MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The remainder when $64^{64^{64}}$ is divided by 7 is equal to

(1) 1　　(2) 2

(3) 3　　(4) 4

Answer (1)

Sol. $64^{64} \Rightarrow (63+1)^{64} = 63\lambda + 1$

$64^{64^{64}} \Rightarrow (63+1)^{64^{64}}$

$63\lambda_1 + 1$

Required remainder when divided 7 is 1.

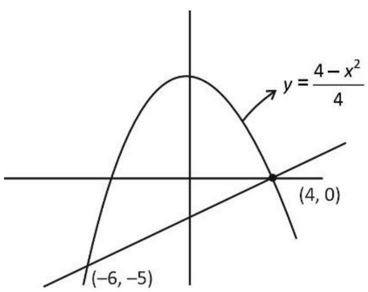
2. Area bounded by the curves $y = 4 - \frac{x^2}{4}$ and $y = \frac{x-4}{2}$ (in square units) is

(1) $\frac{125}{3}$　　(2) $\frac{20}{3}$

(3) $\frac{80}{3}$　　(4) $\frac{120}{3}$

Answer (1)

Sol. $y = \frac{4-x^2}{4}$ and $y = \frac{x-4}{2}$



$$\text{Area} = \int_{-6}^{4} \left(\left(4 - \frac{x^2}{4}\right) - \left(\frac{x-4}{2}\right) \right) dx$$

$$= \int_{-6}^{4} \left(-\frac{x^2}{4} - \frac{x+6}{2} \right) dx$$

$$= \left(\frac{-x^3}{12} - \frac{x^2}{4} + 6x \right)_{-6}^{4}$$

$$\Rightarrow \frac{125}{3} \text{ square unit}$$

3. If x_1, x_2, x_3, x_4 are in GP, then we subtract 2, 4, 7, 8 from x_1, x_2, x_3, x_4 respectively, then the resultant numbers are in AP, then the value of $\frac{1}{24}$ $(x_1 \cdot x_2 \cdot x_3 \cdot x_4)$ is

(1) $\frac{2^4}{3^8}$

(2) $\frac{2^3}{3^9}$

(3) $\frac{2}{3^9}$

(4) $\frac{2}{3^8}$

Answer (2)

Sol.

$$\begin{array}{ccccc} x_1, & x_2, & x_3, & x_4, & \rightarrow \text{GP} \\ \downarrow & \downarrow & \downarrow & \downarrow & \\ a & ar & ar^2 & ar^3 & \end{array}$$

$a - 2,\ ar - 4,\ ar^2 - 7,\ ar^3 - 8 \rightarrow \text{AP}$

$$\left.\begin{array}{l} 2(ar-4) = a + ar^2 - 9 \quad ...(1) \\ 2(ar^2 - 7) = ar + ar^3 - 12 \ ...(2) \end{array}\right\} \begin{array}{l} r = -2 \\ a = \frac{1}{9} \end{array}$$

$$\frac{1}{24}(x_1 x_2 x_3 x_4) = \frac{1}{24} a^4 r^{1+2+3} = \frac{1}{24} \times a^4 \times r^6$$

$$= \frac{1}{24} \times \left(\frac{1}{9}\right)^4 \times (-2)^6$$

$$= \frac{2^3}{3^9}$$

4. If $f(x)=\left[\frac{x^2}{2}\right]-\left[\sqrt{x}\right]\ \forall\ x\in[0,4]$, where [.] denotes the greatest integer function, then number of points of discontinuity of $f(x)$ is

(1) 12 (2) 8

(3) 6 (4) 4

Answer (2)

Sol. $y=\left[\frac{x^2}{2}\right]-\left[\sqrt{x}\right]$, critical points when $\left[\frac{x^2}{2}\right]$ and $\left[\sqrt{x}\right]$ becomes integer.

$\{0, 1, \sqrt{2}, \sqrt{4}, \sqrt{6}, \sqrt{8}, \sqrt{10}, \sqrt{12}, \sqrt{14}, 4\}$

Continues at 0^+, continuous at 4^-.

$\left[\frac{X^2}{2}\right]=\left[\sqrt{x}\right]$, which occurs at $x=\sqrt{2}$

$\Rightarrow$ Not continuous

The function is discontinuous at 8 points.

