\beta - 2\alpha = 5$

Question: Let the straight line AB : x + y - 2 = 0, AC : 3y - x = 2 intersects x-axis at B and C respectively. If P is the orthocentre of the triangle ABC, ten area of the triangle CPB is
Options:
(a) 10 sq. units
(b) 8 sq. units
(c) 6 sq. units
(d) 7 sq. units
Answer: (c)

$y = \frac{x}{3} + \frac{2}{3}$

$CP \equiv (y-0) = 1(x+2)$

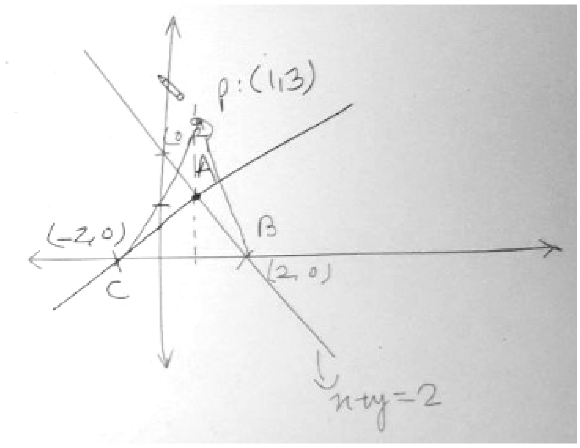
$BP \equiv (y-0) = -3(x-2)$

$x + 2 = -3x + 6$

$4x = 4$

$x = 1, y = 3$

$\frac{1}{2} \times 4 \times 3 = 6$



Question: Consider two statements:

Statement 1 : If $\left(\frac{z+i}{z-i}\right)$ **is purely real, then there are exactly 2 complex numbers z.**

Statement 2 : If $\left(\frac{z+1}{z-1}\right)$ **is purely imaginary, then there are infinite such complex numbers z. Then**

Options:

(a) Statement 1 is true

(b) Statement 2 is true

(c) Both statement 1 and statement 2 are true

(d) Both statement 1 and statement 2 are false

Answer: (b)

$I. \arg\left(\frac{z+i}{z-i}\right) = 0, \pi$

All the complex number lying on $y- < x$ is

satisfy it except $-i, i$

$II. \arg\left(\frac{z+1}{z-1}\right) = \pm\frac{\pi}{2}$

Z lies on a circle

Statement 2 is true

Question: If $\vec{a}$ and $\vec{b}$ **are unit vectors such that angle between** $\vec{a}$ and $\vec{b}$ is $\sin^{-1}\left(\frac{\sqrt{65}}{9}\right)$ and $\vec{c} = 3\vec{a} + 4\vec{b} + 9\left(\vec{a} \times \vec{b}\right)$. **Then, the value of** $\vec{c}\,.\,\vec{a} - 3\vec{c}\,.\,\vec{b}$ is

Options:

(a) $\frac{101}{3}$

(b) $\frac{-101}{9}$

(c) $\frac{101}{9}$

(d) $\frac{-101}{3}$

Answer: (b)

$\vec{c} = 3\vec{a} + 4\vec{b} + a\left(\vec{a} \times \vec{b}\right)$ take $\vec{a}$

$\vec{c} \cdot \vec{a} = 3 + 4\,\vec{a} \cdot \vec{b}$

$\vec{c} \cdot \vec{b} = 3\,\vec{a} \cdot \vec{b} + 4$

$\vec{c} \cdot \vec{b} - 3\vec{c} \cdot \vec{b} = 3 + 4\,\vec{a} \cdot \vec{b} - 9\,\vec{a} - \vec{b} - 12$

$= -9 - 5\,\vec{a} \cdot \vec{b}$

$\sin\theta = \frac{\sqrt{65}}{9}$ $\qquad l = \sqrt{9^2 - 65}$

$\cos\theta = \frac{4}{9}$ $\qquad = \sqrt{81 - 65}$

$\vec{a} \cdot \vec{b} = \frac{4}{9}$ $\qquad = \sqrt{16} = 4$

$= -9 - 5\frac{4}{9} = \frac{-81-20}{9} = \frac{-101}{9}$

Question: Let ‘P’ be the parabola, whose focus is (-2, 1) & directrix is 2x + y + 2 = 0. Then the sum of the ordinates of the points on P, whose abscissa is -2 is ____ .

Options:

(a) $\frac{5}{2}$

(b) $\frac{1}{4}$

(c) $\frac{3}{4}$

(d) $\frac{3}{2}$

Answer: (d)

$pf = pm$

$\sqrt{0 + (k-1)^2} = \left|\frac{4+k+2}{\sqrt{5}}\right|$

$k^2 + 1 - 2k = \frac{k^2+4-4k}{5}$

$5k^2 + 5 - 10k = k^2 + 4 - 4k$

$4k^2 - 6k + 1 = 0$

$k_1 + k_2 = -\frac{-6}{4} = \frac{3}{2}$