5. Solve: $\int_0^{\pi}\frac{(x+3)\sin x}{1+3\cos^2 x}dx$

(1) $\frac{\pi(\pi+6)}{3\sqrt{3}}$ (2) $\frac{\pi(\pi+6)}{2\sqrt{3}}$

(3) $\frac{\pi^2}{2\sqrt{3}}$ (4) $\frac{\pi^2}{4\sqrt{3}}$

Answer (1)

Sol. $I=\int_0^{\pi}\frac{(x+3)\sin x}{1+3\cos^2 x}dx$

$=\int_0^{\pi/2}\left(\frac{(x+3)\sin x}{1+3\cos^2 x}+\frac{(\pi-x+3)\sin(\pi-x)}{1+3\cos^2(\pi-x)}\right)dx$

$=\int_0^{\pi/2}\frac{(\pi+6)\sin x}{1+3\cos^2 x}dx$

$=(\pi+6)\int_0^{\pi/2}\frac{\sin x}{1+3\cos^2 x}dx$

Let $\cos x = t \Rightarrow \sin x\, dx = -dt$

$=(\pi+6)\int_1^{0}\frac{-dt}{1+3t^2}dx$

$=\frac{\pi+6}{3}\int_0^{1}\frac{dt}{\left(\frac{1}{\sqrt{3}}\right)^2+t^2}dx$

$=\frac{\pi+6}{3}\cdot\sqrt{3}\left[\tan^{-1}\sqrt{3}t\right]_0^1=\frac{\pi+6}{\sqrt{3}}\cdot\frac{\pi}{3}=\frac{\pi(\pi+6)}{3\sqrt{3}}$

6. If α and β are negative real roots of the quadratic equation $x^2-(p+2)x+(2p+9)=0$ and $p\in(\alpha,\beta)$. Then the value of $\beta^2-2\alpha$ is

(1) 11

(2) 13

(3) 7

(4) 5

Answer (2)

Sol. $\alpha\beta>0$

$\Rightarrow 2p+9>0$

$\Rightarrow p>\frac{-9}{2}$

and $\alpha+\beta<0$

$p+2<0$

$\Rightarrow p<-2$

$\Rightarrow p\in\left(\frac{-9}{2},-2\right)$

$\beta^2-2\alpha=4-2\left(\frac{-9}{2}\right)$

$=4+9$

$=13$

7. Let the straight line $AB: x+y-2=0$, $AC: 3y-x=2$ intersects x-axis at B and C respectively. If P is the orthocentre of the triangle ABC, then area of the triangle CPB is

(1) 10 sq. units

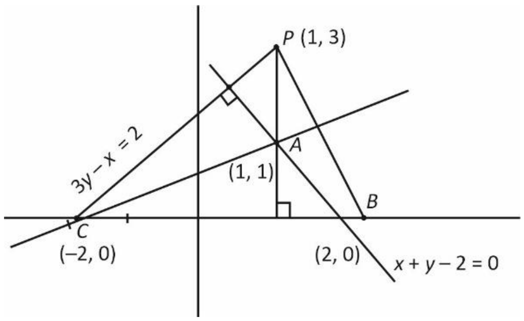
(2) 8 sq. units

(3) 6 sq. units

(4) 7 sq. units

Answer (3)

Sol.



$X = 1$ and $x - y + 2 = 0$

$\Rightarrow$ Area of Δ PCB = 6 sq. units

8. If $\vec{a}$ and $\vec{b}$ are unit vectors such that angle between $\vec{a}$ and $\vec{b}$ is $\sin^{-1}\left(\frac{\sqrt{65}}{9}\right)$ and $\vec{c} = 3\vec{a} + 4\vec{b} + 9(\vec{a}\times\vec{b})$. Then, the value of $\vec{c}\cdot\vec{a} - 3\vec{c}\cdot\vec{b}$ is

(1) $\frac{101}{3}$ (2) $\frac{-101}{9}$

(3) $\frac{101}{9}$ (4) $\frac{-101}{3}$

Answer (2)

Sol. $\vec{c} = 3\vec{a} + 4\vec{b} + 9(\vec{a}\times\vec{b})$

angle between $\vec{a}$ and $\vec{b}$ is $\sin^{-1}\left(\frac{\sqrt{65}}{9}\right)$

$\Rightarrow \sin\theta = \frac{\sqrt{65}}{9}$

$\Rightarrow \cos\theta = \frac{4}{9}$

Also, $\vec{c}\cdot\vec{a} = 3|\vec{a}| + 4\vec{a}\cdot\vec{b}$

$= 3 + 4\times\frac{4}{9}$

$= 3 + \frac{16}{9}$

$= \frac{43}{9}$

$\vec{c}\cdot\vec{b} = 3\vec{a}\cdot\vec{b} + 4|\vec{6}|^2$

$= 3\times\frac{4}{9} + 4 = \frac{16}{3}$

$\vec{c}\cdot\vec{a} - 3\vec{c}\cdot\vec{b} = \frac{43}{9} - 3\left(\frac{16}{3}\right)$

$= \frac{-101}{9}$

9. Consider two statements:

Statement 1: $\left(\frac{z+i}{z-i}\right)$ is purely real and $|z| = 1$, then there are exactly 2 complex numbers z.

Statement 2: $\left(\frac{z+1}{z-1}\right)$ is purely imaginary, then there are infinite such complex numbers z.

Then

(1) Statement 1 is true

(2) Statement 2 is true

(3) Both statement 1 and statement 2 are true

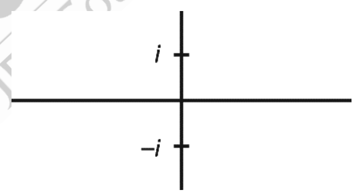
(4) Both statement 1 and statement 2 are false

Answer (3)

Sol. Statement 1: $\left(\frac{-i-z}{i-z}\right)$

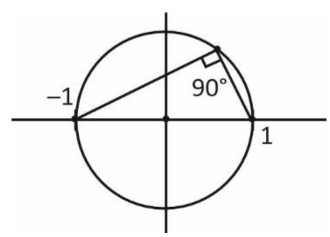
Using rotation

$\arg\left(\frac{-i-z}{i-z}\right)$ is 0 or π



$\Rightarrow$ z lies on the same line as $\pm i$.

Statement 2: $\left(\frac{-1-z}{1-z}\right)$ is imaginary



⇒ (z) lies on the circle

⇒ again infinite such complex numbers.

$z = \cos\theta + i\sin\theta$, satisfies.

10. The value of the limit

$$\lim_{x\to 0^+}\frac{(\tan^{-1}5x^{1/3})^2\cdot\log_e(1+3x^2)\cdot(e^{5x^{4/3}}-1)}{(\sin^{-1}(3\sqrt{x}))^8}$$ is

(1) $\frac{5^3}{3^7}$ (2) $\frac{5^3}{3^6}$

(3) $\frac{5^2}{3^7}$ (4) $\frac{5^2}{3^6}$

Answer (1)

Sol. $$\lim_{x\to 0^+}\left(\frac{\tan^{-1}5x^{1/3}}{5x^{1/3}}\right)^2\cdot(25x^{2/3})\cdot\frac{\log(1+3x^2)}{(3x^2)}\cdot(3x^2)\cdot\frac{(e^{5x^{4/3}}-1)}{(5\cdot x^{4/3})}$$

$$(5x^{4/3})\left(\frac{(3\sqrt{x})}{\sin^{-1}3\sqrt{x}}\right)^8\frac{1}{(3\sqrt{x})^8}$$

$$\lim_{x\to 0^+}(1)(25x^{2/3})(1)\cdot(3x^2)\cdot(1)\cdot 5x^{4/3}\cdot\frac{1}{3^8\cdot x^4}$$

$$\lim_{x\to 0^+}\left(\frac{25.3\times 5}{3^8}\right)=\frac{5^3}{3^7}$$

11. Line L passes through (1, 1, 1) and Line L intersects L_1 and L_2 where

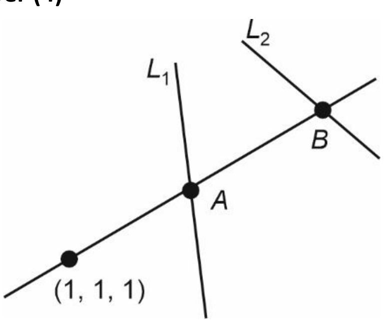
$L_1:\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{4}$ and $L_2:\frac{x-1}{1}=\frac{y+3}{4}=\frac{z}{1}$, then L passes through

(1) (0, 0, 0) (2) (1, 2, 3)

(3) (–1, 3, 4) (4) (9, 15, 18)

Answer (4)

Sol.



$L:\frac{x-1}{a}=\frac{y-1}{b}=\frac{z-1}{c}$

As L passes through (1, 1, 1)

$L_1:\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{4}=\lambda$ (say)

Any point on L_1 be $A(2\lambda+1, 3\lambda-1, 4\lambda)$

$L_2:\frac{x-1}{1}=\frac{y-3}{4}=\frac{z}{1}=\mu$ (say)

Any point on L_2 be $B(\mu+1, 4\mu+3, \mu)$

Dr of L be $\langle 2\lambda, 3\lambda-2, 4\lambda-1\rangle$ or $\langle \mu, 4\mu+2, \mu-1\rangle$

Now,

$$\underbrace{\frac{2\lambda}{\mu}=\frac{3\lambda-2}{4\mu+2}=\frac{4\lambda-1}{\mu-1}}$$

⇒ $\lambda = -4$

$\mu=\frac{-8}{9}$

⇒ Dr of L: be $\langle -8, -14, -17\rangle$

$\therefore\ L:\frac{X-1}{8}=\frac{Y-1}{14},\frac{Z-1}{17}$

12. If $x(x^2+e^x)dy+(e^x(x-2)y-x^3)dx=0$. Such that $f(1)=1$. Then which of the following is correct.

(1) $y=\frac{(x-e)x^2}{x^2-e^x}$ (2) $y=\frac{(ex+1)x^3}{e^x+x^2}$

(3) $y=\frac{(x+e)x^2}{e^x+x^2}$ (4) $y=\frac{(3x+e)x^2}{e^x+x^3}$

Answer (3)

Sol. $x\left(x^2+e^x\right)dy+\left(e^x(x-2)y-x^3\right)dx=0$

$$\frac{dy}{dx}+\frac{e^x(x-2)}{x(x^2+e^x)}y=\frac{x^3}{x(x^2+e^x)}$$

$$\text{If}=e^{\int\frac{e^x(x-2)}{x(x^2+e^x)}dx}$$

$$=e\int\frac{e^x+2x^2-2e^x-2x^2}{x(x^2+e^x)}dx$$

$$= e^{\int \frac{e^x+2x}{e^x+x^2}dx - \int \frac{2}{x}dx}$$

$$= e^{\ln|e^x+x^2| - 2\ln x} = \frac{e^x+x^2}{x^2}$$

$$\therefore y\left(\frac{e^x+x^2}{x^2}\right) = \int \frac{x^3}{x(x^2+e^x)} \times \frac{\left(e^x+x^2\right)}{x^2}dx$$

$$y\left(\frac{e^x+x^2}{x^2}\right) = x + c$$

$\because$ $y(1) = 1$

$$\Rightarrow \left(\frac{e+1}{1}\right) = 1 + C$$

$\Rightarrow c = e$

$$\therefore y = \frac{(x+e)x^2}{e^x+x^2}$$

13. Consider a squad of 15 players consisting of 7 batsman, 6 bowler, 1 captain and 1 vice captain. *A* team of 10 players to be selected such that team has at least 4 batsman and 4 bowler and out of captain and vice captain atleast 1 must be present in the team. Then number of ways to select such team is

(1) 1475 (2) 1575

(3) 1075 (4) 1500

Answer (2)

Sol.

Batsman	Bowler	Captain	Vice	Number of ways
4	4	1	1	${}^7C_4 \cdot {}^6C_4\ 1\times1$
4	5	1	0	${}^7C_4 \cdot {}^6C_4 \cdot {}^1C_1$
4	5	0	1	${}^7C_4 \cdot {}^6C_5 \cdot {}^1C_1$
5	4	1	0	${}^7C_5 \cdot {}^6C_4 \cdot {}^1C_1$
5	4	0	1	${}^7C_5 \cdot {}^6C_4 \cdot {}^1C_1$

$\Rightarrow$ ${}^7C_4 \cdot {}^6C_4 [1] + \cdot {}^7C_4 [2] + \cdot {}^7C_5 \cdot {}^6C_4 [2]$

= 35.15 + 35 × 12 + 21.15 × 2

= 525 + 420 + 630 = 1575

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let *A* be a set defined as *A* = {2, 3, 6, 9}. Find the number of singular matrices of order 2 × 2 such that elements are from the set *A*.

Answer (36)

Sol. $\begin{vmatrix} a & d \\ b & c \end{vmatrix} = ab - bc = 0 \quad \Rightarrow \quad ad = bc$

Case I: exactly 1 number is used

$\Rightarrow$ All singular $\Rightarrow$ ${}^4C_1$

Case II: exactly 2 number is used

${}^4C_2 \cdot 2 \times 2$

Case III: exactly 3 number is used

None will be singular

Case IV: exactly 4 number is used

$ab = cd$ $\Rightarrow$ 2 × 9 = 3 × 6

$\begin{bmatrix} 9 & \square \\ \square & 2 \end{bmatrix}$ ${}^4C_1 \times 2!$ $\Rightarrow$ 8 matrices

$\Rightarrow$ 4 + 6 × 4 + 0 + 8 = 36 matrices

22.

23.

24.

25.

❑ ❑ ❑